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On topologies induced by the soft topology *

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Abstract

Molodtsov introduced soft sets to deal with uncertainty. Shabir introduced soft topological spaces and established that every soft topological space induce a parameterized family of topological spaces. In this paper some concepts in soft topological spaces are characterized by using the analogous concepts in the parameterized family of topological spaces induced by the soft topology.

Keywords and Phrases: *Soft sets, soft topology, soft function, soft continuous, soft locally finite, parameterized family of topological spaces.*

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1. Introduction

The real life problems in Social sciences, Engineering, Medical sciences, Economics and Environment deal with the unpredictability and it is imprecise in nature. There are several theories in the literature such as Fuzzy set theory[16], Rough set theory[12], Interval Mathematics[2] etc dealing with uncertainties, but they have their own limitations. A new Mathematical tool dealing with uncertainty is introduced by Molodtsov[9] in 1999. Molodtsov initiated the theory of soft sets for modeling vagueness and uncertainty. Maji et.al[7, 8] studied the basic operations on soft sets and applied soft sets in decision making problems. Kharal et.al.[6] studied soft functions and their application in Medical expert system. Shabir et.al.[10] initiated the study of soft topological spaces. Moreover, theoretical studies of soft topological spaces have also been studied in [14],[3],[17],[15],[4]. Soft continuous functions are studied in [17]. Soft connectedness and soft Hausdorff spaces were introduced in [13]. In this paper some concepts in soft topological spaces are characterized by using the analogous concepts in the parametrized family of topological spaces induced by the soft topology.

2. Preliminaries

Throughout this paper X,Y are universal sets and E,K are parameter spaces.

Definition 2.1 ([9]). *A pair (F, E) is called a soft set over X where $F : E \to 2^X$ is a mapping.*

$S(X, E)$ denotes the collection of all soft sets over X with parameter space E. We denote (F, E) by $\widetilde{F}$ in which case we write $\widetilde{F} = \{(e, F(e)) : e \in E\}$. In some occasions, we use $\widetilde{F}(e)$ for $F(e)$.

Definition 2.2 ([8]). *For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ in $S(X, E)$, $\widetilde{F}$ is a soft subset of $\widetilde{G}$ if $F(e) \subseteq G(e)$ for all $e \in E$. If $\widetilde{F}$ is a soft subset of $\widetilde{G}$ then we write $\widetilde{F} \subseteq \widetilde{G}$.*

$\widetilde{F}$ and $\widetilde{G}$ are equal if and only if $F(e) = G(e)$ for all $e \in E$. That is $\widetilde{F} = \widetilde{G}$ if $\widetilde{F} \subseteq \widetilde{G}$ and $\widetilde{G} \subseteq \widetilde{F}$.

Definition 2.3 ([8]). *(i)* $\widetilde{\Phi} = \{(e, \phi) : e \in E\} = \{(e, \Phi(e)) : e \in E\} = (\Phi, E)$.
(ii) $\widetilde{X} = \{(e, X) : e \in E\} = \{(e, X(e)) : e \in E\} = (X, E)$.

Definition 2.4 ([8]). *The union of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cup \widetilde{G} = (F \cup G, E)$ where $(F \cup G)(e) = F(e) \cup G(e)$ for all $e \in E$.*

Definition 2.5 ([1]). *The intersection of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cap \widetilde{G} = (F \cap G, E)$ where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$.*

If $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is a collection of soft sets in $S(X, E)$ then the arbitrary union and the arbitrary intersection of soft sets are defined below.
$\bigcup\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcup\{F_\alpha : \alpha \in \Delta\}, E)$ and $\bigcap\{\widetilde{F}_\alpha : \alpha \in \Delta\} = (\bigcap\{F_\alpha : \alpha \in \Delta\}, E)$ where $(\bigcup\{F_\alpha : \alpha \in \Delta\})(e) = \bigcup\{F_\alpha(e) : \alpha \in \Delta\}$ and $(\bigcap\{F_\alpha : \alpha \in \Delta\})(e) = \bigcap\{F_\alpha(e) : \alpha \in \Delta\}$, for all $e \in E$.

Definition 2.6 ([10]). *The complement of a soft set $\widetilde{F}$ is denoted by $(\widetilde{F})' = (F', E)$ (relative complement in the sense of Ifran Ali.et.al([1]) where $F' : E \to 2^X$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.*

It is noteworthy to see that with respect to above complement De Morgan's laws hold for soft sets as stated below.

