Tamsui Oxford Journal of Mathematical Sciences **23(4)** (2007) 393-403
Aletheia University

New Generalizations of Certain Ostrowski Type Inequalities*

B. G. Pachpatte†

57 Shri Niketan Colony, Near Abhinay Talkies,
Aurangabad 431 001 (Maharashtra) India

Received January 3, 2006, Accepted June 7, 2006.

Abstract

In the present paper, new generalizations of certain Ostrowski type inequalities involving two functions are given via some integral identities.

Keywords and Phrases: *Ostrowski type inequalities, Integral identities, Harmonic sequence, Lipschitz function , Bounded variation.*

1. Introduction

In [1] , Cerone, Dragomir and Roumeliatis proved the following identity :

$$\int_a^b h(t)\,dt = \sum_{k=0}^{n-1}\left[\frac{(b-x)^{k+1}+(-1)^k (x-a)^{k+1}}{(k+1)!}\right] h^{(k)}(x)$$

$$+(-1)^n \int_a^b E_n(x,t)\, h^{(n)}(t)\,dt, \tag{1.1}$$

*2000 *Mathematics Subject Classification.* 26D15, 26D20.

†E-mail: bgpachpatte@hotmail.com

for $x \in [a,b]$, where $h : [a,b] \to R$ be a mapping such that $h^{(n-1)}$ is absolutely continuous on $[a,b]$, $E_n : [a,b]^2 \to R$ is given by

$$E_n(x,t) = \begin{cases} \frac{(t-a)^n}{n!} & if \;\; t \in [a,x] \\ \frac{(t-b)^n}{n!} & if \;\; t \in (x,b], \end{cases}$$

for $x \in [a,b]$ and $n \geq 1$ is a natural number .

Let $[P_n]$ be a harmonic sequence of polynomials, that is $P_n' = P_{n-1}, n \geq 1, P_0 = 1$. Furthermore, let $I \subset R$ be a segment and $h : I \to R$ be such that $h^{(n-1)}$ is Lipschitz function or is a continuous function of bounded variation on I , for some $n \geq 1$. In [3] , Dedić ,Pečarić and Ujević proved the following identity :

$$\frac{1}{n}\left[h(x) + \sum_{k=1}^{n-1}(-1)^k P_k(x) h^{(k)}(x) + \sum_{k=1}^{n-1}\bar{H}_k\right] - \frac{1}{b-a}\int_a^b h(t)\,dt$$

$$= \frac{(-1)^{n-1}}{n(b-a)}\int_a^b P_{n-1}(t)\, e(x,t)\, dh^{(n-1)}(t), \tag{1.2}$$

where

$$\bar{H}_k = \frac{(-1)^k (n-k)}{b-a}\left[P_k(a) h^{(k-1)}(a) - P_k(b) h^{(k-1)}(b)\right], \tag{1.3}$$

and

$$e(x,t) = \begin{cases} t-a & if \;\; t \in [a,x] \\ t-b & if \;\; t \in (x,b], \end{cases} \tag{1.4}$$

for $x \in [a,b]$. The sums above are defined to be zero for $n = 1$.

In the same papers [1] and [3] based on the identities (1.1) and (1.2) the authors pointed out the inequalities which provide approximation formulae for the integral $\int_a^b h(t)\,dt$ whose errors can be estimated as follows :

$$\left|\int_a^b h(t)\,dt - \sum_{k=0}^{n-1}\frac{(b-x)^{k+1} + (-1)^k (x-a)^{k+1}}{(k+1)!} h^{(k)}(x)\right| \leq \left\|h^{(n)}\right\|_\infty H_n(x), \tag{1.5}$$

where $h^{(n)} \in L_{\infty}[a,b]$,

$$H_n(x) = \int_a^b |E_n(x,t)|\, dt \tag{1.6}$$

and

$$\left| \frac{1}{n}\left[h(x) + \sum_{k=1}^{n-1} (-1)^k P_k(x)\, h^{(k)}(x) + \sum_{k=1}^{n-1} \bar{H}_k \right] - \frac{1}{b-a}\int_a^b h(t)\, dt \right| \le \frac{L}{b-a} D_n(x), \tag{1.7}$$

where

$$D_n(x) = \frac{1}{n}\int_a^b |P_{n-1}(t)\, e(x,t)|\, dt, \tag{1.8}$$

$\bar{H}_k$ and $e(x,t)$ are as given in (1.3) and (1.4) and $h : [a,b] \to R$ be such that $h^{(n-1)}$ is L-Lipschitz function for some $n \ge 1$ and $L \ge 0$ is a constant.

