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The Orlicz Space of Entire Sequence of Fuzzy Numbers*

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Abstract

In this paper, we introduce and study Orlicz space of entire sequence of fuzzy numbers generated by non negative regular matrix $A=(a_{nk})(n,k=1,2,\cdots)$.

Keywords and Phrases: *Fuzzy numbers, Orlicz space, Entire sequence, Analytic sequence, Paranorm.*

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1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh[13] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces , similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming.

Orlicz [15] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [16] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(1 \leq p < \infty)$. , where $w = \{\text{all complex sequences}\}$.

In this paper, we introduce and examine the concepts of Orlicz space of entire sequence of fuzzy numbers generated by non negative regular matrix.

Definitions and preliminaries

Let D be the set of all bounded intervals $A = \left[\underline{A}, \overline{A}\right]$ on the real line R .

For $A, B \in D$, define $A \leq B$ if and only if $\underline{A} \leq \underline{B}$ and $\overline{A} \leq \overline{B}$, $d(A, B) = \max\{\underline{A} - \underline{B}, \overline{A} - \overline{B}\}$.

Then it can be easily see that d defines a metric on D (cf[1]) and (D, d) is complete metric space.

A fuzzy number is fuzzy subset of the real line R which is bounded, convex and normal. Let $L(R)$ denote the set of all fuzzy numbers which are upper semi continuous and have compact support, i.e. if $X \in L(R)$ then for any $\alpha \in [0,1]$, X^α is compact where

$$X^\alpha = \begin{cases} t : X(t) \geq \alpha \text{ if } 0 < \alpha \leq 1, \\ t : X(t) > 0 \text{ if } \alpha = 0. \end{cases}$$

For each $0 < \alpha \leq 1$, the $\alpha-$level set X^α is a nonempty compact subset of R. The linear structure of $L(R)$ includes addition $X + Y$ and scalar multiplication λX, (λ a scalar) in terms of α-level sets, by $[X+Y]^\alpha = [X]^\alpha + [Y]^\alpha$ and $[\lambda X]^\alpha = \lambda [X]^\alpha$, for each $0 \leq \alpha \leq 1$.

Define a map $\overline{d} : L(R) \times L(R) \rightarrow R$ by $\overline{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha)$.

For $X, Y \in L(R)$ define $X \le Y$ if and only if $X^{\alpha} \le Y^{\alpha}$ for any $\alpha \in [0,1]$. It is known that $(L(R), \bar{d})$ is a complete metric space (cf [7]).

A sequence $X = (X_k)$ of fuzzy numbers is a function X from the set N of natural numbers into $L(R)$. The fuzzy number X_n denotes the value of the function at $n \in N$ and is called the n^{th} term of the sequence.

We denote by $w(F)$ the set of all sequences $X = (X_k)$ of fuzzy numbers.

Recall that ([15],[21]) an Orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0, M(x) > 0,$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \le M(x) + M(y),$ then this function is called modulus function.

In this paper we define Orlicz space of entire sequence of fuzzy numbers by using regular matrices $A = (a_{nk}), (n,k = 1,2,3,\cdots)$. By the regularity of A we mean that the matrix which transform convergent sequence into a convergent sequence leaving the limit (c.f.Maddox [28]).

2. Orlicz space of entire sequence

Let $X = (X_K)$ be a sequence of fuzzy numbers, let $A = (a_{nk})(n,k = 1,2,3,\cdots)$ be a non negative regular matrix and M be a Orlicz function. In this paper we define the following

$$\Gamma_M[F,A,p] = \left\{ X = (X_k) : \sum_k a_{nk} \left[\bar{d} \left(M \left(\frac{|X_k|^{1/k}}{\rho}, 0 \right) \right) \right]^{p_k} \to 0 \text{ as } k \to \infty, \text{ for some } \rho > 0 \right\}$$

$$\wedge_M[F,A,p] = \left\{ X = (X_k) : \sup_{(n)} \left(\sum_k a_{nk} \left[\bar{d} \left(M \left(\frac{|X_k|^{1/k}}{\rho}, 0 \right) \right) \right]^{p_k} \right) < \infty \text{ , for some } \rho > 0 \right\}$$

and call them respectively the spaces of strongly A-Orlicz space of entire sequences and strongly A-Orlicz space of analytic sequences of fuzzy numbers $X = (X_k)$. We can specialize these spaces as follows.

