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A Replenishment Policy for Non-instantaneous Deteriorating Inventory System with Partial Backlogging*

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Abstract

This paper deals with the development of an inventory model for non-instantaneous deteriorating items. This paper investigates an instantaneous replenishment inventory model for the above type of items under cost minimization, considering stock-dependent consumption rate under inflation and time discounting over a finite planning horizon. In the proposed model, shortages are allowed and partially backlogged. We also show that the total cost function is convex. To determine the optimal order quantity and the optimal interval of the total cost function, a solution algorithm is presented. Finally, the model is illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to various parameters of the system is carried out and the results are discussed in detail.

Keywords and Phrases: *Inventory, Stock-dependent demand, Inflation, Partial backlogging, Non-instantaneous deterioration, Shortages.*

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1. Introduction

It has been observed in supermarkets that the demand rate is usually influenced by the amount of the stock level, that is, the demand rate may go up or down with the on-hand stock level. As pointed out by Levin et.al. [11] "at times, the presence of inventory has a motivational effect on the people around it. It is common belief that large piles of goods displayed in a supermarket will lead the customers to buy more". Silver and Peterson [22] also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Due to these facts, a number of authors have developed the Economic Order Quantity (EOQ) models that focused on stock-dependent demand rate patterns. Gupta and Vrat [8] assumed that the demand rate was a function of initial stock level. Mandal and Phaujdar [12] then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Some of the works in this area may refer to Padmanabhan and Vrat [15], Sarker et al [20], Mandal and Maiti [13], Dye [4] and Wu et.al. [26].

It is important to control and maintain the inventories of deteriorating items for the modern corporation. In general, deterioration is defined as the damage, spoilage, dryness, vaporization, etc., that result in decrease of usefulness of the original one. In recent years, inventory problems for deteriorating items have been widely studied after Ghare and Schrader [5] .They presented an EOQ model for an exponentially decaying inventory. Later Covert and Philip [3] formulated the model with variable deterioration rate with two-parameter Weibull distribution. Philip [17] then developed the inventory model with a three-parameter Weibull distribution rate without shortages. Shah [21] extended Philip's model and considered that shortage was allowed. Goyal and Giri [6] provided a detailed review of deteriorating inventory literatures. Sana, Goyal and Chaudhuri [19] developed a production inventory model for deteriorating items. In all the above literatures, almost all the inventory models for deteriorating items assume that the deterioration occurs as soon as the retailer receives the commodities. However, in real life, most of the goods would have a span of maintaining quality or the original condition, for some period. That is during that period there was no deterioration occurring. We term the phenomenon as "non - instantaneous deterioration". Recently, Wu et.al. [26] developed an inventory model for non-instantaneous deteriorating items with stock–dependent demand.

Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But practically some customers are willing to wait for backorder and others would turn to buy from other sellers. Researchers such as Park [16], Hollier and Mak [9] and Wee [25] developed inventory models with partial backorders. Goyal and Giri [7] developed production inventory model with shortages partially backlogged.

In all the models mentioned above, the inflation and time value of money were disregarded. It has happened most because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. However, in the last several years most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was by Buzacott [1] who developed an EOQ model with inflation subject to different types of pricing policies. Vrat and Padmanaban [23] developed an inventory model under a constant inflation rate for initial stock-dependent consumption rate. Later, Chung and Lin [2] and Wee and Law[24], Sana and Chaudhuri [18] all have investigated the effects of inflation, time value of money and deterioration on inventory models. Recently Hou [10] developed an inventory model for deteriorating items with stock dependent demand under inflation. He considered that shortages are completely backordered.

To fit into realistic circumstances we have developed a finite planning horizon inventory model for non-instantaneous deteriorating items with stock-dependent consumption rate. Here, shortages are allowed and partially backlogged. In addition, the effects of inflation and time value of money on replenishment policy under instantaneous replenishment with zero lead-time are also considered. An optimization frame work is presented to derive optimal replenishment policy when the present value of total cost is minimized. Numerical examples are provided to illustrate the optimization procedure. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out.

2. Assumptions and Notations

The following assumptions are made:

1. The Consumption rate D(t) at time t is assumed to be $a+bI(t)$ where a is a positive constant, b is the stock-dependent consumption rate parameter, $0 \le b \le 1$, and $I(t)$ is the inventory level at time t.
2. The replenishment rate is infinite and lead time is zero.
3. The system operates for a prescribed period of a planning horizon.
4. Shortages are allowed to occur. Only a fraction δ $(0 \le \delta \le 1)$ of it is backlogged. The remaining fraction $(1 - \delta)$ is lost.

5. It is assumed that during certain period of time the product has no deterioration (i.e., fresh product time). After this period, a constant fraction, θ (0<θ<1), of the on-hand inventory deteriorates and there is no repair or replacement of the deteriorated units.
6. Product transactions are followed by instantaneous cash flow.