Lemma 2.7 ([17]). *Let I be an arbitrary index set and $\{\widetilde{F}_i : i \in I\} \subseteq S(X, E)$. Then $(\cup\{\widetilde{F}_i : i \in I\})' = \cap\{(\widetilde{F}_i)' : i \in I\}$ and $(\cap\{\widetilde{F}_i : i \in I\})' = \cup\{(\widetilde{F}_i)' : i \in I\}$.*

Definition 2.8 ([10]). *Let $\widetilde{F}$ be a soft set over X and $x \in X$. We say that $x \in \widetilde{F}$ whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.9 ([10]). *Let Y be a non-empty subset of X. Then $\widetilde{Y}$ denotes the soft set (Y, E) over X if $Y(e) = Y$ for all $e \in E$.*

From definition 2.3(ii) it is easy to see that every soft set in S(X,E) is a soft subset of $\widetilde{X}$

Definition 2.10 ([10]). *Let $\widetilde{\tau}$ be a collection of soft subset of $\widetilde{X}$. Then $\widetilde{\tau}$ is said to be a soft topology on X with parameter space E if*
(i) $\widetilde{\Phi}, \widetilde{X} \in \widetilde{\tau}$, (ii) $\widetilde{\tau}$ is closed under arbitrary union, and (iii) $\widetilde{\tau}$ is closed under finite intersection.

If $\widetilde{\tau}$ is a soft topology on X with a parameter space E then the triplet $(X, E, \widetilde{\tau})$ is called a soft topological space over X with parameter space E. Identifying (X, E) with $\widetilde{X}$, $(\widetilde{X}, \widetilde{\tau})$ is a soft topological space.
The members of $\widetilde{\tau}$ are called soft open sets in $(X, E, \widetilde{\tau})$. A soft set $\widetilde{F}$ in $S(X, E)$ is soft closed in $(X, E, \widetilde{\tau})$, if its complement $(\widetilde{F})'$ belongs to $\widetilde{\tau}$. $(\widetilde{\tau})'$ denotes the collection of all soft closed sets in $(X, E, \widetilde{\tau})$. The soft closure of $\widetilde{F}$ is the soft set, $\widetilde{s}cl(\widetilde{F}) = \cap\{\widetilde{G} : \widetilde{G}$ is soft closed and $\widetilde{F} \subseteq \widetilde{G}\}$. The soft interior [14] of $\widetilde{F}$ is the soft set, $\widetilde{s}int(\widetilde{F}) = \cup\{\widetilde{O} : \widetilde{O}$ is soft open and $\widetilde{O} \subseteq \widetilde{F}\}$. It is easy to see that $\widetilde{F}$ is soft open $\Leftrightarrow \widetilde{F} = \widetilde{s}int\widetilde{F}$ and $\widetilde{F}$ is soft closed $\Leftrightarrow \widetilde{F} = \widetilde{s}cl\widetilde{F}$

Lemma 2.11 ([10]). *Let $(X, E, \widetilde{\tau})$ be a soft space over X. Then the collection $\widetilde{\tau}_e = \{F(e) : \widetilde{F} \in \widetilde{\tau}\}$ is a topology on X for each $e \in E$.*

Thus a soft topology on X gives a parameterized family of topologies on X but the converse is not true as shown in Example 1 of [10].

Definition 2.12 ([10]). *Let Z be a non-empty subset of X. Then $\widetilde{Z} \subseteq \widetilde{X}$. Let $(X, E, \widetilde{\tau})$ be a soft topological space. Define $\widetilde{\tau}\big|_Z = \{(Z\big|_F, E) : Z\big|_F(e) = F(e) \cap Z$ for every $e \in E$ and $\widetilde{F} \in \widetilde{\tau}\}$.*

Then $\widetilde{\tau}\big|_Z$ is a soft topology on Z with parameter space E and $(Z, E, \widetilde{\tau}\big|_Z) = (\widetilde{Z}, \widetilde{\tau}\big|_Z)$ is a soft subspace of $(X, E, \widetilde{\tau})$. Clearly $\widetilde{Z}\big|_F = \widetilde{Z} \cap \widetilde{F}$ for all $\widetilde{F} \in \widetilde{\tau}$.

