Tamsui Oxford Journal of Informational and Mathematical Sciences 32(1) (2018) 11-20
Aletheia University

Ricci solitons in N(K)-manifold

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Received April, 11, 2016, Accepted January, 31, 2018

Mathematics Subject Classi cation 2000, 53C15, 53C25.
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Abstract

Recently, the present authors have introduced the notion of *generalized quasi-conformal curvature tensor which bridges Conformal curvature tensor, Concircular curvature tensor, Projective curvature tensor and Conharmonic curvature tensor. The object of the present paper is to find out curvature conditions for Ricci solitons to be shrinking or steady or expanding.*

1 Introduction

Recently, in tune with Yano and Sawaki [24], the first two authors [19] have introduced and studied *generalized quasi-conformal curvature tensor* $\mathcal{W}$ *in the context of* $N(k,\mu)$-manifold. The components of *quasi-conformal like curvature tensor* $\mathcal{W}$ in a Riemannian manifold $(M^{2n+1}, g)(n > 1)$, are given by

$$\begin{aligned}\mathcal{W}(X,Y)Z \quad = \quad & \frac{2n-1}{2n+1}\Big[(1+2na-b)-\{1+2n(a+b)\}c\Big]C(X,Y)Z \\ & +\Big[1-b+2na\Big]E(X,Y)Z+2n(b-a)P(X,Y)Z \\ & +\frac{2n-1}{2n+1}(c-1)\{1+2n(a+b)\}\hat{C}(X,Y)Z \qquad (1.1)\end{aligned}$$

for all X, Y & $Z \in \chi(M)$, the set of all vector field of the manifold M, where scalar triple $(a,\ b,\ c)$ are real constants. The beauty of such *curvature tensor* lies in the fact that it has the flavour of Riemann curvature tensor R if the scalar triple $(a,\ b,\ c) \equiv (0,0,0)$, Conformal curvature tensor C [13] if $(a,\ b,\ c) \equiv (-\frac{1}{2n-1}, -\frac{1}{2n-1}, 1)$, Conharmonic curvature tensor $\hat{C}$ [16] if $(a,\ b,\ c) \equiv (-\frac{1}{2n-1}, -\frac{1}{2n-1}, 0)$, Concircular curvature tensor E ([2], p. 84) if $(a,\ b,\ c) \equiv (0,0,1)$, Projective curvature tensor P([2], p. 84) if $(a,\ b,\ c) \equiv (-\frac{1}{4n}, 0, 0)$ and m-Projective curvature tensor H [20], fi $(a,\ b,\ c) \equiv (-\frac{1}{4n}, -\frac{1}{4n}, 0)$. The equation

Key Words: $N(k)$-contact metric manifold; *quasi-conformal like curvature tensor*; Ricci solitons Shrinking, Steady and Expanding.

(1.1) can also be written as

$$\begin{aligned}\mathcal{W}(X,Y)Z &= R(X,Y)Z + a\Big[S(Y,Z)X - S(X,Z)Y\Big] \\ &+b\Big[g(Y,Z)QX - g(X,Z)QY\Big] \\ &-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\Big]g(Y,Z)X - g(X,Z)Y\Big]. \quad (1.2)\end{aligned}$$

R, S, Q & r being Christoffel Riemannian curvature tensor, Ricci tensor, Ricci operator and scalar curvature respectively.

The study of the Ricci solitons in contact geometry has begun with the work of Ramesh Sharma ([22], [14]). Ricci solitons in contact metric manifolds has also been extensively studied by Mukut Mani Tripathi [23], Cornelia Livia Bejan and Mircea Crasmareanu ([7], [6]) and the references therein. Ricci solitons are introduced as triples (M,g,V), where (M,g) is a Riemannian manifold and V is a vector field so that the following equation is satisfied:

$$\frac{1}{2}\pounds_V g + S + \lambda g = 0 \quad (1.3)$$

where $\pounds$ denotes the Lie derivative, S is the Ricci tensor and λ is real constant on *ik*. A Ricci soliton is said to be shrinking, steady or expanding according to λ negative, zero, and positive respectively. During the last two decades, the geometry of Ricci solitons has been the focus of attention of many mathematicians [5]. It has become more important after Grigory Perelman applied Ricci solitons to solve the long standing Poincaré conjecture posed in 1904.

