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Inventory Model with Price Related Demand, Weibull Distributed Deterioration & Uncertainty

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Abstract:

This article deals with an inventory model for deteriorating items where the rate of deterioration follows the two parameters Weibull distribution. Demand rate depends on the price. The shortages are allowed and are fully backlogged. An uncertain cycle time is considered and described by a (symmetric) triangular fuzzy number. The model is developed analytically by maximizing the total profit. The optimum results are defuzzified by a signed distance method. The numerical examples are given to illustrate the developed model and all results are illustrated numerically and graphically.

Keywords:

Inventory, price dependent demand, weibull distribution, deterioration, triangular fuzzy number.

1.1 Introduction:

To adapt more realistic features in the literature of inventory theory researchers have been continuously improving inventory models. Fuzzy set theory in fuzzy decision making processes was first introduced by Bellman and Zadeh (1970). Zadeh (1965, 1973) showed that for new products and seasonal items it is better to use fuzzy numbers rather than probabilistic approaches. Dubois and Prade (1978) described some operations on fuzzy numbers. Kacpryzk and Staniewski (1982) introduced a fuzzy decision-making inventory model for long-term inventory policy. Zimmerman (1983) also used fuzzy sets in their research. Park (1987) discussed the fuzzy set theoretical interpretation of an EOQ problem. Vujosevic, et.al (1996) developed an EOQ formula by introducing inventory cost as a fuzzy number. Yao and Lee (1999) studied a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number. Kao and Hsu (2002) discussed a single period inventory model with fuzzy demand. Hsieh (2002) studied an inventory model by introducing optimization approach of fuzzy production. Yao and Chiang (2003) introduced an inventory model without backorders and defuzzified the fuzzy holding cost by introducing signed distance and centroid methods. Kumar, et. al (2003) studied an economic production quantity model with fuzzy demand and deterioration rate. Syed and Aziz (2007) considered the signed distance method to introduce a fuzzy inventory model without shortages. De and Sana (2013) developed a backorder *Economic Order Quantity* model with promotional index for fuzzy decision variables. Pal, et. al (2015) considered a production inventory model for deteriorating items with ramp type demand rate where deterioration rate was represented by a two-parameter Weibull distribution and solved under fuzzy environment to evaluate the optimum solution of the model. Roy, (2015) developed a fuzzy inventory model for deteriorating items with price dependent demand.

Selling price plays an essential role in inventory systems. Burwell, Dave, Fitzpatrick, and Roy (1997) studied an economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system for price dependent demand rate was developed by Mondal, Bhunia, and Maiti (2003). You (2005) introduced an inventory model with time and price dependent demand. Begum, Sahoo, and Sahu (2012) studied a pricing and replenishment policy for an inventory system with price-sensitive demand rate and time-proportional deterioration rate. Shah, Soni and Gupta (2014) revised Begum et. al’s (2012) model and showed a complete model formulation for Begum et al. (2012) with price-sensitive demand rate. Roy, (2015) developed a fuzzy inventory model for deteriorating items with price dependent demand where deterioration rate and holding cost were linearly increasing functions of time. To the best of the author's knowledge, no one has developed any inventory model considering fuzzy cycle time with selling price dependent demand rate where deterioration rate follows weibull distribution.

In this article, we focus on the implementation of fuzzy set theory in the inventory control of deteriorating items where deterioration rate follows weibull distribution and the demand rate is a function of selling price. During time t_1, inventory is depleted due to demand and deterioration of the item. At time t_1 the inventory becomes zero and shortages start occurring. To capture the real life situation, we consider that the cycle time is uncertain and that it is possible to describe it by a (symmetric) triangular fuzzy number. Below is the list of assumptions we have considered in order to develop the model.

1.1.1 Assumptions and Notations:

The fundamental assumptions of the model are as follows.

(a) The demand rate is a function of selling price.

b) Shortages are allowed and are fully backlogged.

(c) The deterioration rate follows weibull distribution.

(d) Holding cost h per item per time-unit.

(e) Replenishment is instantaneous and lead time is zero.

(f) T is the length of the cycle.

(g) A is the cost of placing an order.

(h) The selling price per unit item is p.

(i) C is the unit cost of an item.

(j) C_1 is the shortage cost per unit per unit time.

(k) $\theta(t) = \alpha\beta(t)^{\beta-1}$, $0 \le \alpha < 1$, $\beta > 0$, is the deterioration rate.