Definition 2.13 ([10]). *Let $(X, E, \widetilde{\tau})$ be a soft topological space over X, $\widetilde{G}$ be a soft set over X and $x \in X$. Then $\widetilde{G}$ is said to be a soft neighbourhood of x if there exist a soft open set $\widetilde{F}$ such that $x \in \widetilde{F} \subseteq \widetilde{G}$.*

Definition 2.14 ([11]). *A family $\{A_\alpha : \alpha \in \Delta\}$ of sets in a topological space X is called locally finite(nbd-finite in the sense of [5]) if each point of X has a neighborhood V such that $V \cap A_\alpha \neq \phi$ for at most finitely many indices α.*

3. Soft locally finite

In this section, soft locally finite family of soft sets is introduced and its properties are investigated. The following propositions are useful in sequel.

Proposition 3.1 *Let $(X, E, \widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X, E)$ and $e \in E$. If $\widetilde{F}$ is soft open in $(X, E, \widetilde{\tau})$ then $\widetilde{F}(e)$ is open in $(X, \widetilde{\tau}_e)$. Conversely if G is open in $(X, \widetilde{\tau}_e)$ then $G = \widetilde{F}(e)$ is soft open in $(X, E, \widetilde{\tau})$ for some soft open set $\widetilde{F}$ in $(X, E, \widetilde{\tau})$.*

Proof. Suppose $\widetilde{F} \in \widetilde{\tau}$. Then by using the Lemma 2.11, $\widetilde{F}(e) \in \widetilde{\tau}_e$.
Conversely suppose $G \in \widetilde{\tau}_e$. Again by using the same Lemma 2.11, $G = \widetilde{F}(e)$ for some $\widetilde{F} \in \widetilde{\tau}$.

Proposition 3.2 *Let $(X, E, \widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X, E)$ and $e \in E$. If $\widetilde{F}$ is soft closed in $(X, E, \widetilde{\tau})$ then $\widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$. Conversely if G is closed in $(X, \widetilde{\tau}_e)$ then $G = \widetilde{F}(e)$ is soft closed in $(X, E, \widetilde{\tau})$ for some soft closed set $\widetilde{F}$ in $(X, E, \widetilde{\tau})$*

Proof. Suppose $\widetilde{F}$ is soft closed. Then $(\widetilde{F})'$ is soft open. Then by using the Lemma 2.11, $(\widetilde{F})'(e)$ is open in $(X, \widetilde{\tau}_e)$. That is $X - \widetilde{F}(e)$ is open in $(X, \widetilde{\tau}_e)$. That is $\widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$ for all $e \in E$.
Conversely suppose G is closed in $(X, \widetilde{\tau}_e)$. Then G' is open in $(X, \widetilde{\tau}_e)$. Again by

using the same Lemma 2.11, $G' = (\widetilde{F})'(e)$ for some $\widetilde{F} \in \widetilde{\tau}$. That is $G = \widetilde{F}(e)$ is soft closed in $(X, E, \widetilde{\tau})$ for some soft closed set $\widetilde{F}$ in $(X, E, \widetilde{\tau})$.

Proposition 3.3 *Let $(X, E, \widetilde{\tau})$ be a soft topological space. Let $\widetilde{F} \in S(X, E)$. Then $\widetilde{s}cl\widetilde{F}(e) = cl(F(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.*

Proof.

$$\begin{aligned}
(\widetilde{s}cl\widetilde{F})(e) &= \bigcap\{\widetilde{H}(e) : \widetilde{H} \supseteq \widetilde{F}, \widetilde{H} \text{ is soft closed}\} \\
&= \bigcap\{\widetilde{H}(e) : \widetilde{H}(e) \supseteq \widetilde{F}(e), \widetilde{H} \text{ is soft closed}\} \\
&= \bigcap\{\widetilde{H}(e) : \widetilde{H}(e) \supseteq \widetilde{F}(e), \widetilde{H}(e) \text{ is closed in } \widetilde{\tau}_e\} \\
&= cl(\widetilde{F}(e)).
\end{aligned}$$

This shows that $\widetilde{s}cl\widetilde{F}(e) = cl(F(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.

