Multi-objective Inventory Model of Deteriorating Items with Space Constraint in a Fuzzy Environment

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Abstract

Multi-objective inventory models of deteriorating items have been developed with vague and imprecise information about available storage area. Here, the objectives are (i) to maximize the profit, (ii) to minimize the wastage cost due to deterioration and (iii) to minimize the total production cost. These objectives are also fuzzy in nature. In these models, production rate is a decision variable along with the usual decision parameters - inventory quantities. The impreciseness in inventory parameters and objective goals has been expressed by linear membership functions. We have solved the proposed model for a particular unit production cost function using different fuzzy non-linear goal programming techniques based on gradient method. To incorporate the relative importance of the objectives, the cardinal weights (both fuzzy or crisp) have been assigned. The models are illustrated with numerical examples and results of different models are compared.

1. Introduction

In most of the manufacturing systems, it is generally assumed that, the production rate of a machine is predetermined and inflexible [1]. A number of research papers have already been published in this direction by Adler and Nanda [2], Rosenblat and Lee [3], etc. However, in a realistic situation, machine production rate is not always constant but can be changed taking some measures like employing experienced and efficient machine man, introducing new technology etc. In other words, production rate in many cases may be treated as a decision variable. As the production rate is increased, some costs such as labour, material and energy costs also increase and as a result, per unit machine cost increases [4]. As these costs are spread over all production units, the net result is that unit production cost decreases until ideal "design production rate" of the machine is reached.

An important assumption in inventory models found in the existing literature is that the lifetime of an item is infinite while it is in storage. But the effect of deterioration plays an important role in the storage of some commonly used decaying items like, breakable items, (glass, china clay, ceramic goods etc.), radioactive substances, perishable goods etc. In these cases, a certain fraction of these goods are either damaged or decayed and are not in a perfect condition to satisfy the future demand of customers for good items. Deterioration in such items is continuous and time independent or time dependent and/or dependent on on-hand inventory. A number of research papers have already been published on above type of items by Datta and Pal [5] Goswami and Chowdhury [6], Kar et. al.[7] and others.

Multi-item classical inventory models under resource constraints such as capital investment, available storage area, number of orders and available set-up time etc. are presented in well-known books [8-12]. In 1982 Worrall and Hall [13] have discussed the application of posinomial geometric programming to a multi-item classical inventory model with several simultaneous constraints.

In multi-objective mathematical programming problems, a decision maker is required to maximize/minimize two or more objectives simultaneously over a feasible region determined by a given set of decision variables. In general, the decision maker selects a compromise solution from a set of possible solutions. A number of methods like weighting method, assigning priorities to the objectives, setting aspiration 2 levels for the objectives etc., exist for finding compromise solutions [14]. Among these various methods the method based on goal programming is found to be useful in many real life problems. Padmanabhan and Prem Vrat [15] solved a multi-objective inventory model of deteriorating items with stock-dependent demand by a non-linear

goal programming method. A methodology based on the use of a nested hierarchy of priorities for each goal was presented by Rubin and Narasimhan [16]. The importance of multiple objectives in the design of practical engineering systems has been established by Rao.[17].

In many realistic situations, it is difficult to assign precise aspiration levels to objectives. Moreover, in some cases, it is not even possible to articulate precise boundaries of the constraints. In such situations a fuzzy goal model is more appropriate. In these cases, normally both linear and non-linear shapes for the membership functions of the fuzzy objective and constraint goals are proposed. To reflect the decision makers' performances regarding the relative importance of each objective goal, crisp/fuzzy weights are used following Narasimhan [16]. The fuzzy priorities may be "linguistic variables" such as "very important", "moderately important" and "important". Membership functions can be defined for these fuzzy priorities in order to develop a combined measure of the degree to which the different goals are attended. Recently, Kar et. al.[18] presented a multi-objective inventory model of deteriorating items under imprecise and chance constraints.

In this paper, under imprecise storage area, a multi-objective inventory model of deteriorating items with production rate dependent on unit cost function is formulated in fuzzy environment. Here, the objectives are to maximize the average profit, to minimize wastage cost and to minimize the production cost, where profit goal, wastage goal, total production cost and storage area are fuzzy in nature. In this model, fuzzy parameters are represented by linear membership functions and after fuzzification it is solved by fuzzy multi-objective nonlinear programming method. Here, both the crisp and fuzzy models are solved by Zimmermann, Additive, Square Additive, Exponential Square Additive and Productive methods. Crisp and fuzzy weights are also used for relative importance of the objective and constraint goals. The models are illustrated with numerical example.

2.Assumptions and Notations

To develop the inventory model of deteriorating items with variable production rate, the following notations are used:

n = numbers of items,

W = available floor or shelf-space.