If A = I, the unit marix, then we get another set of new sequence spaces for fuzzy number, for some arbitrarily fixed $\rho > 0$,

$$\Gamma_M[F,p]=\left\{X=(X_k):\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\to 0 \text{ as } k\to\infty\right\}$$

$$\wedge_M[F,p]=\left\{X=(X_k):\sup_k\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}<\infty\right\};$$

which on further taking $p_k=p$ for all k , are reduced to $\Gamma_M[F,p]$ and $\wedge_M[F,p]$ respectively .

$$\Gamma_M[F,p]=\left\{X=(X_k):\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p}\to 0 \text{ as } k\to\infty\right\}$$

$$\wedge_M[F,p]=\left\{X=(X_k):\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p}<\infty\right\};$$

and taking $p_k=1$ for all k , are reduced to $\Gamma_M[F]$ and $\wedge_M[F]$ respectively which are called The Orlicz space of entire sequences of fuzzy numbers and Orlicz space of analytic sequences of fuzzy numbers.

If $A=(a_{nk})$ is a Cesaro matrix of order 1, ie

$$a_{nk}=\begin{cases}\frac{1}{n}, & k\le n\\ 0, & k>n\end{cases}$$

then we get

$$\Gamma_M(F,p)=\left\{X=(X_k):\frac{1}{n}\sum_{k=1}^{n}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\to 0 \text{ as } k\to\infty\right\}$$

$$\wedge_M(F,p)=\left\{X=(X_k):\sup_{(n)}\frac{1}{n}\sum_{k=1}^{n}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}<\infty\right\}$$

and further on taking $p_k = p$ for all k, these are reduced to $\Gamma_M(F,p)$ and $\wedge_M(F,p)$,

$$\Gamma_M(F,p) = \left\{ X = (X_k) : \frac{1}{n}\sum_{k=1}^{n}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p} \to 0 \text{ as } n \to \infty \right\}$$

$$\wedge_M(F,p) = \left\{ X = (X_k) : \sup_{(n)}\left(\frac{1}{n}\sum_{k=1}^{n}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p}\right) < \infty \right\}$$

A metric $\bar{d}$ on $L(R)$ is said to be a translation invariant if $\bar{d}(X+Z,Y+Z) = \bar{d}(X,Y)$ for $X,Y,Z \in L(R)$.

Proposition 2.1. If $\bar{d}$ is a translation invariant metric on $L(R)$ then

$$(i)\ \bar{d}(X+Y,0) \leq \bar{d}(X,0) + \bar{d}(Y,0).$$
$$(ii)\ \bar{d}(\lambda X,0) \leq |\lambda|\,\bar{d}(X,0),\ |\lambda| > 1.$$

Theorem 2.2. $\Gamma_M(F,p)$ *is a complete metric space under the metric*

$$d(X,Y) = \sup_{(n)}\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{|X_k - Y_K|^{\frac{1}{k}}}{\rho}\right),0\right)\right]^{p},\ \textit{where}$$

$X = (X_k) \in \Gamma_M(F,p)$ and $Y = (Y_k) \in \Gamma_M(F,p)$ are the sequence of *sequence of fuzzy numbers.*

Proof. Let $\{X^{(n)}\}$ be a cauchy sequence in $\Gamma_M(F,p)$.

Then given any $\varepsilon > 0$ there exists a positive integer N depending on ε such that $d(X^{(n)}, X^{(m)}) < \varepsilon,\ \forall n \geq N$ and $\forall m \geq N$.

Hence

$$\sup_{(n)}\left(\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k^{(n)}-X_k^{(m)}\right|^{1/k}}{\rho},0\right)\right)\right)^p<\varepsilon \quad \forall\ \mathrm{n}\geq \mathrm{N}\ \text{ and } \forall\ \mathrm{m}\geq \mathrm{N}.$$

Consequently $\left\{X_k^{(n)}\right\}$ is a cauchy sequence in the metric space $L(R)$.

But $L(R)$ is complete . So , $X_k^{(n)}\to X_k$ as $n\to\infty$.

