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Entropy based Multi-objective Matrix Game Model with Fuzzy Goals

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Abstract: Game theory is the study of human conflict and cooperation within a competitive situation. Many real conflict problems can be modeled as games. Here we have considered a Multi-Objective Matrix Game model with an additional entropy objective function and fuzzy goals. The said model can be reduced to a generalized regular Max-Min problem and applying fuzzy optimization with entropy disturbance approach for smooth and uniform distribution to solve the said problem. Numerical illustration is included in support of theory.

Keywards: Multi-objective Matrix Game; Entropy; Fuzzy Goal.

1. Introduction

Game theory deals with situation where there are conflicts of interests between persons. This is game of a strategy that is which strategy a person should adopt for a possible move of other persons (Player). Here person may be people, companies etc, the game may be chess game or a game may involve conflict situation in military activities etc. At the end of game there are payoffs to each player. As a collection of mathematical model of conflict and co-operation, game theory was established as a systematic theory by Von Neumann and Morgenstern [20] in 1944. As the name two persons zero sum game, with only two players in which losses of one player is equal to the gain of another so that the sum of their net gain is zero. A mathematical formulation of a game is based upon the 'minimax (maxmin) criteria' of J. Von Neumann. But G.B.Dantzing[7] was first to apply simplex method to solve a game problem. Fuzziness in game problem may occur in goals and payoff. Such types of game were first studied by Campos[4]. Sakawa and Nishizaki[15] , Kumar [12]studied single and multi objective matrix games with fuzzy goals and fuzzy payoff by using max-min principle of game theory. Games with fuzzy payoff introduced by Aubin[1]. Bector et. al. [2] studied fuzzy mathematical programming and fuzzy matrix games. To solve fuzzy multiple attributes decision making approach by Chen and Larbani[6]. Cevikel and Ahlatcioglu[5] used new concept for multiobjective two persons zero sum games with fuzzy goals and payoff using linear membership functions.

Fuzziness, a feature of uncertainty, results from the lack of sharp distinction of the boundary of a set, i.e., an individual is neither definitely a member of the set nor definitely not a member of it. Fuzziness also arises in logic when a proposition can be treated as neither definitely true nor definitely false. Fuzzy entropy, a term used to represent the degree of uncertainty, was first mentioned in 1965 by Zadeh [21]. The first attempt to quantify the fuzziness was made in 1968 by Zadeh [22], who introduced a probabilistic framework and defined the entropy of a fuzzy event as weighted Shannon entropy. The maximum-entropy principle initiated by Jaynes'[9] is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available about the system. This principle has now been broadened and extended and has found wide applications in different fields of science and technology (Kapur [10],[11]). Tsao et. al. [18] introduced a linear programming with inequality constraints via entropic perturbation. Majumder et. al.[14] ,Samanta et. al.([16], [17]) applying the entropy function in various multi-objective mathematical models.

In this paper we deal with fuzzy goals in multi-objective matrix game, assuming that each player has a fuzzy goal of the payoffs. In section 2, we have discussed the basic concepts of matrix game, section 3, entropy principle and its application, section 4,5,6 represents fuzzy goals and membership in Multi-objective game , section 7 ,max-min value equivalent Fuzzy LPP game, section 8, 9 fuzzy optimization, entropy disturbance approach for smooth and uniform distribution to solve the said problem. Lastly, a numerical illustration is to compare the results in two cases (with and without entropy).