For i-th (i = 1, 2, 3,···,n) item

P_i = production rate (a decision variable),

Q_i = production lot-size (a decision variable),

T_i = cycle length,

T_{1i} = production cycle length,

S_i = set-up cost per cycle,

H_i = inventory holding cost per unit item,

D_i = constant rate of demand,

$f_i(P_i)$ = function of production rate representing unit production cost,

s_i = selling price per unit item, which is fixed on the basis of production cost with a mark-up rate given by $s_i = m_i f_i(P_i)$, $m_i > 1$,

i = constant rate of deterioration,

w_i = storage space required per unit item,

$PF(P, Q)$ = total average profit of the system,

$WC(P, Q)$ = average wastage cost,

$PC(P, Q)$ = total production cost per cycle.

(where P and Q are n-dimensional vectors with components as the decision variables P_i (i = 1, 2, $\cdots$,n) and Q_i (i = 1, 2,$\cdots$,n) respectively).

2.1 Basic Assumptions about the Model

(i) Production rate is finite,

(ii) Shortages are not allowed,

(iii) Lead time is zero,

(iv) The unit production cost $f_i(P_i)$ is related to the production rate P_i as:

$$f_i(P_i) = r_i + \frac{g_i}{P_i} + b_i P_i^{\beta_i}$$

where r_i, g_i, b_i, and β_i (i = 1, 2,$\cdots$, n) are non-negative real numbers to be chosen to provide the best fit for the estimated unit production cost function. Here,

(a) r_i = cost component independent of production rate. This cost component includes raw materials cost.

(b) $\dfrac{g_i}{P_i}$ = cost component per unit that decreases with the increase of production rate.

This cost component includes labour cost. For example, if more units are produced per unit time by a worker needed to tend the machine, then the wages of the worker are spread over more units. In other words, g_i is what the literature on optimizing machining rates refers to as cost of operating time.

(c) $b_i P_i^\beta$ = cost component per unit that increases with increase of production rate. This cost includes tool cost and rework cost that might result from increased tool wear-out at higher production rate.

3. Mathematical Formulation

The differential equations describing the inventory level $q_i(t)$ of ith item in the interval, $0 \le t \le T_i$ is given by

$$\frac{dq_i(t)}{dt} + \theta_i q_i(t) = P_i - D_i, \qquad\qquad 0 \le t \le T_{1i}, \tag{1}$$

$$\frac{dq_i(t)}{dt} + \theta_i q_i(t) = -D_i, \qquad\qquad T_{1i} \le t \le T_i. \tag{2}$$

The conditions are $q_i(t) = 0$ at $t = 0$, $q_i(t) = Q_i$ at $t = T_{1i}$, $q_i(t) = 0$ at $t = T_i$ and $q_i(t)$ is continuous at $t = T_{1i}$.

Using the conditions, the solutions of (1) and (2) are

$$q_i(t) = \frac{P_i - D_i}{\theta_i}\left\{ 1 - e^{-\theta_i t} \right\}, \qquad\qquad 0 \le t \le T_{1i}, \tag{3}$$

$$q_i(t) = \frac{D_i}{\theta_i}\left\{ e^{\theta_i(T_i - t)} - 1 \right\}, \qquad\qquad T_{1i} \le t \le T_i. \tag{4}$$

As $q_i(t)$ is continuous at $t = T_{1i}$ and $q_i(T_{1i}) = Q_i$ we have

$$\frac{P_i - D_i}{\theta_i} \left\{ 1 - e^{-\theta_i T_{1i}} \right\} = Q_i = \frac{D_i}{\theta_i} \left\{ e^{\theta_i (T_i - T_{1i})} - 1 \right\}.$$

From this, total production period and total time cycle for i[th] item are obtained as

$$T_{1i} = \frac{1}{\theta_i} \log \left\{ \frac{P_i - D_i}{P_i - D_i - \theta_i Q_i} \right\} \tag{5}$$

and

$$T_i = \frac{1}{\theta_i} \log \left\{ \frac{1 + \dfrac{\theta_i Q_i}{D_i}}{1 - \dfrac{\theta_i Q_i}{P_i - D_i}} \right\}. \tag{6}$$

The holding cost of ith item in each cycle is $C_{Hi} = C_{1i} G_i(P_i, Q_i)$

where

$$G_i(P_i, Q_i) = \int_0^{T_{1i}} q_i(t) dt + \int_{T_{1i}}^{T_i} q_i(t) dt$$

$$= \frac{P_i - D_i}{\theta_i} \left[T_{1i} + \frac{e^{-\theta_i T_{1i}} - 1}{\theta_i} \right] - \frac{D_i}{\theta_i} \left[\frac{1 - e^{\theta_i (T_i - T_{1i})}}{\theta_i} + (T_i - T_{1i}) \right]. \tag{7}$$