In recent years, a number of authors have written about generalizatins, extensions and variants of the well known Ostrowski's inequality ([6, p.468]), see [1-6 , 9 , 10] and the references cited therein . In this paper we will give new generalizations of the inequalities (1.5) and (1.7) involving two functions, as well as some related new inequalities. The analysis used in the proofs is elementary and based on the use of the integral identities (1.1) and (1.2).

2. Statement of Results

In what follows, R denotes the set of real numbers and $[a,b]$ for $a < b$ be a given subset of R . First we give the following notations used to simplify the details of presentations. For a suitable function $h : [a,b] \to R$ we set

$$A[h(x)] = h(x) + \frac{1}{b-a}\sum_{k=1}^{n-1} \frac{(b-x)^{k+1} + (-1)^k (x-a)^{k+1}}{(k+1)!} h^{(k)}(x),$$

$$B\left[h\left(x\right)\right]=\frac{1}{n}\left[h\left(x\right)+\sum_{k=1}^{n-1}\left(-1\right)^{k}P_{k}\left(x\right)h^{(k)}\left(x\right)+\sum_{k=1}^{n-1}\bar{H}_{k}\right],$$

for $x\in[a,b]$ and define $\|h\|_{\infty}=\sup_{t\in[a,b]}|h(t)|<\infty$, for $h\in L_{\infty}[a,b]$. .

Our main results are given in the following theorems.

Theorem 1. *Let $f,g:[a,b]\to R$ be functions such that $f^{(n-1)},g^{(n-1)}$ are absolutely continuous on $[a,b]$ and $f^{(n)},g^{(n)}\in L_{\infty}[a,b]$. Then*

$$\left|g\left(x\right)A\left[f\left(x\right)\right]+f\left(x\right)A\left[g\left(x\right)\right]-\frac{1}{b-a}\left[g\left(x\right)\int_{a}^{b}f\left(t\right)dt+f\left(x\right)\int_{a}^{b}g\left(t\right)dt\right]\right|$$
$$\leq\frac{1}{b-a}\left[\left|g\left(x\right)\right|\left\|f^{(n)}\right\|_{\infty}+\left|f\left(x\right)\right|\left\|g^{(n)}\right\|_{\infty}\right]H_{n}\left(x\right),\tag{2.1}$$

and

$$\left|A\left[f\left(x\right)\right]A\left[g\left(x\right)\right]-\frac{1}{b-a}\left[A\left[g\left(x\right)\right]\int_{a}^{b}f\left(t\right)dt+A\left[f\left(x\right)\right]\int_{a}^{b}g\left(t\right)dt\right]\right.$$
$$\left.+\frac{1}{\left(b-a\right)^{2}}\left(\int_{a}^{b}f\left(t\right)dt\right)\left(\int_{a}^{b}f\left(t\right)dt\right)\right|$$
$$\leq\frac{1}{\left(b-a\right)^{2}}\left\|f^{(n)}\right\|_{\infty}\left\|g^{(n)}\right\|_{\infty}H_{n}^{2}\left(x\right),\tag{2.2}$$

for all $x\in[a,b]$, where $H_{n}(x)$ is given by (1.6).

Remark 1. *We note that in [1] the authors have evaluated the integral in (1.6) and obtained*

$$H_{n}\left(x\right)=\frac{1}{\left(n+1\right)!}\left[\left(x-a\right)^{n+1}+\left(b-x\right)^{n+1}\right],\tag{2.3}$$

for $x\in[a,b]$. If we take $g(x)=1$ and hence $g^{(n)}(x)=0$ for $n\geq1$, in the inequality (2.1), then we recapture the inequality established in (1.5), that is, the inequality in [1], when $H_{n}(x)$ is given by (2.3). One can also obtain bounds on the right hand sides in the inequalities in (2.1) and (2.2) when $f^{(n)},g^{(n)}$ belongs to $L_{p}[a,b]$ or $L_{1}[a,b]$.