2. Game in LPP form

Game theory and LPP are very closely associated. A two person zero sum matrix game is defined as

$\mathrm{MG} = \{\mathrm{S_m}, \mathrm{S_n}, \mathrm{A}, \mathrm{V}\}$ where $(\mathrm{p_1}, \mathrm{p_2}, \dots\dots, \mathrm{p_m})$ and $(\mathrm{q_1}, \mathrm{q_2}, \dots\dots, \mathrm{q_m})$ are the probabilities with which Players I & II adopt their strategies space $\mathrm{S_m} = (\mathrm{I_1}, \mathrm{I_2}, \dots\dots, \mathrm{I_m})$ and $\mathrm{S_n} = (\mathrm{II_1}, \mathrm{II_2}, \dots\dots, \mathrm{II_n})$ respectively, $A = (\mathrm{a_{ij}})_{\mathrm{mXn}}$is a payoff matrix and V is the value of the game, then expected gain to player I for this game when Player II select strategies $\mathrm{II_j}(\mathrm{j} = 1,2, \dots\dots, \mathrm{n})$ one by one and Player I is the gainer, so expects at least V,

$$\sum_{\mathrm{I}=1}^{\mathrm{m}} \mathrm{a_{ij}}\, \mathrm{p_i} \geq \mathrm{v}, \mathrm{j} = 1,2, \dots\dots\dots, \mathrm{n} \text{ and } \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{p_i} = 1, \mathrm{p_i} \geq 0\ \forall\, \mathrm{i}$$

For **player I:** The objective of Player I is to maximize the value of the game v which equivalent to minimizing $\frac{1}{\mathrm{v}}$. The LPP can be stated as

$$\text{Minimize } \mathrm{Z_I} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{x_i}$$

$$\text{subject to } \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{a_{ij}}\, \mathrm{x_i} \geq 1, \mathrm{j} = 1,2, \dots\dots\dots, \mathrm{n},$$

$$\mathrm{x_i} \geq 0\ \forall\, \mathrm{i} = 1,2, \dots\dots\dots, \mathrm{m}. \text{ where } x_i = \frac{\mathrm{p_i}}{\mathrm{v}}. \text{ , v be the value of the game.}$$

Similarly, for **Player II,**

$$\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{a_{ij}}\, \mathrm{q_j} \leq \mathrm{v}, \mathrm{i} = 1,2, \dots\dots\dots, \mathrm{m} \text{ where } \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{q_i} = 1, \mathrm{q_i} \geq 0\ \forall\, \mathrm{i}$$

The Player II , minimizes the expected loss i.e. maximizing $\frac{1}{v}$. The LPP can be formulated as

Maximize $Z_{II} = \sum_{i=1}^{n} y_i$

subject to $\sum_{j=1}^{n} a_{ij} y_j \leq 1, i = 1,2, \dots\dots\dots, m,$

$y_j \geq 0 \; \forall \; j = 1,2, \dots\dots\dots, n.$ where $y_j = \frac{q_j}{v}$. .

A multi-objective game is represented by multiple payoff matrices $A^k = (a_{ij}^k)_{mXn}$ and goals to each objective (each Player has same r objectives) are V^k for $k = 1,2, \dots\dots, r$.

3. Maximum Entropy principle

The principle of maximum entropy is a method for assigning values to probability distributions on the basis of partial information that is, when one has only partial information about the possible outcomes one should choose the probabilities so as to maximize the uncertainty about the missing information. By applying the principle of maximum entropy, one obtains the most random distribution subject to the satisfaction of the given constraints. We might also say that if there is insufficient information about a distribution, the optimum estimate is as unbiased as possible, and so choose the most random possible distribution. Consider a random process which can be described by discrete random variable X with n possible outcomes $\{x_1, \; x_2, \; \dots\dots\dots \; x_n\}$. Define p_i, (i = 1,2,….n.) to be the probability that X has the values x_i, i = 1,2,….n that is $P(X = x_i) = p_i$.The probabilities are not known. Some information is available about the random process in the form of m expectation functions

$\sum_{i=1}^{n} a_j(x_i)p_i = E(a_j)$, j = 1,2,….m., $\sum_{i=1}^{n} p_i = 1$.

Our objective is that of finding a probability assignment which avoids bias while agreeing with whatever information is given. The distribution random process possesses great deal of disorder or chaos. The measure of disorder / diversification was given by Shannon as $En = -\sum_{i=1}^{n} p_i \ln p_i$.