The total quantity of ith item deteriorated per cycle is

$$S_{d_i} = \theta_i G_i(P_i, Q_i). \tag{8}$$

Therefore, the total average profit for i^{th} item is

$$PF_i(P_i, Q_i) = \frac{1}{T_i} \left[P_i T_{1i} \{ s_i - f(P_i) \} - C_{Hi} - S_i - s_i S_{d_i} \right]$$

$$= \frac{1}{T_i} \left[P_i T_{1i} (m_i - 1) f(P_i) - (C_{1i} - s_i \theta_i) G_i(P_i, Q_i) - S_i \right] \quad (9)$$

Total average wastage and total production costs for i^{th} item are respectively

$$WC(P_i, Q_i) = \frac{\theta_i G_i(P_i, Q_i) f(P_i)}{T_i}, \quad (10)$$

$$PC(P_i, Q_i) = P_i T_{1i} f(P_i). \quad (11)$$

(i) Crisp model

Our problem is to (i) maximize the total average profit, (ii) minimize the average wastage cost and (iii) minimize the total production cost under the limitation of total space area i.e.

$$\text{Maximize } PF(P, Q) = \sum_{i=1}^{n} PF(P_i, Q_i), \quad (12)$$

$$\text{Minimize } WC(P, Q) = \sum_{i=1}^{n} WC(P_i, Q_i),$$

$$\text{Minimize } PC(P, Q) = \sum_{i=1}^{n} P_i T_{1i} f(P_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} w_i Q_i \leq W,$$

where $P = (P_1, P_2, \cdots, P_n)^T$, $Q = (Q_1, Q_2, \cdots, Q_n)^T$ are decision vectors with all components $P_i > 0$ and $Q_i > 0$.

(ii) Fuzzy model

When the above total average profit, average wastage cost, total production cost and availability of space area become fuzzy, the said crisp model (12) is transformed to a fuzzy model as:

Maximize PF(P, Q),   (13)

Minimize WC(P, Q),

Minimize PC(P, Q)

Subject to $\sum_{i=1}^{n} w_i Q_i \leq \tilde{W}$,

P, Q are decision vectors as in (12)

4. Multi-objective Mathematical Programming

A general multiple objective non-linear programming problem is of the following form:

Minimize f(x) = [f1(x), f2(x), $\cdots$, fn(x)]

subject to $x \in S$, where $S = [x \ / \ x \in R^n, g_i(x) \leq a_i, h_j(x) = b_j]$.

Here, $x = [x_1, x_2, \cdots, x_n]^T$ is an n-dimensional vector of decision variables, f1(x), f2(x), $\cdots$, fk(x) are k distinct objective functions, and S is the set of feasible solutions. An optimal solution, for a single objective problem is defined as one that minimizes the objective function fi(x) subject to the constraint set $x \in S$. Attempting to define a vector minimal point as one at which all components of the objective function vector f are simultaneously minimized is not an adequate generalisation since such points are seldom attainable. Zimmermann[20] showed that fuzzy programming technique can be used nicely to solve the multi-objective programming problem.

4.1 Fuzzy Programming Technique to Solve Crisp Multi-objective Problem

The above multi-objective programming problem (12) is defined completely in crisp environment. To solve this crisp problem by fuzzy technique we first have to assign two values U_k and L_k as upper and lower bounds of the kth objective for each k = 1, 2, 3. Here, L_k = aspired level of achievement, U_k = higher acceptable level of achievement and $d_k = U_k - L_k$ = the degradation allowance. The steps of the fuzzy programming technique are as follows:

Step-1:

Each objective function PF(P, Q), WC(P, Q) and PC(P, Q) of the multi-objective programming problem (12) is optimized separately subject to the constraints of the problem (12). Let these optimum values be PF*(P^1, Q^1), WC*(P^2, Q^2) and PC*(P^3, Q^3).

Step-2:

At each optimal solution of the three single-objective programming problem solved in step-1 find the value of the remaining objective functions and construct a pay-off matrix of order 3×3 as follows:

	PF(P, Q)	WC(P, Q)	PC(P, Q)
(P^1, Q^1)	PF*(P^1, Q^1)	WC(P^1, Q^1)	PC(P^1, Q^1)
(P^2, Q^2)	PF(P^2, Q^2)	WC*(P^2, Q^2)	PC(P^2, Q^2)
(P^3, Q^3)	PF(P^3, Q^3)	WC(P^3, Q^3)	PC*(P^3, Q^3)

From the Pay-off matrix, find lower bounds L_{PF}, L_{WC}, L_{PC} and upper bounds U_{PF}, U_{WC}, U_{PC} as follows.