Hence there exists a positive integer n_0 such that

$$\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k^{(n)}-X_k^{(m)}\right|^{1/k}}{\rho},0\right)\right)\right]^p<\varepsilon\ \ \forall\ \ \mathrm{n}\geq \mathrm{n}_0.$$

In particular, we have

$$\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k^{(n_0)}-X_k\right|^{1/k}}{\rho},0\right)\right)\right]^p<\varepsilon\ .$$

Now

$$\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k\right|^{1/k}}{\rho},0\right)\right)\right]^p\leq\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k-X_k^{(n_0)}\right|^{1/k}}{\rho},0\right)\right)\right]^p+\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k^{(n_0)}\right|^{1/k}}{\rho},0\right)\right)\right]^p$$

$$\leq\varepsilon+0 \text{ as } n\to\infty.$$

Thus

$$\left[\frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(M\left(\frac{\left|X_k\right|^{1/k}}{\rho},0\right)\right)\right]^p<\varepsilon \text{ as } n\to\infty\ .$$

That is $\left(X_k\right)\in\Gamma_M\left(F,p\right)$.

Therefore $\Gamma_M(F,p)$ is a complete metric space.

This completes the proof.

Theorem 2.3. *If $\bar{d}$ transition invariant metric and M is a modulus function, that $\Gamma_M(F,p)$ is a linear set over the set of complex numbers.*

Proof. $\bar{d}$ translation invariant implies that

$(i)\ \bar{d}(X+Y,0)\le \bar{d}(X,0)+\bar{d}(Y,0)$

$(ii)\ \bar{d}(\lambda X,0)\le |\lambda|\,\bar{d}(X,0)$, λ a scalar.

Let $X=(X_k)$, $\mathrm{Y}=(\mathrm{Y_k})\in\Gamma_{\mathrm{M}}(F,p)$ and $\alpha,\beta\in \mathrm{C}$.

In order to prove the result , we need to find some ρ_3 such that

$$\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{|(\alpha X_k+\beta Y_k)|^{1/k}}{\rho_3},0\right)\right)\right]^p \to 0 \text{ as } k\to\infty \ . \tag{2.1}$$

Since $X=(X_k), Y=(Y_k)\in\Gamma_M(F,p)$, there exists some positive ρ_1 and ρ_2 such that

$$\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho_1},0\right)\right)\right]^p \to 0 \text{ as } k\to\infty \text{ and}$$

$$\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{|Y_k|^{1/k}}{\rho_2},0\right)\right)\right]^p \to 0 \text{ as } k\to\infty \ . \tag{2.2}$$

Since *M* is a non decreasing modulus function , we have

$$\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|(\alpha X_k+\beta Y_k)\right|^{1/k}}{\rho_3},0\right)\right)\right]^p \le \sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|\alpha X_k\right|^{1/k}}{\rho_3}+\frac{\left|\beta Y_k\right|^{1/k}}{\rho_3},0\right)\right)\right]^p$$

$$\le \sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{|\alpha|^{1/k}\left|X_k\right|^{1/k}}{\rho_3}+\frac{|\beta|^{1/k}\left|Y_k\right|^{1/k}}{\rho_3},0\right)\right)\right]^p$$

$$\le \sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{|\alpha|\left|X_k\right|^{1/k}}{\rho_3}+\frac{|\beta|\left|Y_k\right|^{1/k}}{\rho_3},0\right)\right)\right]^p$$

Take ρ_3 such that

$$\frac{1}{\rho_3}=\min\left\{\frac{1}{|\alpha|^p}\frac{1}{\rho_1},\frac{1}{|\beta|^p}\frac{1}{\rho_2}\right\} \tag{2.3}$$

Then

$$\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|(\alpha X_k+\beta Y_k)\right|^{1/k}}{\rho_3},0\right)\right)\right]^p \le \sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|X_k\right|^{1/k}}{\rho_1}+\frac{\left|Y_k\right|^{1/k}}{\rho_2},0\right)\right)\right]^p$$

$$\le \sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|X_k\right|^{1/k}}{\rho_1},0\right)\right)\right]^p+\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|Y_k\right|^{1/k}}{\rho_2},0\right)\right)\right]^p$$

$$\to 0 \ (\text{by } (2.2))$$

Hence $\sum_{k=1}^{n}\frac{1}{n}\left[\bar{d}\left(M\left(\frac{\left|(\alpha X_k+\beta Y_k)\right|^{1/k}}{\rho_3},0\right)\right)\right]^p \to 0$ as $k\to\infty$. (2.4)

So $(\alpha X+\beta Y)\in\Gamma_M(F,p)$.