Jaynes[9] Maximum entropy principle casts the problem of determining the discrete probabilities p_i into the form of an optimization problem. Modified form of Maximum entropy principle is used to generate smooth and uniform distribution of the solution to wider, more general problems where the available information is not complete.

4. Game with fuzzy goal

A two person zero sum matrix game with fuzzy goal is defined as

$$FMG = \{S_m, S_n, A, V_+, V_-, \approx\}$$

where S_m and S_n are strategy spaces for the player I & II respectively and A is the payoff matrix and V_+, V_- are scalars representing the aspiration levels of Player I & II and symbol $\approx$ represents fuzzy field versions of usual $\geq$ and $\leq$.

A multi-objective fuzzy goal is represented by fuzzy multiple payoff matrices A^k and fuzzy goals to each objective is V_+^K (or V_-^K), $k = 1,2, \dots \dots, r$ to the player I (or Player II).

5. Aggregated Fuzzy Goals in Multi-Objective Game (MOG)

Let $D^k \subseteq R$ be the domain of k^{th} payoff for Player I & II , then a fuzzy goal G_I^k, G_{II}^k and with respect to k^{th} payoff for Player I & II is a fuzzy set on D^k characterized by the membership functions $\mu_{G_I^k}: D^k \to [0,1]$ and $\mu_{G_{II}^k}: D^k \to [0,1]$.

A value of membership function for a fuzzy goal can be interpreted as the degree of attainment of fuzzy goal for the payoff. Therefore when a player has two different payoffs, he prefers the payoff possessing higher membership function value. That means player I aims to maximize his degree of attainment.

The Max-Min value with respect to the degree of attainment of an aggregated fuzzy goal to Player I is $\underset{x \in S_m}{\mathrm{Max}} \; \underset{y \in S_n}{\mathrm{Min}} \; \underset{k}{\mathrm{Min}}\{\mu_{G_I^k}(x^T A^k y)\}$ ....(i)

Similarly, for Player II ,

$$\underset{y \in S_n}{\mathrm{Max}} \; \underset{x \in S_m}{\mathrm{Min}} \; \underset{k}{\mathrm{Min}}\{\mu_{G_{II}^k}(x^T A^k y)\} \qquad \dots\dots\dots\dots\dots\dots\text{(ii)}$$

6. Membership value of MOG and Entropy Objective:

Here the membership function for fuzzy goal G_I^k for the Player I are given by

$$\mu_{G_I^k}(x^T A^k y) = \begin{cases} 0 & \text{if } x^T A^k y \le V_-^k \\ \dfrac{x^T A^k y - V_-^k}{V_+^k - V_-^k} & \text{if } V_-^k < x^T A^k y < V_+^k \\ 1 & \text{if } x^T A^k y \ge V_+^k \end{cases}$$

Similarly, for Player II

$$\mu_{G_{II}^k}(x^T A^k y) = \begin{cases} 1 & \text{if } x^T A^k y \le V_-^k \\ \dfrac{V_+^k - x^T A^k y}{V_+^k - V_-^k} & \text{if } V_-^k < x^T A^k y < V_+^k \\ 0 & \text{if } x^T A^k y \ge V_+^k \end{cases}$$

and the membership function for Entropy goal is given by

$$\mu_{En}(En) = \begin{cases} 0 & \text{if } En \le En_- \\ \dfrac{En - En_-}{En_+ - En_-} & \text{if } En_- < En < En_+ \\ 1 & \text{if } En \ge En_+ \end{cases}$$

Where En_- , En_+ represents minimum and maximum value of Entropy goal favorable for the players and V_+^K and V_-^K are the kth payoffs for which degree of attainment of player I are 1 and 0. The values of V_+^K and V_-^K can be obtain [2]as follows:

$$V_-^k = \underset{x}{Min}\,\underset{y}{Min}(x^T A^k y) = \underset{i}{Min}\,\underset{j}{Min}(a_{ij}^k)$$

$$V_+^k = \underset{x}{Max}\,\underset{y}{Max}(x^T A^k y) = \underset{i}{Max}\,\underset{j}{Max}(a_{ij}^k)$$

where $i \in (1,2,\ldots\ldots,m) = I$, is the pure strategy for player I and $j \in (1,2,\ldots\ldots,n) = J$, is the pure strategies for player II.