L_{PF} = Min{PF(P^1, Q^1), PF(P^2, Q^2), PF(P^3, Q^3)},
L_{WC} = Min{WC(P^1, Q^1), WC(P^2, Q^2), WC(P^3, Q^3)},
L_{PC} = Min{PC(P^1, Q^1), PC(P^2, Q^2), PC(P^3, Q^3)},

and the upper bounds

$U_{PF} = \text{Max}\{PF(P^1, Q^1), PF(P^2, Q^2), PF(P^3, Q^3)\}$,
$U_{WC} = \text{Max}\{WC(P^1, Q^1), WC(P^2, Q^2), WC(P^3, Q^3)\}$,
$U_{PC} = \text{Max}\{PC(P^1, Q^1), PC(P^2, Q^2), PC(P^3, Q^3)\}$.

Step-3:

To solve this crisp problem by Zimmermann [20] method, we take the membership functions $\mu_{PF}(PF(P, Q))$, $\mu_{WC}(WC(P, Q))$, and $\mu_{PC}(PC(P, Q))$ respectively of the objective functions PF(P, Q), WC(P, Q), PC(P, Q) in the linear form as follows:

$$
\mu_{PF} = \begin{cases} 1 & , \quad \text{for} \quad PF(P, Q) > U_{PF}; \\ \dfrac{PF(P, Q) - L_{PF}}{U_{PF} - L_{PF}} & , \quad \text{for} \quad L_{PF} \leq PF(P, Q) \leq U_{PF}; \\ 0 & , \quad \text{for} \quad PF(P, Q) < L_{PF}, \end{cases}
$$

$$
\mu_{WC} = \begin{cases} 1 & , \quad \text{for} \quad WC(P, Q) < L_{WC}; \\ \dfrac{U_{WC} - WC(P, Q)}{U_{WC} - L_{WC}} & , \quad \text{for} \quad L_{WC} \leq WC(P, Q) \leq U_{WC}; \\ 0 & , \quad \text{for} \quad WC(P, Q) > U_{WC}, \end{cases}
$$

$$
\mu_{PC} = \begin{cases} 1 & , \quad \text{for} \quad PC(P, Q) < L_{PC}; \\ \dfrac{U_{PC} - PC(P, Q)}{U_{PC} - L_{PC}} & , \quad \text{for} \quad L_{PC} \leq PC(P, Q) \leq U_{PC}; \\ 0 & , \quad \text{for} \quad PC(P, Q) > U_{PC}. \end{cases}
$$

Step-4:

Using above membership functions formulate and solved the crisp non-linear programming model following the methods due to Zimmermann (1978) and others.

4.2 Crisp Weights

Sometimes decision makers are able to provide crisp relative weights for objective goals to reflect their relative importance. Here, positive crisp weights w^i (i = 1, 2, $\cdots$, m.) for crisp model are used (which can be normalised by taking $\sum_{i=1}^{m} w^i = 1$.

The decision makers assign different weights to reflect their relative importance. To achieve more importance of the objective goal we choose suitable inverse weight in the fuzzy non-linear programming technique. Similarly, in fuzzy inventory model we may choose the smallest of the inverse weighted membership function corresponding to the most important objective goal.

4.3 Fuzzy Weights

When the decision maker can only provide linguistic or imprecise weights (e.g. profit goal is very important, wastage cost goal is moderately important etc.) we may use fuzzy weights according to Narashiman[19]. Here, membership functions of fuzzy weights are introduced to develop a combined measure of the degree to which objective goals are attained.

Let $\mu_{w^i}(\mu_i(x))$ represent the weighted contribution of the ith goal to the overall aggregated objective, where $\mu_{w^i}(\mu_i(x))$ is the membership function corresponding to the fuzzy weights associated with the ith goal. Then by using min operation, the membership function $\mu_D(x)$ of the decision (D) is:

$$\mu_D(x) = \mu_{w^1}(\mu_1(x)) \wedge \mu_{w^2}(\mu_2(x)) \wedge \cdots \wedge \mu_{w^m}(\mu_m(x))$$

$$= \min\{\mu_{w^1}(\mu_1(x)), \mu_{w^2}(\mu_2(x)), \cdots, \mu_{w^m}(\mu_m(x))\}.$$

The maximized decision x* is obtained by:

$$\mu_D(x*) = \max \{\min\{\mu_{w^i}(\mu_i(x))\}.$$

Note that the membership functions of fuzzy weights are functions of the membership function of the goal. The rationality for constructing these membership functions is that the more important the goals are, the higher are the degrees of their membership, and so the higher are the membership grade of their fuzzy weights.