The following notations are used:

r discount rate, representing the time value of money.

i inflation rate

R $r-i$, representing the net discount rate of inflation (which is constant.)

H planning horizon.

T replenishment cycle.

m the number of replenishment during the planning horizon, $m=H/T$

T_j the total time that elapsed up to and including the j^{th} replenishment cycle ($j=1,2,\ldots,m$). where $T_0=0$, $T_1=T$, ..., $T_m=H$.

t_j the time at which the inventory level in the j^{th} replenishment cycle drops to zero ($j=1,2,\ldots,m$).

t_d the length of time in which the product has no deterioration (Fresh product time).

T_j-t_j time period when shortage occurs ($j=1,2,\ldots,m$).

Q the 2^{nd}, 3^{rd},…, m th replenishment lot size.

I_m maximum Inventory level.

I_b maximum amount of shortage demand to be backlogged.

θ deterioration rate.

A cost per replenishment.($/order.)

p per unit cost of the item.($/unit.)

h per unit inventory holding cost per unit time.($/unit/unit time.)

s per unit shortage cost per unit time.($/unit/unit time.)

π per unit opportunity cost due to lost sales.($/unit)

3. Model Formulation

Suppose that the planning horizon H is divided into m equal parts of length $T=H/m$. Hence the reorder times over the planning horizon H are $T_j=jT$ $(j=0,1,2,\ldots,m)$. When the inventory is positive, demand rate is dependent on stock levels, whereas for negative inventory, the demand is partially backlogged. The period for which there is no-shortage in each interval $[jT,(j+1)T]$ is a fraction of the scheduling period T and is equal to kT $(0<k<1)$. Shortages occur at time $t_j=(k+j-1)T,$ $(j=1,2,\ldots,m)$ and are accumulated until time $t=jT$ $(j=1,2,\ldots,m)$ before they are backordered. This model is illustrated in Fig.1. The first replenishment lot size of I_m is replenished at $T_0=0$. During the interval $[0,t_d]$, the inventory level decreases due to time-dependent demand rate. The inventory level drops to zero due to time-dependent demand and deterioration during the time interval $[t_d,t_1]$. During the interval $[t_1, T]$, shortages occur and are accumulated until $t=T_1$ before they are partially backlogged. $I_1(t)$ denotes the inventory level at time t $(0\le t\le t_d)$ in which the product has no deterioration, $I_2(t)$ is the inventory level at time t $(t_d\le t\le t_1)$ in which the product has deterioration. $I_3(t)$ denotes the inventory level at time t $(t_1\le t\le T)$ in which the product has shortage.

Therefore the inventory system at any time t can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt}=-(a+bI_1(t)) \quad 0\le t\le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-(a+bI_2(t)) \quad t_d\le t\le t_1 \tag{2}$$

$$\frac{dI_3(t)}{dt}=-a\delta \qquad t_1\le t\le T \qquad 3)$$

The solutions of the above differential equations after applying the boundary conditions

$I_1(0)=I_m;\quad I_2(t_1)=0;\quad I_3(t_1)=0,$ are

$$I_1(t)=e^{-bt}I_m-\frac{a}{b}\left[1-e^{-bt}\right] \qquad 0\le t\le t_d \tag{4}$$

$$I_2(t)=\frac{a}{\theta+b}\left[e^{(\theta+b)(t_1-t)}-1\right] \qquad t_d\le t\le t_1 \tag{5}$$

$$I_3(t) = -\delta a(t - t_1) \qquad t_1 \le t \le T \tag{6}$$

Considering the continuity of $I(t)$ at $t = t_d$ it follows that $I_1(t_d) = I_2(t_d)$ which implies that

$$I_m = \frac{a}{\theta + b}\left[e^{(\theta+b)(t_1 - t_d)} - 1\right]e^{bt_d} + \frac{a}{b}\left[e^{bt_d} - 1\right] \tag{7}$$

Substituting Eq (7) into Eq (4) it gives

$$I_1(t) = \frac{a}{\theta + b}\left[e^{(\theta+b)(t_1 - t_d)} - 1\right]e^{-b(t - t_d)} + \frac{a}{b}\left[e^{-b(t - t_d)} - 1\right] \tag{8}$$

Therefore, the maximum inventory level and maximum amount of shortage demand to be backlogged during the first replenishment cycle are

$$I_m = \frac{a}{\theta + b}\left[e^{(\theta+b)(kH/m - t_d)} - 1\right]e^{bt_d} + \frac{a}{b}\left[e^{bt_d} - 1\right] \tag{9}$$

$$I_b = \delta a(H/m)(1 - k) \tag{10}$$

respectively.