7. Max-Min Value Game Problem Equivalent Fuzzy LPP Game

The Max Min problem (i) for Player I can be transformed to

$$\underset{x\in S_m}{Max}\ \underset{y\in S_n}{Min}\ \underset{k}{Min}\{\frac{x^T A^k y - V_-^k}{V_+^k - V_-^k}\}$$

$$= \underset{x\in S_m}{Max}\ \underset{y\in S_n}{Min}\ \underset{k}{Min}\{\sum_{i=1}^m \sum_{j=1}^n \tilde{a}_{ij}^k x_i y_j + c^k\}$$

$$= \underset{x\in S_m}{Max}\ \underset{k}{Min}\ \underset{y\in S_n}{Min}\ \sum_{j=1}^n \{\sum_{i=1}^m \tilde{a}_{ij}^k x_i + c^k\} y_j$$

$$= \underset{x\in S_m}{Max}\ \underset{k}{Min}\underset{j\in J}{Min}\sum_{i=1}^m \tilde{a}_{ij}^k x_i + c^k$$

$$= \underset{x\in S_m}{Max}\ \underset{k}{Min}\mu_k(x) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)} \quad \text{where } \tilde{a}_{ij}^k = \frac{a_{ij}^k}{v_+^k - v_-^k} \text{ and } c^k = \frac{v_-^k}{v_+^k - v_-^k}$$

Let $\underset{j\in J}{Min}\sum_{i=1}^m \tilde{a}_{ij}^k x_i + c^k = \mu_k(x)$ and further let $\underset{k}{Min}\mu_k(x) = \mu$, the Max Min problem for player I reduces to the LPP form whichever described in section 9.

Equivalent Fuzzy Linear Programming of the above problem is

Find $x \in R^m$

such that $\sum_{i=1}^m a_{i1}^k x_i \gtrsim v_+^k$,(k = 1,2,..r)

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)}$$

$\sum_{i=1}^{m} a_{i2}^{k} x_i \gtrsim v_+^k$,(k = 1,2,..r)

…………………

$\sum_{i=1}^{m} a_{in}^{k} x_i \gtrsim v_+^k$,(k = 1,2,..r)

$\sum_{i=1}^{m} x_i = 1, x_i \geq 0$ (i = 1,2,...., m)

where $\sum_{i=1}^{m} a_{ij}^{k} x_i \gtrsim v_+^k$ means that $\sum_{i=1}^{m} a_{ij}^{k} x_i$ is essentially greater than or equal to v_+^k with tolerance $v_+^k - v_-^k$.

Similarly, for **Player II**,(from(ii))

$$\underset{y \in S_n}{\mathrm{Max}} \ \underset{x \in S_m}{\mathrm{Min}} \ \underset{k}{\mathrm{Min}}\{\frac{V_+^k - x^T A^k y}{V_+^k - V_-^k}\}$$

$$= \underset{y \in S_n}{\mathrm{Max}} \ \underset{x \in S_m}{\mathrm{Min}} \ \underset{k}{\mathrm{Min}}\{\sum_{i=1}^{m} \sum_{j=1}^{n} \breve{a}_{ij}^{k} x_i y_j + c^k\}$$

$$= \underset{y \in S_n}{\mathrm{Max}} \ \underset{k}{\mathrm{Min}} \ \underset{x \in S_m}{\mathrm{Min}} \ \{\sum_{i=1}^{m} \{\sum_{j=1}^{n} \breve{a}_{ij}^{k} y_j + c^k\} x_i$$

$$= \underset{y \in S_n}{\mathrm{Max}} \ \underset{k}{\mathrm{Min}} \underset{i \in I}{\mathrm{Min}} \sum_{j=1}^{n} \breve{a}_{ij}^{k} y_j + c^k$$

$$= \underset{y \in S_n}{\mathrm{Max}} \ \underset{k}{\mathrm{Min}} \nu_k(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)} \quad \text{where } \breve{a}_{ij}^{k} = \frac{a_{ij}^{k}}{v_-^k - v_+^k} \text{ and } \hat{c}^k = \frac{v_+^k}{v_+^k - v_-^k}$$