1.1.2 Model Formulation:

The length of the cycle is T. during time t_1 inventory is depleted due to deterioration and demand of the items. At the time t_1 the inventory level becomes zero and shortages occur in the period (t_1, T), which is completely backlogged. Let I(t) be the inventory level at time t ($0 \le t < T$). The differential equation can be defined when the instantaneous state over (0, T) are given by

$$dI(t)/_{dt} + \alpha\beta t^{(\beta-1)} I(t) = -(a-p), 0 \le t \le t_1 \quad (1)$$

$$dI(t)/_{dt} = -(a-p), \qquad t_1 \le t \le T \quad (2)$$

With $I(t_1) = 0 \; at \; t = t_1$

The total crisp profit P(T) per unit time is

$$p(a-p) - \frac{1}{T}[Ordering\ cost + Purchase\ cost + Shortage\ cost + Holding\ cost]$$

$$= p(a-p) - \frac{1}{T}[A + c(a-p)\left(\frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha^2 t_1^{\beta+1}}{2(2\beta+1)} + \frac{1}{2}C_1(a-p)(T-t_1)^2 + T\right) - h(a-p)\left(\frac{\alpha}{\beta+1}\left(\frac{t_1^{\beta+2}}{\beta+2} - t_1^{\beta+2}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(\frac{t_1^{2\beta+2}}{2(\beta+1)} - t_1^{2\beta+2}\right)\right)]$$

Let $t_1 = \mu T$, $0 < \mu < 1$

$$= p(a-p) - \frac{1}{T}[A + c(a-p)\left(\frac{\alpha(\mu T)^{\beta+1}}{\beta+1} + \frac{\alpha^2(\mu T)^{\beta+1}}{2(2\beta+1)} + T\right) + \frac{1}{2}C_1(a-p)(T-\mu T)^2 - h(a-p)\left(\frac{\alpha}{\beta+1}\left(\frac{(\mu T)^{\beta+2}}{\beta+2} - (\mu T)^{\beta+2}\right) + \frac{\alpha^2}{2(2\beta+1)}\left(\frac{(\mu T)^{2\beta+2}}{2(\beta+1)} - (\mu T)^{2\beta+2}\right)\right)] \quad (3)$$

1.1.3 Fuzzy Continuous Review Inventory Model:

Let us consider that the cycle time is uncertain and it is possible to describe it with triangular fuzzy number (symmetric). Then the cycle time is = $\tilde{T}$ = $(T-\Delta, \Delta, T+\Delta)$

So from (3) the profit function P(T) with fuzzy cycle time is:

$$p(a-p)-\frac{1}{\tilde{T}}[A+c(a-p)\left(\frac{\alpha(\mu\tilde{T})^{\beta+1}}{\beta+1}+\frac{\alpha^2(\mu\tilde{T})^{\beta+1}}{2(2\beta+1)}+\frac{1}{2}C_1(a-p)\left(\tilde{T}-\mu\tilde{T}\right)^2+\tilde{T}\right)$$
$$-h(a-p)\left(\frac{\alpha}{\beta+1}\left(\frac{(\mu\tilde{T})^{\beta+2}}{\beta+2}-(\mu\tilde{T})^{\beta+2}\right)\right.$$
$$\left.+\frac{\alpha^2}{2(2\beta+1)}\left(\frac{(\mu\tilde{T})^{2\beta+2}}{2(\beta+1)}-(\mu\tilde{T})^{2\beta+2}\right)\right)]$$

To defuzzify the cost function we will introduce the signed distance. We know for any a and 0$\in$ R, the signed distance from a to 0 is $d_0(a,0)=a$. If a$\langle$ 0, the distance from a to 0 is –a = $-d_0(a,0)$.

Let Ψ be the family of all fuzzy sets $\tilde{B}$ defined on R for which the α -cut B(α) $=[B_L(\alpha), B_U(\alpha)]$ exists for every $\alpha\in[0, 1]$. Both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on $\alpha\in[0, 1]$. Then we can say for any $\tilde{B}\in\Psi$ we have $\tilde{B}=\bigcup_{0\le\alpha\le1}[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$.

So for $\tilde{B}\in\Psi$ we can define the signed distance of $\tilde{B}$ to $\tilde{0}$ (y axis) as

$$d(\tilde{B},\tilde{0}_1)=\frac{1}{2}\int_0^1[B_L(\alpha)+B_U(\alpha)]d\alpha$$

For the triangular fuzzy number $\tilde{A}$=(a, b, c), the α -cut of $\tilde{A}$ is A(α)=[$A_L(\alpha), A_U(\alpha)$], for $\alpha\in[0,1]$, where $A_L(\alpha)=a+(b-c)\alpha$ and $A_U(\alpha)=c-(c-b)\alpha$, the signed distance of $\tilde{A}$ to $\tilde{0}$ (y axis) is