There are m cycles during the planning horizon. Since, inventory is assumed to start and end at zero, an extra replenishment at $T_m = H$ is required to satisfy the backorders of the last cycle in the planning horizon. Therefore there are $m+1$ replenishments in the entire planning horizon H. The first replenishment lot size is I_m.

The 2nd, 3rd, …, m th replenishment lot size is

$$Q = I_m + I_b \tag{11}$$

and the last or $(m+1)$th replenishment lot size is I_b.

Since replenishment in each cycle is done at the start of each cycle, the present value of ordering cost during the first cycle is

$$C_r = A \tag{12}$$

Inventory occurs during period t_1, therefore the present value of holding cost during the first replenishment cycle is

$$C_h = h\int_0^{t_d} I_1(t)e^{-Rt}\,dt + h\int_{t_d}^{t_1} I_2(t)e^{-Rt}\,dt$$

$$C_h = \frac{ah}{\theta + b}\left\{\frac{\theta + b}{bR}\left[e^{-Rt_d} - 1\right] + \frac{1}{b + R}\left(e^{bt_d} - e^{-Rt_d}\right)\left[e^{(\theta+b)(kH/m - t_d)} + \theta/b\right]\right.$$

$$+\frac{1}{R}\left[e^{-RkH/m}-e^{-Rt_d}\right]+\frac{1}{\theta+b+R}\left[e^{(\theta+b)(kH/m-t_d)-Rt_d}-e^{-RkH/m}\right]\Bigg\} \tag{13}$$

Shortages are partially backlogged. Therefore, the present value of shortage cost during the first replenishment cycle is

$$C_s=s\int_{t_1}^{T}\left[-I_3(t)\right]e^{-Rt}dt=\frac{s\delta a}{R^2}\left\{e^{-RkH/m}-e^{-RH/m}\left[R(H/m)(1-k)+1\right]\right\} \tag{14}$$

The present value of opportunity cost due to lost sales during the first replenishment cycle is

$$C_0=\pi\int_{t_1}^{T}a(1-\delta)e^{-Rt}dt=\frac{\pi\ a(1-\delta)}{R}\left[e^{-RkH/m}-e^{-RH/m}\right] \tag{15}$$

Replenishment is done at $t=0$ and T. The present value of material cost during the first replenishment cycle is

$$C_p=pI_m+pe^{-RT}I_b$$

$$=p\left\{\frac{a}{\theta+b}\left[e^{(\theta+b)(kH/m-t_d)}-1\right]e^{bt_d}+\frac{a}{b}\left[e^{bt_d}-1\right]+e^{-RH/m}\left[\delta a(H/m)(1-k)\right]\right\} \tag{16}$$

Consequently, the present value of total cost of system during the first replenishment cycle can be formulated as

$$TRC=C_r+C_h+C_s+C_0+C_p \tag{17}$$

So, the present value of total cost of the system over a finite planning horizon H is

$$TC(m,k)=\sum_{j=0}^{m-1}TRCe^{-RjT}+Ae^{-RH}=TRC\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)+Ae^{-RH} \tag{18}$$

where $T=H/m$ and TRC derived by substituting Equations. (12) to (16) in Equation (17).

On simplification we get

$$TC(m,k)=Ga\left\{\frac{\mathrm{A}}{a}+\frac{h}{\theta+b}\left\{\frac{\theta+\mathrm{b}}{\mathrm{bR}}\left[e^{-Rt_d}-1\right]+\frac{1}{b+R}\left(e^{bt_d}-e^{-Rt_d}\right)\left[e^{(\theta+b)(kH/m-t_d)}+\frac{\theta}{b}\right]\right\}\right.$$

$$\left.+\frac{h}{\theta+b}\left\{\frac{1}{R}\left[e^{-RkH/m}-e^{-Rt_d}\right]+\frac{1}{\theta+b+R}\left[e^{(\theta+b)(kH/m-t_d)-Rt_d}-e^{-RkH/m}\right]\right\}\right.$$

$$+\frac{s\delta}{R^2}\left\{e^{-RkH/m}-e^{-RH/m}\left[RH/m\,(1-k)+1\right]\right\}$$

$$+\frac{\pi(1-\delta)}{R}\left[e^{-RkH/m}-e^{-RH/m}\right]$$

$$+p\left\{\frac{1}{\theta+b}\left[e^{(\theta+b)(kH/m-t_d)}-1\right]e^{bt_d}+\frac{1}{b}\left[e^{bt_d}-1\right]\right.$$

$$\left.+e^{-RH/m}\,\delta(H/m)(1-k)\right\}+Ae^{-RH}$$

where $G=\left(\dfrac{1-e^{-RH}}{1-e^{-RH/m}}\right)$

3.1 Solution Procedure

The present value of total cost $TC(m,k)$ is a function of two variables m and k where m is a discrete variable and k is a continuous variable. For a given value of m, the necessary condition for $TC(m,k)$ to be minimized is $dTC(m,k)/dk=0$ which gives