Let $\underset{i \in I}{\mathrm{Min}} \sum_{j=1}^{n} \breve{a}_{ij}^{k} y_j + c^k = \nu_k(y)$ and further let $\underset{k}{\mathrm{Min}} \nu_k(y) = \nu$, the Max Min problem for player II reduces to the LPP form whichever described in section 9.

Equivalent Fuzzy Linear Programming of the above problem is

Find $y \in R^n$

Such that $\sum_{j=1}^{n} a_{1j}^{k} y_j \lesssim v_-^k$,(k = 1,2,..r)

……………………(vi)

$\sum_{j=1}^{n} a_{2j}^{k} y_j \lesssim v_-^k$,(k = 1,2,..r)

…………………

$\sum_{j=1}^{n} a_{mj}^{k} y_j \lesssim v_-^k$,(k = 1,2,..r)

$\sum_{j=1}^{n} y_j = 1, y_j \geq 0$ (j = 1,2,...., n)

where $\sum_{j=1}^{n} a_{ij}^{k} y_j \lesssim v_-^k$ means that $\sum_{j=1}^{n} a_{ij}^{k} y_j$ is essentially less than or equal to v_-^k with tolerance $v_+^k - v_-^k$.

8. Fuzzy MOG with Entropy Objective

For Player I, adding another constraint in (iv) for Entropy objectives, which is

$\sum_{i=1}^{m} x_i \ln x_i \tilde{\geq} En_+$

where $\sum_{i=1}^{m} x_i \ln x_i \tilde{\geq} En_+$ means that $\sum_{i=1}^{m} x_i \ln x_i$ is essentially greater than or equal to En_+ with tolerance $En_+ - En_-$.

Similarly for Player II, adding another constraint in (vi) for Entropy objectives, which is

$\sum_{j=1}^{n} y_j \ln y_j \tilde{\geq} En_+$

where $\sum_{j=1}^{n} y_j \ln y_j \tilde{\geq} En_+$ means that $\sum_{j=1}^{n} y_j \ln y_j$ is essentially greater than or equal to En_+ with tolerance $En_+ - En_-$.

9. Fuzzy Decision making method in MOG with Entropy Objective

For Player I, the entropy based MOG(adding an additional Shannon's entropy objective function) in(iv)and From (iv) which is equivalent to

Maximize μ ……………………………………..(vii)

subject to $\sum_{i=1}^{m} a_{i1}^{k} x_i \geq v_-^k + \mu(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

$\sum_{i=1}^{m} a_{i2}^{k} x_i \geq v_-^k + \mu(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

…………………………………………

$\sum_{i=1}^{m} a_{in}^{k} x_i \geq v_-^k + \mu(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

$\sum_{i=1}^{m} x_i \ln x_i \geq En_- + \mu(En_+ - En_-)$

$\sum_{i=1}^{m} x_i = 1, x_i \mu \leq 1, \mu \geq 0 \ \ (i = 1,2,\ldots., m)$

Similarly, for the Player II, (Using (vi))

Maximize ν ………………………………..(viii)

subject to $\sum_{j=1}^{n} a_{1j}^{k} y_j \leq v_-^k + (1-\nu)(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

$\sum_{j=1}^{n} a_{2j}^{k} y_j \leq v_-^k + (1-\nu)(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