$$d(\tilde{A},\tilde{0}_1)=\frac{1}{4}(a+2b+c)$$

The signed distance of P(T) and 0 is $d(\tilde{P},\tilde{0})=$

$$p(a-p)-\ d(\tfrac{1}{\tilde{T}},0)[A+c(a-p)(\tfrac{\alpha\left(\mu d(\tilde{T},0)\right)^{\beta+1}}{\beta+1}+\tfrac{\alpha^2\left(\mu d(\tilde{T},0)\right)^{\beta+1}}{2(2\beta+1)}+\tfrac{1}{2}C_1(a-p)\left(d(\tilde{T},0)-\mu d(\tilde{T},0)\right)^2+d(\tilde{T},0))-h(a-p)\left(\tfrac{\alpha}{\beta+1}\left(\tfrac{(\mu d(\tilde{T},0))^{\beta+2}}{\beta+2}-(\mu d(\tilde{T},0))^{\beta+2}\right)+\tfrac{\alpha^2}{2(2\beta+1)}\left(\tfrac{(\mu d(\tilde{T},0))^{2\beta+2}}{2(\beta+1)}-(\mu d(\tilde{T},0))^{2\beta+2}\right)\right)] \tag{4}$$

From the definition of signed distance we can write $\mathrm{d}(\tilde{T},\tilde{0})=T$ and as $\tilde{T}\in\Psi$, where Ψ be the family of all fuzzy sets $\tilde{T}$ defined on R for which the α-cut T(α)=[$T_L(\alpha),T_U(\alpha)$] exists for every $\alpha\in[0,1]$ and both $T_L(\alpha),T_U(\alpha)$ are continuous functions on $\alpha\in[0,1]$. Then for $\tilde{T}\in\Psi$, the signed distance is

$$d(1/\tilde{T},\tilde{0})=\frac{1}{2}\int_0^1[(1/T)_L(\alpha)+(1/T)_U(\alpha)]d\alpha$$

$$=\frac{1}{2}\int_0^1[\frac{1}{T+\Delta-\Delta\alpha}+\frac{1}{T-\Delta+\Delta\alpha}]d\alpha\ =\frac{1}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta}) \tag{5}$$

From (3), (4), (5) we can write the defuzzified total profit

P(T) = d($\tilde{P}(\tilde{T}),\tilde{0}$)=

$$(a-p)\left[p-\frac{C_1(1-\mu)^2T}{2}-C\left(\frac{\alpha\mu^{\beta+1}T^{\beta}}{\beta+1}+\frac{\alpha^2\mu^{2\beta+1}T^{2\beta}}{2(2\beta+1)}+1\right)+h\left(\frac{\alpha\mu^{\beta+2}T^{\beta+1}}{\beta+1}\left(\frac{1}{\beta+2}-1\right)+\frac{\alpha^2\mu^{2\beta+2}T^{2\beta+1}}{2(2\beta+1)}\left(\frac{1}{2(\beta+1)}-1\right)\right)\right]-A.\frac{1}{2\Delta}ln\frac{(T+\Delta)}{(T-\Delta)}. \tag{6}$$

Theorem 1.1 : The average system profit function P(T), given by (6), is strictly concave.

Proof. Here

$$\frac{dP(T)}{dT} = (a-p)\left[p - \frac{C_1(1-\mu)^2}{2} - C\left(\frac{\alpha\mu^{\beta+1}\beta T^{\beta-1}}{\beta+1} + \frac{\alpha^2\beta\mu^{2\beta+1}T^{2\beta-1}}{(2\beta+1)}\right)\right.$$
$$\left. + h\left(\alpha\mu^{\beta+2}T^{\beta}\left(\frac{1}{\beta+2}-1\right) + \frac{1}{2}\alpha^2\mu^{2\beta+2}T^{2\beta}\left(\frac{1}{2(\beta+1)}-1\right)\right)\right] - \frac{A}{2\Delta}\left[\frac{1}{(T+\Delta)}\right.$$
$$\left. - \frac{1}{(T-\Delta)}\right]$$

And

$$\frac{d^2P(T)}{dT^2} = -C(a-p)\left(\frac{\alpha\mu^{\beta+1}\beta(\beta-1)T^{\beta-2}}{\beta+1} + \frac{\alpha^2\beta(2\beta-1)\mu^{2\beta+1}T^{2\beta-2}}{(2\beta+1)}\right)$$
$$-h(a-p)\left(\frac{1}{\beta+2}\alpha\mu^{\beta+2}\beta(\beta+1)T^{\beta-1} + \frac{1}{2(\beta+1)}\alpha^2\beta(2\beta+1)\mu^{2\beta+2}T^{2\beta-1}\right)$$
$$-\frac{2AT}{(T^2-A^2)^2} < 0.$$

Hence P(T) is strictly concave.