Proposition 3.4 *Let $(X, E, \widetilde{\tau})$ be a soft topological space. Let $\widetilde{F} \in S(X, E)$. Then $\widetilde{s}int\widetilde{F}(e) = int(F(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.*

Proof.

$$\begin{aligned}
(\widetilde{s}int\widetilde{F})(e) &= \bigcup\{\widetilde{G}(e) : \widetilde{G} \subseteq \widetilde{F}, \widetilde{G} \text{ is soft open}\} \\
&= \bigcup\{\widetilde{G}(e) : \widetilde{G}(e) \subseteq \widetilde{F}(e), \widetilde{G} \text{ is soft open}\} \\
&= \bigcup\{\widetilde{G}(e) : \widetilde{G}(e) \subseteq \widetilde{F}(e), \widetilde{G}(e) \text{ is open in } \widetilde{\tau}_e\} \\
&= int(\widetilde{F}(e)).
\end{aligned}$$

Therefore $\widetilde{s}int\widetilde{F}(e) = int(F(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.

Proposition 3.5 *Let $(X, E, \widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X, E)$. Then $\widetilde{F}$ is soft closed in $(X, E, \widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$.*

Proof.
Suppose $\widetilde{F} \in S(X,E)$ is soft closed in $(X,E,\widetilde{\tau}) \Leftrightarrow \widetilde{s}cl\widetilde{F} = \widetilde{F} \Leftrightarrow (\widetilde{s}cl\widetilde{F})(e) = \widetilde{F}(e)$ for every $e \in E \Leftrightarrow cl(\widetilde{F}(e)) = \widetilde{F}(e)$ for every $e \in E$ by using Proposition 3.3. $\Leftrightarrow \widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$.

Proposition 3.6 *Let $(X,E,\widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X,E)$. Then $\widetilde{F}$ is soft open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is open in $(X,\widetilde{\tau}_e)$.*

Proof.
Suppose $\widetilde{F} \in S(X,E)$ is soft open in $(X,E,\widetilde{\tau}) \Leftrightarrow \widetilde{s}int\widetilde{F} = \widetilde{F} \Leftrightarrow (\widetilde{s}int\widetilde{F})(e) = \widetilde{F}(e)$ for every $e \in E \Leftrightarrow int(\widetilde{F}(e)) = \widetilde{F}(e)$ for every $e \in E$ by using Proposition 3.4 $\Leftrightarrow \widetilde{F}(e)$ is open in $(X,\widetilde{\tau}_e)$.

Definition 3.7 *A family $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ of soft sets over X is soft locally finite in the soft topological space $(X,E,\widetilde{\tau})$ if for every $x \in X$ there exists a soft open set $\widetilde{G}$ such that $x \in \widetilde{G}$ and $\widetilde{G} \cap \widetilde{F}_\alpha \neq \widetilde{\Phi}$ holds for at most finitely many indices α.*

The next proposition gives the connection between soft locally finite family in soft topological spaces and locally finite family in topological spaces induced by the soft topology.

Proposition 3.8 *Let $(X,E,\widetilde{\tau})$ be a soft topological space over X. Let $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ be a family of soft sets over X. Then $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite in $(X,E,\widetilde{\tau})$ iff $\{F_\alpha(e) : \alpha \in \Delta\}$ is locally finite in the topological space $(X,\widetilde{\tau}_e)$ for every $e \in E$.*

Proof.
Suppose $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite in $(X,E,\widetilde{\tau})$. $\Leftrightarrow$ for every $x \in X$ there exists a soft open set $\widetilde{G}$ such that $x \in \widetilde{G}$ and $\widetilde{G} \cap \widetilde{F}_\alpha \neq \widetilde{\Phi}$ holds for at most finitely many indices α. $\Leftrightarrow x \in \widetilde{G}(e)$ and $\widetilde{G}(e) \cap \widetilde{F}_\alpha(e) \neq \phi$ for atmost finitely many indices α, for every $e \in E \Leftrightarrow \{\widetilde{F}_\alpha(e) : \alpha \in \Delta\}$ is locally finite in $(X,\widetilde{\tau}_e)$ for every $e \in E$.