…………………………………………

$\sum_{j=1}^{n} a_{mj}^{k} y_j \leq v_-^k + (1-\nu)(v_+^k - v_-^k)$, $(k = 1,2,\ldots\ldots, r)$

$\sum_{j=1}^{n} y_j \ln y_j \geq En_- + \nu(En_+ - En_-)$

$\sum_{j=1}^{n} y_j = 1, \nu \leq 1, y_j, \nu \geq 0 \ \ (j = 1,2,\ldots., n)$

10. Numerical Example

Let us consider a Multi-Objective matrix game as follows:

Even through there are several manufacturers of motor cycles, two companies with brand names HERO and HONDA, control their market in INDIA. Both companies make changes in model for aiming to enhance the sales and market share of the motor cycle in a targeted market. For this situation, the demand of the motor cycle in the targeted market is basically fixed, the sales and market share of HERO company increase , following the decrease the sales and market share of HONDA company, but the sales is not certain to be proportional to the market share. The two companies are consider three strategies to increase the sales and market share are as follows:

$x_1 \rightarrow$ Publicity / Advertisement; $x_2 \rightarrow$ Reduce/ Minor increase the price;
$x_3 \rightarrow$ Model changes for better handling / looking

Let HERO company be Player I , taking strategy (x_1, x_2, x_3) and HONDA company be Player II, taking the strategy (y_1, y_2, y_3) . For these strategies, the payoff matrices A^1, A^2 of targeted sales quantity k_1 (million) and market share k_2 (Percentage) are separately as follows:

$$A^1 = \begin{bmatrix} 150 & 343 & 520 \\ 246 & 460 & 175 \\ 80 & 144 & 128 \end{bmatrix},\ A^2 = \begin{bmatrix} 22 & 38 & 40 \\ 29 & 25 & 31 \\ 14 & 12 & 24 \end{bmatrix}.$$

Here $v_{-}^{1} = 80$, $v_{+}^{1} =520$ and $v_{-}^{2} = 12$, $v_{+}^{2} =40$, $v_{-}^{1} = 80$, $En_{+} =0.9259259$, $En_{-}=0$ with tolerances $v_{+}^{1} - v_{-}^{1} =460$, $v_{+}^{2} - v_{-}^{2} =28$ and $En_{+} - En_{-} = 0.9259259$.

So from(vii),(For Player I)

Maximize μ

subject to $150x_1+246x_2+80x_3\geq 80+460\mu$, $343x_1+460x_2+144x_3\geq 80+460\mu$,

$520x_1+175x_2+128x_3\geq 80+460\mu$, $22x_1+29x_2+14x_3\geq 12+28\mu$

$38x_1+25x_2+12x_3\geq 12+28\mu$, $40x_1+31x_2+24x_3\geq 12+28\mu$

$x_1 \ln x_1+x_2 \ln x_2+x_3 \ln x_3 \geq 0.9259259\mu$

$x_1+x_2+x_3=1$, $x_1, x_2, x_3,\mu\geq 0$, $\mu\leq 1$.

Table-10.1: Optimal solutions of MOG without Entropy and with Entropy(Player I)

Model	μ^*	x_1^*	x_2^*	x_3^*
MOG (without Entropy)	0.3272700	0.1609977	0.8390023	0
MOG (with Entropy)	0.2934541	0.2788071	0.6956169	0.0255760

We see the above table (10.1) that MOG(without Entropy) model has one variable x_3^* with zero value whereas there are all non-zero values (smooth and uniform solution) of x_1^*, x_2^*, x_3^* in MOG(with Entropy) Model. Here entropy is acted as a measure of dispersal / diversification of game strategies with small changes of aspiration level of μ^*. If a Company wishes to nearly uniform distribute his strategies in various targeted markets, the MOG with Entropy will be more realistic for him.