5. Crisp Weighted Models:

If w^1, w^2 and w^3 are the intuitive crisp weights for the profit goal, wastage cost goal and total production cost goal respectively then for different models the problem (12) can be formulated as follows:

Zimmermann's model

Maximize α (14)

subject to

$$w^1\left(\frac{PF(P,Q)-L_{PF}}{U_{PF}-L_{PF}}\right) \geq \alpha, \qquad w^2\left(\frac{U_{WC}-WC(P,Q)}{U_{WC}-L_{WC}}\right) \geq \alpha,$$

$$w^3\left(\frac{U_{PC}-PC(P,Q)}{U_{PC}-L_{PC}}\right) \geq \alpha, \qquad \sum_{i=1}^{n} w_i Q_i \leq W, \qquad 0 \leq \alpha \leq 1$$

where   P, Q are decision vectors as in (12)   and $w^1 + w^2 + w^3 = 1$.

Additive model

Maximize $V(\alpha_1, \alpha_2, \alpha_3) = w^1\alpha_1 + w^2\alpha_2 + w^3\alpha_3$ (15)

subject to

$$\left(\frac{PF(P,Q)-L_{PF}}{U_{PF}-L_{PF}}\right) = \alpha_1, \qquad \left(\frac{U_{WC}-WC(P,Q)}{U_{WC}-L_{WC}}\right) = \alpha_2,$$

$$\left(\frac{U_{PC}-PC(P,Q)}{U_{PC}-L_{PC}}\right) = \alpha_3, \qquad \sum_{i=1}^{n} w_i Q_i \leq W, \qquad 0 \leq \alpha_i \leq 1, \quad i = 1, 2, 3$$

where   P, Q are decision vectors as in (12) and $w^1 + w^2 + w^3 = 1$.

Square additive model

Maximize $V(\alpha_1, \alpha_2, \alpha_3) = w^1\alpha_1{}^2 + w^2\alpha_2{}^2 + w^3\alpha_3{}^2$ (16)

subject to constraints and restrictions as in (15).

Exponential square additive model

Maximize $V(\alpha_1, \alpha_2, \alpha_3) = \left(\alpha_1^{\,2}\right)^{w1} + \left(\alpha_2^{\,2}\right)^{w2} + \left(\alpha_3^{\,2}\right)^{w3}$ \hfill (17)

subject to constraints and restrictions as in (15).

Exponential weighted product model

Maximize $V(\alpha_1, \alpha_2, \alpha_3) = \alpha_1^{\,w1} \alpha_2^{\,w2} \alpha_3^{\,w3}$ \hfill (18)

subject to constraints and restrictions as in (15).

5.1. Fuzzy Non-linear Programming (FNLP) Algorithm to Solve Fuzzy Multi-objective Inventory Model (13)

In many realistic situations, it is difficult to assign precise aspiration levels to objectives and also in some cases, it is not possible to articulate precise boundaries of the constraint(s). In such situation, a fuzzy goal model is more appropriate to represent the problem. In fuzzy set theory, the fuzzy objectives and fuzzy constraints are defined by their membership functions which may be linear or non-linear.

Taking the profit goal as B_0 with tolerance P_{PF}, the wastage goal as C_0 with tolerance P_{WC}, production cost goal as D_0 with tolerance P_{PC} and space constraint goal as W with tolerance P_W, the linear membership functions - $\mu_{PF}(P, Q)$, $\mu_{WC}(P, Q)$, $\mu_{PC}(P, Q)$ and $\mu_W(Q)$ for three objectives and one constraint are as follows:

$$\mu_{PF} = \begin{cases} 0 & , & \text{for} \quad PF(P,Q) < B_0 - P_{PF}; \\ 1 - \dfrac{B_0 - PF(P,Q)}{P_{PF}} & , & \text{for} \quad B_0 - P_{PF} \le PF(P,Q) \le B_0; \\ 1 & , & \text{for} \quad PF(P,Q) > B_0, \end{cases}$$

$$\mu_{WC} = \begin{cases} 1 & , & \text{for} \quad WC(P,Q) < C_0; \\ 1 - \dfrac{WC(P,Q) - C_0}{P_{WC}} & , & \text{for} \quad C_0 \le WC(P,Q) \le C_0 + P_{WC}; \\ 0 & , & \text{for} \quad WC(P,Q) > C_0 + P_{WC}, \end{cases}$$

$$\mu_{PC} = \begin{cases} 1 & , & \text{for} \quad PC(P,Q) < D_0; \\ 1 - \dfrac{PC(P,Q) - D_0}{P_{PC}} & , & \text{for} \quad D_0 \le PC(P,Q) \le D_0 + P_{PC}; \\ 0 & , & \text{for} \quad PC(P,Q) > D_0 + P_{PC}, \end{cases}$$

$$\text{and} \quad \mu_{W} = \begin{cases} 1 & , & \text{for} \quad \sum_{i=1}^{n} w_i Q_i < W; \\ 1 - \dfrac{\sum_{i=1}^{n} w_i Q_i - W}{P_W} & , & \text{for} \quad W \le \sum_{i=1}^{n} w_i Q_i \le W + P_W; \\ 0 & , & \text{for} \quad \sum_{i=1}^{n} w_i Q_i > W + P_W. \end{cases}$$