Theorem 3.9 *Let $\{\widetilde{F}_\alpha, \alpha \in \Delta\}$ be a soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\{\widetilde{s}cl(\widetilde{F}_\alpha), \alpha \in \Delta\}$ is also soft locally finite.*

Proof.
Suppose $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite. By using Proposition 3.8, $\{\widetilde{F}_\alpha(e) : \alpha \in \Delta\}$ is locally finite in $(X, \widetilde{\tau}_e)$. Then $\{cl(\widetilde{F}_\alpha(e)) : \alpha \in \Delta\}$ is locally finite in $(X, \widetilde{\tau}_e)$ for every $e \in E$. By using Theorem 9.2(1) of [5] page 82, $\{(\widetilde{s}cl\widetilde{F}_\alpha)(e) : \alpha \in \Delta\}$ is locally finite in $(X, \widetilde{\tau}_e)$ for every $e \in E$, that implies $\{\widetilde{s}cl\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite in $(X, E, \widetilde{\tau})$.

Theorem 3.10 *Let $\{\widetilde{F}_i, i \in \Delta\}$ be soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\cup\widetilde{s}cl(\widetilde{F}_i)$ is soft closed set.*

Proof.
Suppose $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite. By using the Proposition 3.8, $\{\widetilde{F}_\alpha(e) : \alpha \in \Delta\}$ is locally finite in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Then by using Theorem 9.2(2) of [5] page 82, $\cup\{cl(\widetilde{F}_\alpha(e)) : \alpha \in \Delta\}$ is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. By using Proposition 3.3, $\cup\{(\widetilde{s}cl\widetilde{F}_\alpha)(e) : \alpha \in \Delta\}$ is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. That is $(\cup\{\widetilde{s}cl\widetilde{F}_\alpha : \alpha \in \Delta\})(e)$ is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Then by using Proposition 3.5, $\cup\{\widetilde{s}cl\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft closed in $(X, E, \widetilde{\tau})$.

Theorem 3.11 *Let $\{\widetilde{F}_\alpha, \alpha \in \Delta\}$ be a soft locally finite in the soft topological space $(X, E, \widetilde{\tau})$. Then $\cup\{\widetilde{s}cl(\widetilde{F}_\alpha) : \alpha \in \Delta\} = \widetilde{s}cl(\cup\widetilde{F}_\alpha : \alpha \in \Delta)$.*

Proof.
Suppose $\{\widetilde{F}_\alpha : \alpha \in \Delta\}$ is soft locally finite. By using Proposition 3.8, $\{\widetilde{F}_\alpha(e) : \alpha \in \Delta\}$ is locally finite in $(X, \widetilde{\tau}_e)$ for every $e \in E$. By using Theorem 3.10 and by using Theorem 9.2(2) of [5] page 82, $\cup\{cl(\widetilde{F}_\alpha(e)) : \alpha \in \Delta\}$ is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Again by using Problem 7 of [5] page 91, $\cup\{cl(\widetilde{F}_\alpha)(e) : \alpha \in \Delta\} = cl\{\cup(\widetilde{F}_\alpha)(e) : \alpha \in \Delta\}$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$. By using Proposition 3.3, $(\cup(\widetilde{s}cl\widetilde{F}_\alpha) : \alpha \in \Delta\})(e) = (\widetilde{s}cl \cup \{\widetilde{F}_\alpha : \alpha \in \Delta\})(e)$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$. That is $(\cup\{\widetilde{s}cl\widetilde{F}_\alpha\})(e) = (\widetilde{s}cl \cup \{\widetilde{F}_\alpha\})(e)$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Therefore $\cup\{\widetilde{s}cl\widetilde{F}_\alpha : \alpha \in \Delta\} = \widetilde{s}cl \cup \{\widetilde{F}_\alpha : \alpha \in \Delta\}$ in $(X, E, \widetilde{\tau})$.

Lemma 3.12 *Let* $(X, E, \widetilde{\tau})$ *be a soft topological spaces. Let* $Z \subseteq X$. *Then* $(Z, E, \widetilde{\tau}\big|_Z)$ *is a soft subspace of* $(X, E, \widetilde{\tau})$. *Then* $(\widetilde{\tau}\big|_Z)_e = \widetilde{\tau}_e\big|_Z$.