Therefore $\Gamma_M(F,p)$ is linear.

This completes the proof.

3. Main Results

Theorem 2.4.
If $X=\left(X_k\right)$ be a sequence of fuzzy numbers . Then $\Gamma_M\left(F,A,p\right)$ *complete with respect to the topology generated by the paranorm* h defined by

$$h\left(X\right)=\sup_{(k)}\left(\sum a_{nk}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\right)^{1/\mu}$$

where μ=max$\left\{1,\sup_{(k)}\left(p_k/\mu\right)\right\}$, where $\bar{d}$ translation invariant .

Proof . Clearly $h(\theta)=0, h(-X)=h(X)$. It con also be seen easily that $h(X+Y)\le h(X)+h(Y)$ for X = (X_k), Y = (Y_k) in $\Gamma_M(F,A,p)$, since $\bar{d}$ translation invariant.

Now for any scalar λ, we have $|\lambda|^{p_k/\mu}<\max\{1,\sup|\lambda|\}$, so that $h(\lambda X)<\max\{1,\sup|\lambda|\}$, λ fixed implies $\lambda X\to\theta$. New let $\lambda\to\theta$, X fixed. for $\sup|\lambda|<1$ we have

$$\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\right]^{1/\mu}<\varepsilon \quad \text{for } N>N\left(\varepsilon\right).$$

Also, for $1\le n\le N$, since $\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\right]^{1/\mu}<\varepsilon$, there exists m such that $\left[\sum_{k=m}^{\infty} a_{nk}\left[\bar{d}\left(M\left(\frac{|\lambda X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\right]^{1/\mu}<\varepsilon$.

Taking λ small enough we then have

$$\left[\sum_{k=m}^{\infty} a_{nk}\left[\bar{d}\left(M\left(\frac{|\lambda X_k|^{1/k}}{\rho},0\right)\right)\right]^{p_k}\right]^{1/\mu}<2\varepsilon \text{ for all } k.$$

Hence $h(\lambda X) \to 0$ as $\lambda \to 0$. Therefore h is a paranorm on $\Gamma_M(F, A, p)$.

To show the completeness, let $\left(X^{(i)}\right)$ be a Cauchy sequence in $\Gamma_M(F, A, p)$.

Then for a given $\varepsilon > 0$ there is $r \in N$ such that

$$\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{\left|X^{(i)} - X^{(j)}\right|^{1/k}}{\rho}, 0\right)\right)\right]^{p_k}\right]^{1/\mu} < \in \text{ for all } i, j > r. \tag{2.5}$$

Since $\bar{d}$ is a translation invariant , So (2.5) implies that

$$\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{\left|X_k^{(i)} - X_k^{(j)}\right|^{1/k}}{\rho}, 0\right)\right)\right]^{p_k}\right]^{1/\mu} < \in \text{ for all } i, j > r \text{ and each } n. \tag{2.6}$$

Hence $\left[\bar{d}\left(M\left(\frac{\left|X_k^{(i)} - X_k^{(j)}\right|^{1/k}}{\rho}, 0\right)\right)\right] < \in$ for all $i, j > r$.

Therefore $\left(X^{(i)}\right)$ is a Cauchy sequence in $L(R)$.

Since L(R) is complete, $\lim_{j\to\infty} X_k{}^j = X_k$, say.

Fixing $r_0 \geq r$ and letting $j \to \infty$, we obtain (2.6) that

$$\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{\left|X_k^{(i)} - X_k\right|^{1/k}}{\rho}, 0\right)\right)\right]\right] < \in \text{ for all } r_0 \geq r, \tag{2.7}$$

since $\bar{d}$ is a translation invariant . Hence

$$\left[\sum a_{nk}\left[\bar{d}\left(M\left(\frac{\left|X^{(i)} - X\right|^{1/k}}{\rho}, 0\right)\right)\right]^{p_k}\right]^{1/\mu} < \in$$

(i.e) $X^{(i)} \to X$ in $\Gamma(F, A, p)$. It is easy to see that $X \in \Gamma_M(F, A, p)$.

Hence $\Gamma_M(F, A, p)$ is complete.