Crisp weighted fuzzy models

In this case, in addition to the weights, w^1, w^2, w^3 attributed to the objectives, if w^4 is the intuitive crisp weight attached to the space constraint goal, then different models of equation (13) are as follows:

Zimmermann's model

Maximize α (19)

subject to

$$w^1\left(1-\frac{B_0-PF(P,Q)}{P_{PF}}\right)>\alpha, \qquad w^2\left(1-\frac{WC(P,Q)-C_0}{P_{WC}}\right)>\alpha,$$

$$w^3\left(1-\frac{PC(P,Q)-D_0}{P_{PC}}\right)>\alpha, \qquad w^4\left(1-\frac{\sum_{i=1}^{n}w_iQ_i-W}{P_W}\right)>\alpha,$$

$$0\le\alpha\le 1,$$

where   P, Q are the decision vectors as in (12)

and $w^1+w^2+w^3+w^4=1.$

Additive model

Maximize $V(\alpha_1,\alpha_2,\alpha_3,\alpha_4)=w^1\alpha_1+w^2\alpha_2+w^3\alpha_3+w^4\alpha_4$ (20)

subject to

$$\left(1-\frac{B_0-PF(P,Q)}{P_{PF}}\right)=\alpha_1, \qquad \left(1-\frac{WC(P,Q)-C_0}{P_{WC}}\right)=\alpha_2,$$

$$\left(1-\frac{PC(P,Q)-D_0}{P_{PC}}\right)=\alpha_3, \qquad \left(1-\frac{\sum_{i=1}^{n}w_iQ_i-W}{P_W}\right)=\alpha_4,$$

$$0\le\alpha_i\le 1, \quad i=1,2,3,4,$$

where   P, Q are the decision vectors as in (12)

and $w^1+w^2+w^3+w^4=1.$

Square additive model

Maximize $V(\alpha_1,\alpha_2,\alpha_3,\alpha_4)=w^1\alpha_1^2+w^2\alpha_2^2+w^3\alpha_3^2+w^4\alpha_4^2$ (21)

subject to constraints and restrictions as in (20).

Exponential square additive model

$$\text{Maximize } V(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \left(\alpha_1^{\,2}\right)^{w^1} + \left(\alpha_2^{\,2}\right)^{w^2} + \left(\alpha_3^{\,2}\right)^{w^3} + \left(\alpha_4^{\,2}\right)^{w^4} \quad (22)$$

subject to constraints and restrictions as in (20).

Exponential weighted product model

$$\text{Maximize } V(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \alpha_1^{\,w^1} \alpha_2^{\,w^2} \alpha_3^{\,w^3} \alpha_4^{\,w^4} \quad (23)$$

subject to constraints and restrictions as in (20).

Fuzzy weighted models

      For fuzzy weights, we consider model-1 (Zimmermann's model) only. In this case, if w^1, w^2, w^3 and w^4 are the intuitive fuzzy weights for the profit goal, wastage cost goal, total production cost goal and constraint goal respectively, and then the fuzzy weighted models of the problems (12) and (13) can be written as:

For crisp model (12) :

$$\text{Maximize } \alpha \quad (24)$$

subject to

$$\mu_{w1}\left(\frac{PF(P,Q) - L_{PF}}{U_{PF} - L_{PF}}\right) \geq \alpha, \qquad \mu_{w2}\left(\frac{U_{WC} - WC(P,Q)}{U_{WC} - L_{WC}}\right) \geq \alpha,$$

$$\mu_{w3}\left(\frac{U_{PC} - PC(P,Q)}{U_{PC} - L_{PC}}\right) \geq \alpha, \qquad \sum_{i=1}^{n} w_i Q_i \leq W,$$

$$0 \leq \alpha \leq 1$$

where and P, Q are the decision vectors as in (12).

For fuzzy model (13) :

Maximize α (25)

subject to

$$\mu_{w1}\left(1-\frac{B_0-PF(P,Q)}{P_{PF}}\right) \geq \alpha, \qquad \mu_{w2}\left(1-\frac{WC(P,Q)-C_0}{P_{WC}}\right) \geq \alpha,$$

$$\mu_{w3}\left(1-\frac{PC(P,Q)-D_0}{P_{PC}}\right) \geq \alpha., \qquad \mu_{w4}\left(1-\frac{\sum_{i=1}^{n} w_i Q_i - W}{P_W}\right) \geq \alpha,$$

$0 \leq \alpha \leq 1,$
where   P, Q are the decision vectors as in (12).