Theorem 2. *Let (P_n) be a harmonic sequence of polynomials and $f, g : [a, b] \to R$ be functions such that $f^{(n-1)}, g^{(n-1)}$ are respectively L-Lipschitz and M-Lipschitz functions for some $n \geq 1$ i.e.*

$$\left|f^{(n-1)}(x) - f^{(n-1)}(y)\right| \leq L\,|x-y|\,,$$

$$\left|g^{(n-1)}(x) - g^{(n-1)}(y)\right| \leq M\,|x-y|\,,$$

for $x, y \in [a, b]$, where L, M are nonnegative constants. Then

$$\begin{aligned} &\left|g(x)\,B[f(x)] + f(x)\,B[g(x)] - \frac{1}{b-a}\left[g(x)\int_a^b f(t)\,dt + f(x)\int_a^b g(t)\,dt\right]\right| \\ \leq\ & \frac{1}{b-a}\left[L\,|g(x)| + M\,|f(x)|\right] D_n(x)\,, \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} &\left|B[f(x)]\,B[g(x)] - \frac{1}{b-a}\left[B[g(x)]\int_a^b f(t)\,dt + B[f(x)]\int_a^b g(t)\,dt\right]\right. \\ &\left. + \frac{1}{(b-a)^2}\left(\int_a^b f(t)\,dt\right)\left(\int_a^b g(t)\,dt\right)\right| \\ \leq\ & \frac{LM}{(b-a)^2} D_n^2(x)\,, \end{aligned} \tag{2.5}$$

for all $x \in [a, b]$, where $D_n(x)$ is given by (1.8).

Remark 2. *By taking $g(x) = 1$ and hence $g^{(n-1)}(x) = 0$ for $n \geq 2$ in (2.4) and by simple calculation, it is easy to observe that the inequality (2.4) reduces to the inequality (1.7) given in [3]. We note that, one can also obtain inequalities similar to that of (2.4) and (2.5) for the functions $f, g : [a, b] \to R$ such that $f^{(n-1)}, g^{(n-1)}$ are continuous functions of bounded variations for some $n \geq 1$ (see [3,Theorem 2]). We believe that the inequalities established in (2.2) and (2.5) are new to the literature.*

3. Proofs of Theorems 1 and 2

From the hypotheses of Theorem 1, we have the following identities (see [1,5]) :

$$A\left[f\left(x\right)\right]-\frac{1}{b-a}\int\limits_a^b f\left(t\right)dt=\frac{\left(-1\right)^{n+1}}{b-a}\int\limits_a^b E_n\left(x,t\right)f^{(n)}\left(t\right)dt, \tag{3.1}$$

$$A\left[g\left(x\right)\right]-\frac{1}{b-a}\int\limits_a^b g\left(t\right)dt=\frac{\left(-1\right)^{n+1}}{b-a}\int\limits_a^b E_n\left(x,t\right)g^{(n)}\left(t\right)dt. \tag{3.2}$$

Multiplying (3.1) and (3.2) by $g(x)$ and $f(x)$ respectively and adding the resulting identities we have

$$\begin{aligned}&g\left(x\right)A\left[f\left(x\right)\right]+f\left(x\right)A\left[g\left(x\right)\right]-\tfrac{1}{b-a}\left[g\left(x\right)\int\limits_a^b f\left(t\right)dt+f\left(x\right)\int\limits_a^b g\left(t\right)dt\right]\\=\;&\tfrac{\left(-1\right)^{n+1}}{b-a}\left[g\left(x\right)\int\limits_a^b E_n\left(x,t\right)f^{(n)}\left(t\right)dt+g\left(x\right)\int\limits_a^b E_n\left(x,t\right)f^{(n)}\left(t\right)dt\right].\end{aligned} \tag{3.3}$$