This completes the proof.

The completeness of $\wedge_M(F, A, p)$ can be similarly obtained.

Theorem 2.5. *Let* $A=(a_{nk})(n,k=1,2,3,\cdots)$ *be an infinite matrix with complex entries. Then* $A\in(\Gamma:\Gamma_M(F,A,p))$ *if and only if given* $\varepsilon>0$ *there exists* $\mathrm{M}=\mathrm{M}(\varepsilon)>0$ *such that* $|a_{nk}|<\varepsilon^n\mathrm{M}^k\ (n,k=1,2,3,\cdots)$, *where* $X=(X_k)$ *be a sequence of fuzzy numbers and* $\overline{d}$ *translation invariant.*

Proof.

Let $X=(X_k)\in\Gamma$ and let $Y_n=\left(\sum_{k=1}^{\infty}a_{nk}\overline{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)^p\right)$, $(n=1,2,3,\ldots)$. Then $(Y_n)\in\Gamma$ if and only if given any $\varepsilon>0\ \exists\ \mathrm{M}=\mathrm{M}(\varepsilon)>0$ such that $|a_{nk}|<\varepsilon^n\mathrm{M}^k$ by using Theorem 4 of [26]. Thus $A\in(\Gamma:\Gamma_M(F,A,p))$ if and only if the condition holds. This completes the proof.

Theorem 2.6. *Let* $A=(a_{nk})$ *transforms* Γ *into* $\Gamma_M(F,A,p)$ *then* $\lim_{n\to\infty}(a_{nk})q^n=0$ *for all integers q>0 and each fixed* $k=1,2,3,\cdots$. *where* $X=(X_k)$ *be a sequence of fuzzy numbers and* $\overline{d}$ *translation invariant.*

Proof. Let $Y_n=\left(\sum_{k=1}^{\infty}a_{nk}\overline{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)^p\right)(n=1,2,3,\cdots)$, for mally.

Let $(X_k)\in\Gamma$ and $(Y_n)\in\Gamma_M(F,A,p)$. Take $(X_k)=\delta^k=(0,0,0,\cdots,1,0,0,\cdots)$, 1 in the k^{th} place and zero's elsewhere. Then $(X_k)\in\Gamma$. Hence $\sum_{k=1}^{\infty}|a_{nk}|q^n<\infty$ for every positive *q*. In particular $\lim_{n\to\infty}(a_{nk})q^n=0$ for all positive integers q and each fixed $k=1,2,3,\cdots$.

This completes the proof.

Theorem2.7. I*f* $A=(a_{nk})$ *transform* $\Gamma_M(F, A, p)$ *into* Γ . *Then* $\lim_{n\to\infty}(a_{nk})q^n=0$ $\forall$ *positive integers q, where* $X=(X_k)$ *be a sequence of fuzzy numbers and* $\overline{d}$ *translation invariant.*

Proof. Let $t_n = \left(\sum_{k=1}^{\infty} a_{nk}\overline{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)^p\right)$ with $(X_k)\in\Gamma_M$, $(t_n)\in\Gamma$.

$$s_n = \left(\sum_{k=1}^{\infty} a_{nk}\overline{d}\left(M\left(\frac{|0|^{1/k}}{\rho},0\right)\right)^p\right) \quad (s_n)\in\Gamma .$$

Then $Y_n = (t_n - s_n) = \left(\sum_{k=1}^{\infty} a_{nk}\overline{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)\right)^p\right)$, and $\overline{d}\left(M\left(\frac{|X_k|^{1/k}}{\rho},0\right)^p\right)\in\Gamma.$

Hence $(Y_n)\in\Gamma.$ There fore $(a_{nk})q^n \to 0$ as $n\to\infty\forall k$, by [27].

This completes the proof .

Theorem 2.8. I*f* $A=(a_{nk})$ *transform* $\Gamma_M(F, A, p)$ *into* $\Gamma_M(F, A, p)$ *then* $a_{nk}q^n \to 0$ *where* $X=(X_k)$ *be a sequence of fuzzy numbers and* $\overline{d}$ *translation invariant.*

Proof. From Theorem (2.6) and (2.7) we have $a_{nk}q^n \to 0$ as $n\to\infty$, for all positive integers q and $\forall k$.

This completes the proof.

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