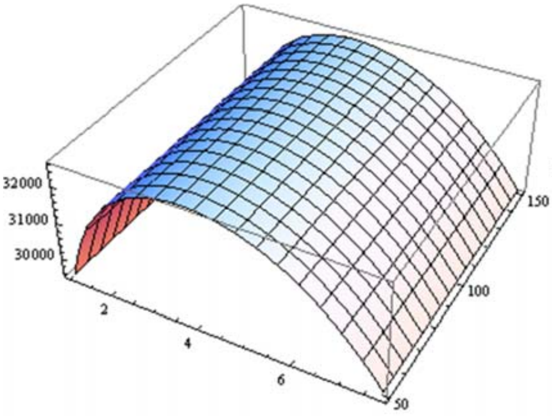
Since P(T) is strictly concave in T, there exists an unique optimal cycle time T^* that maximizes P(T). This optimal cycle time T^* is the solution of the equation dP/dT=0.

1.2 Numerical Solution:

Example 1.1: a = 450, p = 91.59, C_1 = 1, μ = 0.2, C = 1, α = 0.4, β = 2.0, h = 100, A = 4000, Δ = 3.95 in appropriate units. Based on these input data, the computer outputs are as follows.

T* = 4.001, Crisp Profit = 32440.93, Fuzzy Profit = 29884.36.

Figures (I, II, III) are based on the result of Example 1.1, which will give the readers the graphical representation of P(T).



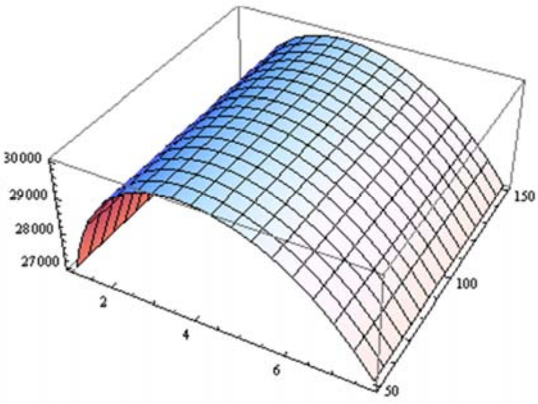


Fig-I: Crisp Profit

Fig-II: Fuzzy Profit

The effects of the changes of the parameters on the optimal profit derived by the proposed method have a key impact on profits. In Example 1 both crisp and fuzzy models are highly sensitive to the changes in the percentage of uncertainty, demand and deterioration. Always success depends on the correctness of the estimation of the input parameters. In real life, these parameters values could change over time due to their complex structures. Therefore, it is more reasonable to assume that these parameters are known only within some given ranges.

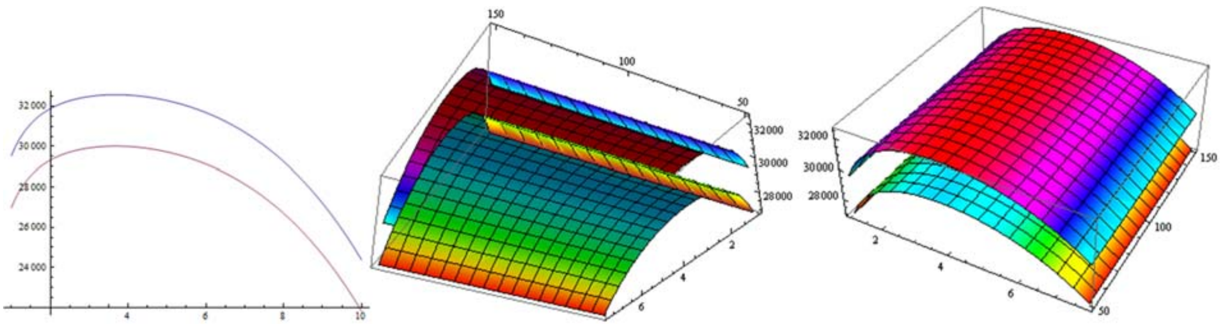


Fig-III: Comparison of Crisp and Fuzzy Profit in 2D and 3D.

1.3 Conclusion:

In this paper we have developed an inventory model for deteriorating items, which follows weibull distribution. To capture the real life situation we have considered uncertain cycle time and we introduced triangular fuzzy number (symmetric) to describe it. The optimum results of the fuzzy model are defuzzified by the signed distance method. In the numerical example, Example 1.1, it is found that the optimum average profit in the crisp model is 8.55% more than that in the fuzzy model. This percentage is highly correlated with the change of uncertainty, as for more uncertainty we achieve less profit in the fuzzy model. Both numerically and graphically we tried to compare the crisp model with fuzzy model and we concluded that if the uncertainties are accounted for in appropriate manner the cycle time would increase. In future different costs such as holding cost and deterioration cost etc. could be replaced by fuzzy numbers to incorporate real life situations.

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