From (3.3) and by using the properties of modulus we have

$$\begin{aligned}&\left|g\left(x\right)A\left[f\left(x\right)\right]+f\left(x\right)A\left[g\left(x\right)\right]-\tfrac{1}{b-a}\left[g\left(x\right)\int\limits_a^b f\left(t\right)dt+f\left(x\right)\int\limits_a^b g\left(t\right)dt\right]\right|\\\leq\;&\tfrac{1}{b-a}\left[\left|g\left(x\right)\right|\int\limits_a^b\left|E_n\left(x,t\right)\right|\left|f^{(n)}\left(t\right)\right|dt+\left|f\left(x\right)\right|\int\limits_a^b\left|E_n\left(x,t\right)\right|\left|g^{(n)}\left(t\right)\right|dt\right]\\\leq\;&\tfrac{1}{b-a}\left[\left|g\left(x\right)\right|\left\|f^{(n)}\right\|_\infty+\left|f\left(x\right)\right|\left\|g^{(n)}\right\|_\infty\right]\int\limits_a^b\left|E_n\left(x,t\right)\right|dt\\=\;&\tfrac{1}{b-a}\left[\left|g\left(x\right)\right|\left\|f^{(n)}\right\|_\infty+\left|f\left(x\right)\right|\left\|g^{(n)}\right\|_\infty\right]H_n\left(x\right).\end{aligned}$$

The proof of (2.1) is complete.

Multiplying the left sides and right sides of (3.1) and (3.2) we get

$$\begin{aligned}&A\left[f\left(x\right)\right]A\left[g\left(x\right)\right]-\tfrac{1}{b-a}\left[A\left[g\left(x\right)\right]\int\limits_a^b f\left(t\right)dt+A\left[f\left(x\right)\right]\int\limits_a^b g\left(t\right)dt\right]\\&+\tfrac{1}{\left(b-a\right)^2}\left(\int\limits_a^b f\left(t\right)dt\right)\left(\int\limits_a^b g\left(t\right)dt\right)\\=\;&\tfrac{\left(-1\right)^{2n+2}}{\left(b-a\right)^2}\left[\int\limits_a^b E_n\left(x,t\right)f^{(n)}\left(t\right)dt\right]\left[\int\limits_a^b E_n\left(x,t\right)g^{(n)}\left(t\right)dt\right].\end{aligned} \tag{3.4}$$

From (3.4) and using the properties of modulus we have

$$\begin{aligned}
&\left|A\left[f\left(x\right)\right]A\left[g\left(x\right)\right]-\tfrac{1}{b-a}\left[A\left[g\left(x\right)\right]\int_a^b f\left(t\right)dt+A\left[f\left(x\right)\right]\int_a^b g\left(t\right)dt\right]\right.\\
&\left.+\tfrac{1}{(b-a)^2}\left(\int_a^b f\left(t\right)dt\right)\left(\int_a^b g\left(t\right)dt\right)\right|\\
\leq\ &\tfrac{1}{(b-a)^2}\left[\int_a^b \left|E_n\left(x,t\right)\right|\left|f^{(n)}\left(t\right)\right|dt\right]\left[\int_a^b \left|E_n\left(x,t\right)\right|\left|g^{(n)}\left(t\right)\right|dt\right]\\
\leq\ &\tfrac{1}{(b-a)^2}\left\|f^{(n)}\right\|_\infty\left\|g^{(n)}\right\|_\infty\left[\int_a^b \left|E_n\left(x,t\right)\right|dt\right]^2\\
=\ &\tfrac{1}{(b-a)^2}\left\|f^{(n)}\right\|_\infty\left\|g^{(n)}\right\|_\infty H_n^2\left(x\right).
\end{aligned}$$

This is the desired inequality in (2.2).

From the hypotheses of Theorem 2, the following identities hold (see [3]):

$$B\left[f\left(x\right)\right]-\frac{1}{b-a}\int_a^b f\left(t\right)dt=\frac{\left(-1\right)^{n-1}}{n\left(b-a\right)}\int_a^b P_{n-1}\left(t\right)e\left(x,t\right)df^{(n-1)}\left(t\right), \quad (3.5)$$

$$B\left[g\left(x\right)\right]-\frac{1}{b-a}\int_a^b g\left(t\right)dt=\frac{\left(-1\right)^{n-1}}{n\left(b-a\right)}\int_a^b P_{n-1}\left(t\right)e\left(x,t\right)dg^{(n-1)}\left(t\right). \quad (3.6)$$

Multiplying (3.5) and (3.6) by $g(x)$ and $f(x)$ respectively and adding the resulting identities we have

$$\begin{aligned}
&g\left(x\right)B\left[f\left(x\right)\right]+f\left(x\right)B\left[g\left(x\right)\right]-\tfrac{1}{b-a}\left[g\left(x\right)\int_a^b f\left(t\right)dt+f\left(x\right)\int_a^b g\left(t\right)dt\right]\\
=\ &\tfrac{(-1)^{n-1}}{n(b-a)}\left[g\left(x\right)\int_a^b P_{n-1}\left(t\right)e\left(x,t\right)df^{(n-1)}\left(t\right)\right.\\
&\left.+f\left(x\right)\int_a^b P_{n-1}\left(t\right)e\left(x,t\right)dg^{(n-1)}\left(t\right)\right]. \quad (3.7)
\end{aligned}$$