Our work is structured as follows. Section 2 is a very brief review of $N(k)$-manifolds. In section 3, we investigate Ricci solitons in a $N(k)$-manifold admitting $\omega(\xi,X)\cdot\mathcal{W}=0$ where ω and $\mathcal{W}$ stand for *quasi conformal like curvature tensor* with the associated scalar triples $(\bar{a},\bar{b},\bar{c})$ and (a,b,c) respectively, the dot means that $\omega(X,Y)$ acts as a derivation on $\mathcal{W}$. Based on this result, we proved by *taking into account, the permutation of different curvature tensors* that the Ricci solitons is expanding or shrinking. In section 4, we determine that Ricci solitons in $N(k)$-manifold satisfying $\mathcal{W}(\xi,X)\cdot S=0$ is shrinking.

2 Preliminaries

In this section, we recall some basic results which we will used later. A $(2n+1)$-dimensional differential manifold $M^{2n+1}$is called a contact manifold if it carries a global differentiable1-form η such that $\eta\Lambda(d\eta)^n \neq 0$ everywhere on M^{2n+1}. This 1-form η is called the contact form on M^{2n+1}. A Riemannian metric g is said to be associated with a contact manifold if there exist a $(1,1)$ tensor field ϕ and a contravariant global vector field ξ, called the characteristic vector field of the manifold such that

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi \cdot \xi = 0, \quad \eta \cdot \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(X, \phi Y) = -g(Y, \phi X), \quad g(X, \xi) = \eta(X), \quad g(X, \phi Y) = d\eta(X, Y), \tag{2.3}$$

for all vector fields X, Y on M. Also,

$$\nabla_X \xi = -\phi X, \tag{2.4}$$

holds in a contact metric manifold.

The k-nullity distribution of a Riemannian manifold (M, g) for a real number k is a distribution

$$N(k) : p \to N_p(k) = \{Z \in T_pM; R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y], \tag{2.5}$$

for any X, $Y \in T_pM$. Hence, if the characteristic vector field ξ of a contact metric manifold belongs to the k-nullity distribution, then we have

$$R(X, Y\xi = k[\eta(Y)X - \eta(X)Y]. \tag{2.6}$$

Thus a contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ satisfying the relation (2.5) is called a $N(k)$-contact metric manifold.

Also, in a $N(k)$-contact metric manifold, the following relations hold:

$$S(X, \xi) = 2nk\eta(X), \tag{2.7}$$

$$S(\phi X, \phi Y) = S(X, Y) - 2nk\eta(X)\eta(Y), \tag{2.8}$$

$$R(\xi, X)Y = k[g(X, Y)\xi - \eta(Y)X], \tag{2.9}$$

$$\eta\,(R(X, Y)Z) = k[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)], \tag{2.10}$$

for any vector field X, Y on M^{2n+1}.

Let(g, V, λ) be a Ricci soliton in an $(2n + 1)$-dimensional $N(k)$-manifold M. From (2.6), we have

$$(\pounds_\xi g)(X, Y) = 0 \tag{2.11}$$

From (1.1) and (2.11), we find that

$$S(X, Y) = -\lambda g(X, Y). \tag{2.12}$$

Thus, for $N(k)$-manifold with Ricci soliton the quasi-conformal like curvature tensor $\mathcal{W}$ takes the form

$$\mathcal{W}(X, Y)Z = R(X, Y)Z - \Big[\lambda(a + b) + \frac{cr}{2n + 1}\Big(\frac{1}{2n} + a + b\Big)\Big] \Big[g(Y, Z)X - g(X, Z)Y\Big]. \tag{2.13}$$

where Q and r are respectively the Ricci operator and scalar curvature on M.

3 Ricci solitons in $N(k)$-manifold satisfying $\omega(X,Y)\cdot\mathcal{W} = 0$

Let us consider a $(2n+1)$-dimensional $N(k)$-contact manifold M, satisfying the condition

$$(\omega(X,Y)\cdot\mathcal{W})(Z,U)V = 0, \tag{3.1}$$

for any vector fields X, Y on the manifold and $\omega(X,Y)$ acts on $\mathcal{W}$ as derivation, where ω and $\mathcal{W}$ stand for *quasiconformal like curvature tensor* with the associated scalar triples $(\bar{a},\bar{b},\bar{c})$ and (a,b,c) respectively. Which is equivalent to

$$\begin{aligned} &g(\omega(\xi,X)\mathcal{W}(Y,Z)U,\xi) - g(\mathcal{W}(\omega(\xi,X)Y,Z)U,\xi)\\ &-g(\mathcal{W}(Y,\omega(\xi,X)Z)U,\xi) - g(\mathcal{W}(Y,Z)\omega(\xi,X)U,\xi) = 0. \end{aligned}\tag{3.2}$$