Proof.
$B \in (\widetilde{\tau}\big|_Z)_e \Leftrightarrow B = \widetilde{F}(e)$ for some $\widetilde{F} \in \widetilde{\tau}\big|_Z \Leftrightarrow B = \widetilde{H}(e) \cap \widetilde{Z}(e)$ where $\widetilde{F} = \widetilde{H} \cap \widetilde{Z}$, $\widetilde{H} \in \widetilde{\tau} \Leftrightarrow B = H(e) \cap Z$ where $H(e) \in \widetilde{\tau}_e \Leftrightarrow B \in \widetilde{\tau}_e\big|_Z$.

Theorem 3.13 *Let* $\{X_\alpha, \alpha \in \Delta\}$ *be a family of sets that cover the set* X. *That is* $X = \cup X_\alpha$. *Let* $(X, E, \widetilde{\tau})$ *be a soft topological space. Assume that all the* $\widetilde{X}_\alpha$ *are soft open sets and let* $B \subseteq X$. *Then* $\widetilde{B}$ *is soft open if and only if each* $\widetilde{B} \cap \widetilde{X}_\alpha$ *is soft open in the soft subspace* $(X_\alpha, E, \widetilde{\tau}\big|_{X_\alpha})$.

Proof.
Suppose $\widetilde{X}_\alpha$ is soft open in $(X, E, \widetilde{\tau})$ for every $\alpha \in \Delta$. Then by using Proposition 3.6, X_α is open in $(X, \widetilde{\tau}_e)$ for every $e \in E$. That is $\{X_\alpha : \alpha \in \Delta\}$ is an open cover for $(X, \widetilde{\tau}_e)$ for every $e \in E$. By using Theorem 9.3 of [5] page 82, $B \subseteq X$ is open in $(X, \widetilde{\tau}_e)$ if and only if each $B \cap X_\alpha$ is open in the subspace $(X_\alpha, \widetilde{\tau}_e\big|_{X_\alpha})$ for every $e \in E$. Since by using Lemma 3.12, we have $B \subseteq X$ is open in $(X, (\widetilde{\tau})_e)$ if and only if each $B \cap X_\alpha$ is open in the subspace $(X_\alpha, (\widetilde{\tau})_e\big|_{X_\alpha})$ for every $e \in E$. That is $\widetilde{B}$ is soft open in $(X, E, \widetilde{\tau})$ if and only if $\widetilde{B} \cap \widetilde{X}_\alpha$ is soft open in the subspace $(X_\alpha, E, \widetilde{\tau}\big|_{X_\alpha})$.

Theorem 3.14 *Let* $\{X_\alpha, \alpha \in \Delta\}$ *be a family of sets that cover the set* X. *That is* $X = \cup X_\alpha$. *Let* $(X, E, \widetilde{\tau})$ *be a soft topological space. Assume that all the* $\widetilde{X}_\alpha$ *are soft closed sets and form a soft locally finite family and let* $B \subseteq X$. *Then* $\widetilde{B}$ *is soft closed if and only if each* $\widetilde{B} \cap \widetilde{X}_\alpha$ *is soft closed in the soft subspace* $(X_\alpha, E, \widetilde{\tau}\big|_{X_\alpha})$.

Proof.
Suppose $\widetilde{X}_\alpha$ is soft closed in $(X, E, \widetilde{\tau})$ for every $\alpha \in \Delta$. Then by using Proposition 3.5, X_α is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Then $\{X_\alpha : \alpha \in \Delta\}$ is locally finite and X_α is closed in $(X, \widetilde{\tau}_e)$ for every $e \in E$. Then using Theorem 9.3 of [5] page 82,$B \subseteq X$ is closed in $(X, \widetilde{\tau}_e)$ if and only if each $B \cap X_\alpha$ is closed in the subspace $(X_\alpha, \widetilde{\tau}_e\big|_{X_\alpha})$ for every $e \in E$. Since by Lemma 3.12,

$B \subseteq X$ is closed in $(X, (\widetilde{\tau})_e)$ if and only if each $B \cap X_\alpha$ is closed in the subspace $(X_\alpha, (\widetilde{\tau})_e\big|_{X_\alpha})$ for every $e \in E$. That implies $\widetilde{B}$ is soft closed in $(X, E, \widetilde{\tau})$ if and only if $\widetilde{B} \, \cap \, \widetilde{X}_\alpha$ is soft closed in the subspace $(X_\alpha, E, \widetilde{\tau}\big|_{X_\alpha})$.

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