6. Illustration of the Model

To illustrate the above crisp model (12) we assume the following input data shown in table -1.

Table-1 : Input data for crisp model

Items	S_i(\$)	H_i(\$)	D_i	m_i	θ_i	r_i	g_i	b_i	β_i	w_i
1	200	8	300	1.2	0.05	85	5000	2	0.0002	4
2	250	6	250	1.18	0.03	90	4000	2	0.0005	3

W = 240 sq. ft.

For the above data, the following pay-off matrix (cf. Table-2) is constructed and then the optimum results for the different representations of the crisp inventory model i.e. (14)-(18) are presented in the tables 3 - 7 respectively.

Table - 2 : Pay-off matrix

	PF(P, Q)	WC(P, Q)	BC(P, Q)
(P_1, Q_1)	10549.83	146.04	245736.0
(P_2, Q_2)	8794.45	127.22	16174.43
(P_3, Q_3)	9022.18	146.50	10197.11

The optimal results of the crisp weighted models are:

Table -3 : Zimmermann's model

Case	w^1	w^2	w^3	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/3	1/3	1/3	9425.37	139.57	15145.40	434.54	1050.1	27.64	36.88	221.20
2	0.2	0.5	0.3	9606.29	140.56	1712280	402.75	1062.7	27.41	37.33	221.63
3	0.5	0.2	0.3	9569.29	133.22	114541.0	504.22	260.48	28.27	38.06	227.28
4	0.5	0.3	0.2	9136.89	140.23	14380.91	1229.2	422.77	25.91	39.98	223.56

Table - 4 : Additive model

Case	w^1	w^2	w^3	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/3	1/3	1/3	10322.44	134.72	101354.6	322.23	265.83	28.40	37.89	227.26
2	0.6	0.2	0.2	10370.85	135.25	101114.4	322.91	269.22	28.52	37.88	227.71
3	0.2	0.6	0.2	9951.41	131.68	69798.78	384.91	269.56	28.09	37.41	224.60
4	0.2	0.2	0.6	10366.95	135.18	101349.7	324.04	268.50	28.49	37.90	227.65

Table - 5 : Square additive model

Case	w^1	w^2	w^3	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/3	1/3	1/3	9758.76	146.50	179866.0	1072.1	256.41	30.07	37.10	231.58
2	0.6	0.2	0.2	10377.60	135.46	101356.6	320.62	270.84	28.65	37.75	227.87
3	0.2	0.6	0.2	10260.85	134.33	101352.2	341.93	264.45	28.36	37.86	226.99
4	0.2	0.2	0.6	10366.95	135.18	101349.7	324.04	268.50	28.49	37.90	227.65

Table - 6 : Exponential square additive model

Case	w^1	w^2	w^3	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/3	1/3	1/3	10343.52	134.90	101345.1	328.67	266.71	28.43	37.90	227.40
2	0.6	0.2	0.2	10369.75	135.23	100334.5	322.67	269.64	28.52	37.87	227.68
3	0.2	0.6	0.2	10227.06	134.11	100454.1	347.01	264.17	28.34	37.82	226.84
4	0.2	0.2	0.6	10021.68	135.12	100394.2	338.09	271.17	28.09	37.92	227.14

Table - 7 : Product model

Case	w^1	w^2	w^3	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/3	1/3	1/3	10162.84	133.72	100954.8	342.23	285.86	28.48	37.88	227.26
2	0.6	0.2	0.2	10221.85	134.15	101174.4	352.91	289.23	28.12	38.01	226.53
3	0.2	0.6	0.2	9953.46	132.28	79398.72	364.94	301.56	28.01	37.32	223.60
4	0.2	0.2	0.6	9838.40	130.04	37408.33	356.93	313.95	27.87	37.41	222.91

Here, optimum results of the crisp model by five different methods are presented. In each method, four different cases have been made out depending upon the importance given among three different objectives. In case -1, equal weightage to all objectives ; in case -2, more importance to profit goal than the other two objectives - wastage cost and production cost ; in case -3, more care to minimization of wastage cost than the others, and similarly in case -4, production cost received more weightage than others. As expected, case -2 model gives maximum return when maximum attention is paid to the profit goal objective. Similarly case-3 and case-4 give better results if the decision maker gives maximum importance to the minimization of wastage cost and production cost.