For integrable function $F:[a,b]\to R$ we have

$$\left|\int_a^b F\left(t\right)df^{(n-1)}\left(t\right)\right|\leq L\int_a^b \left|F\left(t\right)\right|dt, \quad (3.8)$$

$$\left|\int_a^b F(t)\,dg^{(n-1)}(t)\right| \le M\int_a^b |F(t)|\,dt, \tag{3.9}$$

since $f^{(n-1)}$ and $g^{(n-1)}$ are L-Lipschitz and M-Lipschitz functions. From (3.7) , using the properties of modulus and in view of (3.8) , (3.9) we observe that

$$\begin{aligned}
&\left|g(x)B[f(x)] + f(x)B[g(x)] - \tfrac{1}{b-a}\left[g(x)\int_a^b f(t)\,dt + f(x)\int_a^b g(t)\,dt\right]\right| \\
\le\ & \tfrac{1}{n(b-a)}\left[|g(x)|\,L\int_a^b |P_{n-1}(t)\,e(x,t)|\,dt + |f(x)|\,M\int_a^b |P_{n-1}(t)\,e(x,t)|\,dt\right] \\
=\ & \tfrac{1}{(b-a)}\left[L|g(x)| + M|f(x)|\right]\tfrac{1}{n}\int_a^b |P_{n-1}(t)\,e(x,t)|\,dt \\
=\ & \tfrac{1}{(b-a)}\left[L|g(x)| + M|f(x)|\right]D_n(x),
\end{aligned}$$

which is the required inequality in (2.4).

Multiplying the left sides and right sides of (3.5) and (3.6) we have

$$\begin{aligned}
&B[f(x)]\,B[g(x)] - \tfrac{1}{b-a}\left[B[g(x)]\int_a^b f(t)\,dt + B[f(x)]\int_a^b g(t)\,dt\right] \\
&+\tfrac{1}{(b-a)^2}\left(\int_a^b f(t)\,dt\right)\left(\int_a^b g(t)\,dt\right) \\
=\ & \left\{\tfrac{(-1)^{n-1}}{n(b-a)}\right\}^2\left[\int_a^b P_{n-1}(t)\,e(x,t)\,df^{(n-1)}(t)\right] \\
&\times\left[\int_a^b P_{n-1}(t)\,e(x,t)\,dg^{(n-1)}(t)\right].
\end{aligned} \tag{3.10}$$

The rest of the proof of (2.5) can be completed by closely looking at the proofs of (2.2) and (2.4) given above.

Remark 3. *We note that, one can very easily obtain from Theorems 1 and 2, the corollaries similar to those of various corollaries of the corresponding results given in [1,3] with suitable modifications. Here we do not discuss the details.*

4. Some Related Inequalities

In this section, we point out some new inequalities related to the inequalities given in Theorems 1 and 2.

Dividing both sides of (3.3) and (3.4) by $(b-a)$ and then integrating both sides with respect to x over $[a,b]$ and following closely the proof of Theorem 1 we get

$$\begin{aligned}
&\left|\tfrac{1}{b-a}\int_a^b [g(x)\,A[f(x)]+f(x)\,A[g(x)]]dx-\tfrac{2}{(b-a)^2}\left(\int_a^b f(t)\,dt\right)\left(\int_a^b g(t)\,dt\right)\right| \\
\leq\ & \tfrac{1}{(b-a)^2}\int_a^b \left[|g(x)|\left\|f^{(n)}\right\|_\infty+|f(x)|\left\|g^{(n)}\right\|_\infty\right]H_n(x)\,dx, \qquad (4.1)
\end{aligned}$$

and

$$\begin{aligned}
&\left|\tfrac{1}{b-a}\int_a^b [A[f(x)]\,A[g(x)]]dx-\tfrac{1}{(b-a)^2}\left[\left(\int_a^b A[g(x)]\,dx\right)\left(\int_a^b f(x)\,dx\right)\right.\right. \\
&\left.+\left(\int_a^b A[f(x)]\,dx\right)\left(\int_a^b g(x)\,dx\right)\right] \\
&\left.+\tfrac{1}{(b-a)^2}\left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)\right| \\
\leq\ & \tfrac{1}{(b-a)^3}\left\|f^{(n)}\right\|_\infty\left\|g^{(n)}\right\|_\infty\int_a^b H_n^2(x)\,dx. \qquad (4.2)
\end{aligned}$$