Similar results and explanation are obtained in the table **10.2** ,for **Player II** by using (viii):

Table-10.2: Optimal solutions of MOG without Entropy and with Entropy (Player II)

Model	v^*	x_1^*	x_2^*	x_3^*
MOG (without Entropy)	0.4405005	0.6664964	0.3335036	0
MOG (with Entropy)	0.4356052	0.6421903	0.3187611	0.03904868

11. Conclusion

In this paper we have discussed Entropy based multi-objective matrix game with fuzzy goals. Using Max-Min Fuzzy membership value approach, the said Game equivalent to Fuzzy LPP game and applying fuzzy optimization, entropy disturbance approach for smooth and uniform distribution to solve the said problem. The MOG with Entropy will be more realistic for him, if a Player wishes to nearly uniform distribute his strategies in various targeted markets. A numerical illustration is to compare the results in two cases (MOG model with and without entropy).

References

[1] J.P.Aubin.Cooperative fuzzy games, Mathematics of Operartions Research,6(1)(1981)1-13.
[2] C.R. Bector and S.Chandra, Fuzzy Mathematical Programming and Fuzzy Matrix Games, Springer, Berlin, Heidelberg, 2004.
[3] R.E. Bellman and L.A.Zadeh,"Decision-making in a fuzzy environment, Management Science, 17 (4) (1970) 141 -164.
[4] L.Campos, Fuzzy linear programming models to solve fuzzy matrix games, Fuzzy Sets and Systems, 32(1989) 275-289.

[5] A.C.Çevikel and M.Ahlatçýoglu, A linear interactive solution concept for fuzzy multi objective games, European Journal of Pure and Applied Mathematics, 35 (2010) 107-117.

[6] Y.W.Chen and M.Larbani ,Two-person zero-sum game approach for fuzzy multipleattribute decision making problems, Fuzzy Sets and Systems, 157 (2006) 34-51.

[7] G.B.Dantzing, Linear programming and extensions, Princeton University Press, New Jersey, 1963.

[8] J.L.Fan and Y.L.Ma, Some new fuzzy entropy formulas, Fuzzy Sets and Systems, 128(2002) 277-284.

[9] E.T. Jaynes, Information theory and statistical mechanics, Phys. Rev. 106(1957)620- 630.

[10] J.N. Kapur and H. K. Kesavan, Entropy Optimization Principles with Applications, Academic Press, Inc., San Diego, 1992

[11] J. N. Kapur, Maximum-Entropy Models in Science and Engineering, Revised ed., Wiley Eastern Limited, New Delhi, 1993.

[12] S. Kumar,Max-min solution approach for Multi-objective matrix game with fuzzy goals, Yugoslav journal of operations research,DOI:10.2298/YUJOR140415008k.

[13] Y. J. Lai and C.L Hwang, Fuzzy Multi-objective Decision Making, Springer-Verlag,1994

[14] S.K. Mazumder and N.C.Das, Decision-making process-Maximum-Entropy approach, AMSE, 41(2) (1999), 59-68.

[15] M. Sakawa and I.Nishizaki ,Max-min solutions for fuzzy multiobjective matrix games Fuzzy Sets and Systems, 67 (1994) 53-69.

[16] B. Samanta and T.K Roy, Multi-Objective Portfolio Optimization Model,Tamsui Oxford Journal of Mathematical sciences, Taiwan 21(1)(2005)55-70.

[17] B. Samanta, and S.K. Majumder, Entropy based Transportation model use Geometric Programming method, Journal of Applied mathematics and Computation, Elsevier,179 (2006) 431-439

[18] H. S. J. Tsao and S. C. Fang, Linear Programming with inequality constraints Via entropic perturbation, Int. J. of math. and math. Sci, 19(1)(1996)177-184.

[19] V.Vijay, S. Chandra and C.R. Bector, Matrix games with fuzzy goals and fuzzy payoffs, Omega, 33 (2005) 425-429.

[20] J.Von Neumann and D.Morgenstern, The theory of games in economic behavior, Wiley, New York,1944.

[21] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.

[22] L.A. Zadeh, Probability measures of fuzzy events, Journal of Mathematical Analysis and application,23(1968)421-427.