Crisp weighted fuzzy models

For fuzzy model, we consider the input data shown in Table-1 alongwith the following fuzzy data:

$\widetilde{PF}$ =($9000, $12000), $\widetilde{WC}$ =($100, $150), $\widetilde{PC}$ =($40000, $60000), $\widetilde{SC}$ = (230sq.ft, 260sq.ft). For these data, the optimum results of the fuzzy models (20) - (23) are:

Table - 8 : Additive model

Case	w^1	w^2	w^3	w^4	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/4	1/4	1/4	1/4	10016.63	131.16	47845.19	348.46	292.98	28.67	36.20	223.26
2	0.7	0.1	0.1	0.1	10023.95	131.25	47874.36	343.56	297.30	28.66	36.20	223.35
3	0.1	0.7	0.1	0.1	9678.40	129.96	47896.83	428.22	279.05	28.17	36.77	222.99
4	0.1	0.1	0.1	0.7	9245.57	131.46	47906.46	376.63	274.93	29.01	35.53	222.61

Table - 9 : Square additive model

Case	w^1	w^2	w^3	w^4	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/4	1/4	1/4	1/4	10003.43	131.05	48479.43	354.86	288.36	28.59	36.30	223.24
2	0.7	0.1	0.1	0.1	10068.89	131.56	51346.46	340.77	293.30	28.65	36.34	223.63
3	0.1	0.7	0.1	0.1	9962.53	130.07	49881.06	358.87	289.24	28.54	36.25	223.62
4	0.1	0.1	0.1	0.7	9849.40	130.62	49640.01	392.80	279.14	28.94	35.95	222.98

Table - 10 : Exponential square additive model

Case	w^1	w^2	w^3	w^4	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/4	1/4	1/4	1/4	9890.61	130.43	40000.00	356.34	350.05	28.63	36.06	222.69
2	0.7	0.1	0.1	0.1	10124.7	132.05	56412.35	337.50	288.34	28.82	36.23	223.98
3	0.1	0.7	0.1	0.1	9878.72	130.09	52122.37	352.51	327.16	28.71	36.11	223.01
4	0.1	0.1	0.1	0.7	10002.6	131.02	5000.11	327.58	295.38	28.89	36.25	222.62

Table – 11: Product model

Case	w^1	w^2	w^3	w^4	PF	WC	PC	P_1	P_2	Q_1	Q_2	SC
1	1/4	1/4	1/4	1/4	9892.31	130.43	40000.00	354.94	306.32	28.65	36.04	222.70
2	0.7	0.1	0.1	0.1	9955.05	131.16	43788.46	343.60	308.55	29.30	35.29	223.04
3	0.1	0.7	0.1	0.1	9721.42	129.46	40000.00	426.43	286.39	28.08	36.92	222.88
4	0.1	0.1	0.1	0.7	9633.12	129.49	40000.00	425.74	287.33	28.23	36.52	222.48

Here, results have been presented for the fuzzy model by four different methods with the different crisp weights to the fuzzy objectives and the fuzzy constraint. As before, four cases are presented with different priorities to objectives and constraint and the results almost follows the pattern of crisp results presented in Tables – (3 - 7).

Now, we consider fuzzy weights for both crisp and fuzzy objectives of Zimmermann's model and the optimum results are displayed in Table -12 along with the fuzzy input weights.

Table - 12 : Fuzzy weighted model

Zimmer-mann	$\widetilde{w}^1$	$\widetilde{w}^2$	$\widetilde{w}^3$	PF	WC	PC	P_1	P_2	Q_1	Q_2
Crisp	[0, .5]	[.5, 1]	[.5, 1]	9492.44	130.43	28483.38	354.51	412.76	27.74	37.18
Fuzzy	[0, .5]	[.5, 1]	[.5, 1]	10405.2	131.56	51346.46	340.77	293.30	28.65	36.34

Here, fuzzy model gives more profit than the crisp one though it accounts for more wastage and production costs.

7. Concluding Remarks

Till now, in the field of inventory, very few multi-objective models with two objectives only are available in crisp environment. To the best of our knowledge, no inventory model with three or more objectives have been formulated even in crisp environment. Here, for the first time, inventory models with three objectives have been presented in both crisp and fuzzy environments and solved by FNLP and different fuzzy goal programming techniques. The results have been presented with different types of weights admissible to objectives. Each weight, which implies the relative importance of the objective goals, can be determined through the practical experiences. Though the problem has been formulated in the field of inventory, the present methodology in formulation and solution can be adopted for a fuzzy non-linear decision making problem in any discipline. Moreover, in this paper, model has been formulated with constant demand, infinite replenishment, and no shortages. The present analysis can be easily extended to other types of inventory models with finite replenishment, fully or partially backlogged shortages, fixed time horizon, etc. Hence,

the determination of the exact weights for the multi-objective fuzzy inventory models and their solutions may be the topics of the future research.

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