Putting $X = Y = e_i$ in (3.2) where $\{e_1,\ e_2,\ e_3, ..., e_{2n},\ e_{2n+1} = \xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M and taking the summation over i, $1 \le i \le 2n+1$,we get

$$\begin{aligned} &\sum_{i=1}^{2n+1}\Big[g(\omega(\xi,e_i)\mathcal{W}(e_i,Z)U,\xi) - g(\mathcal{W}(\omega(\xi,e_i)e_i,Z)U,\xi)\\ &-g(\mathcal{W}(e_i,\omega(\xi,e_i)Z)U,\xi) - g(\mathcal{W}(e_i,Z)\omega(\xi,e_i)U,\xi)\Big] = 0. \end{aligned}\tag{3.3}$$

From the equation (2.13), we can easily bring out the followings

$$\begin{aligned} &\eta(\mathcal{W}(\xi,U)Z)\\ =\ &\left[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\Big[g(Z,U) - \eta(Z)\eta(U)\Big] \end{aligned}\tag{3.4}$$

$$\begin{aligned} &\sum_{i=1}^{2n+1}\bar{\mathcal{W}}(e_i,Z,U,e_i)\\ =\ &\left[-\lambda - 2n\left\{\lambda(a+b) + \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}\right]g(Z,U) \end{aligned}\tag{3.5}$$

$$\begin{aligned} &\text{Now, } \sum_{i=1}^{2n+1} g(\omega(\xi,e_i)\mathcal{W}(e_i,Z)U,\xi)\\ =\ &\left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\left\{\eta(\mathcal{W}(\xi,Z)U) - \sum_{i=1}^{2n+1}\bar{\mathcal{W}}(e_i,Z,U,e_i)\right\} \end{aligned}\tag{3.6}$$

In view of (3.4) & (3.5), (3.6) becomes

$$\begin{aligned} & g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi) \\ = & \left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\times \\ & \left[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\left[g(Z,U) - \eta(Z)\eta(U)\right] \\ & -\left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\times \\ & \left[-\lambda - 2n\left\{\lambda(a+b) + \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}\right]g(Z,U). \end{aligned} \tag{3.7}$$

In consequence of of (3.4)-(3.5), we obtain the followings

$$\begin{aligned} & g(\mathcal{W}(\omega(\xi, X)Y, Z)U, \xi) \\ = & \left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\times \\ & \left[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\times \\ & \left[g(Z,U)g(Y,X) - \eta(Z)\eta(U)g(Y,X) + g(X,U)\eta(Z)\eta(Y) - g(Z,U)\eta(X)\eta(Y)\right] \end{aligned} \tag{3.8}$$

$$\begin{aligned} & \sum_{i=1}^{2n+1} g(\mathcal{W}(\omega(\xi, e_i)e_i, Z)U, \xi) \\ = & 2n\left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\times \\ & \left[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\left[g(Z,U) - \eta(Z)\eta(U)\right]. \end{aligned} \tag{3.9}$$

$$\begin{aligned} & g(\mathcal{W}(Y, \omega(\xi, X)Z)U, \xi) \\ = & \left[k - \lambda(\bar{a}+\bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\times \\ & \left[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\times \\ & \Big[- g(Y,U)g(X,Z) + \eta(Y)\eta(U)g(X,Z) - g(X,U)\eta(Y)\eta(Z) \\ & + g(Y,U)\eta(Z)\eta(X)\Big] \end{aligned} \tag{3.10}$$

$$
\begin{aligned}
& g(\mathcal{W}(e_i, \omega(\xi, e_i)Z)U, \xi) \\
= \ & -\Big[k - \lambda(\bar{a} + \bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\Big] \times \\
& \Big[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\Big] \times \\
& \Big[g(Z,U) - \eta(Z)\eta(U)\Big] \qquad (3.11)
\end{aligned}
$$

$$
\begin{aligned}
& g(\mathcal{W}(Y, Z)\omega(\xi, X)U, \xi) \\
= \ & \Big[k - \lambda(\bar{a} + \bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\Big] \times \\
& \Big[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\Big] \times \\
& \Big[g(Y,X)\eta(Z)\eta(U) - g(X,Z)\eta(Y)\eta(U)\Big]. \qquad (3.12)
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi) \\
= \ & 2n\Big[k - \lambda(\bar{a} + \bar{b}) - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\Big] \times \\
& \Big[k - \lambda(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\Big]\eta(Z)\eta(U). \qquad (3.13)
\end{aligned}
$$