Similarly, dividing both sides of (3.7) and (3.10) by $(b-a)$ and then integrating both sides with respect to x over $[a,b]$ and following the arguments as in the proof of Theorem 2, we get

$$\begin{aligned}
&\left|\tfrac{1}{b-a}\int_a^b [g(x)\,B[f(x)]+f(x)\,B[g(x)]]\,dx-\tfrac{2}{(b-a)^2}\left(\int_a^b f(t)\,dt\right)\left(\int_a^b g(t)\,dt\right)\right| \\
\leq\ & \tfrac{1}{(b-a)^2}\int_a^b [L\,|g(x)|+M\,|f(x)|]\,D_n(x)\,dx, \qquad (4.3)
\end{aligned}$$

and

$$\begin{aligned}&\Big| \frac{1}{b-a}\int_a^b B[f(x)]B[g(x)]\,dx-\frac{1}{(b-a)^2}\Bigg[\left(\int_a^b B[g(x)]\,dx\right)\left(\int_a^b f(x)\,dx\right)\\&+\left(\int_a^b B[f(x)]\,dx\right)\left(\int_a^b g(x)\,dx\right)\Bigg]+\frac{1}{(b-a)^2}\left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)\Big|\\ \le\ &\frac{LM}{(b-a)^3}\int_a^b D_n^2(x)\,dx. \qquad (4.4)\end{aligned}$$

Remark 4. *We note that the inequalities obtained in (4.1)-(4.4) are similar to the well known inequalities due to Grüss (see [7, Chapter X]) and Čebyšev (see [7, p.297]) . For various other inequalities of the above type, see [7,8] .*

Finally we note that the method presented in this paper can be used in order to obtain the further extensions of the results given in Theorems 1 and 2 involving n functions via the integral identities (1.1) and (1.2). In this case we shall meet only some technical complications. Here we do not discuss the details.

References

[1] P. Cerone, S. S. Dragomir and J. Roumelotis, Some Ostrowski type inequalities for n-time differentiable mappings and applications, *Demonstrato Math.* **32(4)** (1999), 697-712.

[2] P. Cerone and S. S. Dragomir, Three point quadrature rules involving, at most, a first derivative, *RGMIA Res. Rep. Coll.* **2(4)** (1999), 511-583.

[3] Lj. Dedić, M. Matić and J. Pečarić, On some generalizations of Ostrowski inequality for Lipschitz functions and functions of bounded variation, *Math. Inequal. Appl.* **3(1)** (2000), 1-14.

[4] S. S. Dragomir and Th. M. Rassias (Eds.), *Ostrowski type Inequalities and Applications in Numerical Integration*, Kluwer Academic Publishers, Dordrecht, 2002.

[5] M. Matić, J. Pečarić and N. Ujević , Improvement and further generalization of inequalities of Ostrowski-Grüss type, *Computers Math. Appl.* **39** (2000),161-175.

[6] D. S. Mitrinović, J. E. Pečarić and A. M. Fink, *Inequalities Involving Functions and Their Integrals and Derivatives*, Kluwer Academic Publishers, Dordrecht, 1991.

[7] D. S. Mitrinović, J. E. Pečarić and A. M. Fink, *Classical and New Inequalities* in Analysis, Kluwer Academic Publishers, Dordrecht, 1993.

[8] B. G. Pachpatte, On Grüss type integral inequalities, *J. Inequal.Pure and Appl. Math.* **3(1)** Art.11, 2002.

[9] B. G. Pachpatte, On a new generalization of Ostrowski's inequality, *J. Inequal. Pure and Appl. Math.* **5(2)**, Art.36, 2004.

[10] B. G. Pachpatte, New inequalities of Ostrowski and Trapezoid type for n-time differentiable functions, *Bull. Korean Math. Soc.* **41(4)** (2004), 633-639.