By virtue of (3.7), (3.9), (3.11) & (3.13), the equation (3.3) yields for $Z = U = \xi$ that

$$
\begin{aligned}
& \lambda[1 + 4n(1-c)(a+b) - 2c] = 2nk, \\
\text{or,} \ \ k \ = \ & \lambda[(\bar{c} - 1)(\bar{a} + \bar{b}) - \frac{\bar{c}}{2n}]. \qquad (3.14)
\end{aligned}
$$

By taking permutation and combination of different values of $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) (like 0, $-\frac{1}{2n-1}$, $-\frac{1}{2n}$, $-\frac{1}{4n}$ etc.,) one will get 36 curvature conditions and we find that Ricci soliton (M, g, ξ) in a $N(k)$-manifold for each curvature restriction is either expanding or shrinking.

Theorem 3.1 *Ricci soliton (M, g, ξ) in a $N(k)$-manifold satisfying $\omega(\xi, X) \cdot \mathcal{W} = 0$ is either expanding or shrinking.*

4 $N(k)$-contact manifolds with $\mathcal{W}\cdot S=0$

Let $M^{2n+1}(\phi,\xi,\eta,g)(n>1)$, be a $N(k)$-contact metric manifold, satisfying the condition

$$\mathcal{W}(\xi,X)\cdot S=0. \tag{4.1}$$

$$\begin{aligned} i.e.\ \mathcal{W}(\xi,X)S(Y,Z)-S(\mathcal{W}(\xi,X)Y,Z)-S(Y,\mathcal{W}(\xi,X)Z) &= 0.\\ i.e.\ S(\mathcal{W}(\xi,X)Y,Z)+S(Y,\mathcal{W}(\xi,X)Z) &= 0. \end{aligned} \tag{4.2}$$

Taking $Z=\xi$ in (4.2) and using (2.7), we get

$$2nk\eta(\mathcal{W}(\xi,X)Y)+S(Y,\mathcal{W}(\xi,X)\xi)=0. \tag{4.3}$$

In view of (2.9) and (2.13), we have

$$\begin{aligned} &\eta(\mathcal{W}(\xi,X)Y)\\ =\ &\left[k-\lambda(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\Big[g(X,Y)-\eta(X)\eta(Y)\Big] \end{aligned} \tag{4.4}$$

Using (4.4) in (4.3), we have

$$\left[k-\lambda(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\Big[2nkg(X,Y)-S(X,Y)\Big]=0. \tag{4.5}$$

Putting $X=Y=e_i$ in (4.5) where $\{e_1,\ e_2,\ e_3,...,e_{2n},\ e_{2n+1}=\xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M and taking the summation over i, $1\le i\le 2n+1$,we get

$$k=\lambda\left[(1-c)(a+b)-\frac{c}{2n}\right]\ \text{or}\ \Big[\lambda=-2nk. \tag{4.6}$$

Curvature condition	Value of λ
$R(\xi,X)\cdot S=0$ (Obtain by $a=b=c=0$)	$\lambda=-2nk,$
$E(\xi,X)\cdot S=0$ (Obtain by $a=b=0,\ c=1$)	$\lambda=-2nk,$
$C(\xi,X)\cdot S=0$ (Obtain by $a=b=-\frac{1}{2n-1},c=1$)	$\lambda=-2nk,$
$\hat{C}(\xi,X)\cdot S=0$ (Obtain by $a=b=-\frac{1}{2n-1},c=0$)	$\lambda=-2nk,-\frac{2n-1}{2}$
$P(\xi,X)\cdot S=0$ (Obtain by $a=-\frac{1}{2n},b=c=0$)	$\lambda=-2nk,$
$H(\xi,X)\cdot S=0$ (Obtain by $a=b=-\frac{1}{4n},\ c=0$)	$\lambda=-2nk.$

From the above table, we can state the following-

Theorem 4.1 *Ricci soliton (M,g,ξ) in a $N(k)$-manifold admitting $\mathcal{W}\cdot S=0$ is shrinking.*

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