$$\frac{dTC(m,k)}{dk}=\frac{1-e^{-RH}}{1-e^{-RH/m}}\,a\,(H/m)\left\{\frac{h}{b+R}\left(e^{bt_d}-e^{-Rt_d}\right)e^{(\theta+b)(kH/m-t_d)}\right.$$

$$+\frac{h}{(\theta+b+R)}\left[e^{(\theta+b)(k\,H/m-t_d)-Rt_d}-e^{-RkH/m}\right]$$

$$\left.+\frac{\delta\,s}{R}\left(e^{-RH/m}-e^{-RkH/m}\right)-\pi(1-\delta)e^{-RkH/m}+p\left[e^{(\theta+b)k(H/m)-\theta t_d}-\delta\,e^{-RH/m}\right]\right\}=0 \tag{19}$$

Furthermore, Theorem in appendix shows that *TC* (*m, k*) is convex with respect to *k*. Now, the procedure is to locate the optimal solution k^* of TC(m,k) when m is fixed. Since m is a discrete variable, the following algorithm can be used to determine the optimal values of m and k for the proposed model. To acquire the optimal TC (m, k), we follow the algorithm proposed by Montgomery [14] which is given below.

Algorithm

Step1: Start with m=1.

Step 2: Using (19) solve for k using Newton-Raphson method. Then substitute the solution obtained for (19) into (18) to compute the total inventory cost.

Step 3: Increase m by 1 and repeat Step 2.

Step 4: Repeat Step 2 and Step 3 until TC (m, k) increases. The value of m which corresponds to the increase of TC for the first time is taken as the optimal value of m (denoted by m*) and the corresponding k (denoted by k*) is the optimal value for k.

Using the optimal solution procedure described above, we can find the optimal order quantity and maximum Inventory level to be

$$Q^* = \frac{a}{\theta+b}\left[e^{(\theta+b)\left(k^* H/m^* - t_d\right)} - 1\right]e^{b\,t_d} + \frac{a}{b}\left[e^{bt_d} - 1\right] + \delta a\left(H/m^*\right)\left(1-k^*\right)$$

$$I_m{}^* = \frac{a}{\theta+b}\left[e^{(\theta+b)\left(k^* H/m^* - t_d\right)} - 1\right]e^{bt_d} + \frac{a}{b}\left[e^{bt_d} - 1\right]$$

3.2 Numerical Examples

Example 1:

In order to illustrate the above solution procedure, let us consider an inventory system with the following data:

Let A = 250, h = 1.2, d=1.5, s=2.2, π = 1.8, p=2, θ = 0.08, t_d=0.0833, δ=0.56 , H=10, a=1000, b = 0.2, R=0.2 in appropriate units.

Using the solution procedure described above, the results are presented in Table 1. From this table we see that when the number of replenishments m = 12, the total cost TC becomes minimum. Hence, the optimal values of m and k are m^* = 12, k^* = 0.2898 respectively, and the minimum total cost TC (m^*, k^*) =10,974. We then have, $T^*=H/m^*=10/12=0.8333$, $t_1^*=k^*H/m^*=0.2415$, $Q^*=579.91$.

Example 2:

We consider another inventory system with the following data:

Let A = 350, h=1.5, d=1.2, s=2.4, π=1.2, p=2, θ=0.02, t_d=0.0833, δ=0.5 , H=10, a=800, b=0.25, R=0.2 in appropriate units.

Using the solution procedure described above the results are presented in Table 2. From this table we see that when the number of replenishments m = 9, the total cost TC becomes minimum. Hence, the optimal values of m and k are $m^* = 9$, $k^* = 0.1902$ respectively and the minimum total cost TC $(m^*, k^*) = 8{,}676.5$. We then have, $T^*=H/m^*=10/9 = 1.1111$, $t_1^*=k^*H/m^*=0.2113$, $Q^*=533.67$.

We now study the effects of changing the parameters θ, R, t_d and δ on the optimal replenishment policy of the Example 1. The results are summarized in Table 3. Based on Table 3, the observations can be made as follows:

- When the deterioration rate θ is increasing, the optimal cost is increasing and the order quantity is decreasing.
- When the net discount rate of inflation R is increasing, the optimal cost is decreasing.
- When the length of fresh product time 't_d' is increasing, the total cost is decreasing and the order quantity is increasing.
- When the backlogging rate δ is increasing, the total cost and the order quantity is increasing.

3.3 Sensitivity Analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. Using the numerical examples given in the preceding section, the sensitivity analysis of various parameters has been done. Let the estimated values of order quantity and total cost be Q' and TC' respectively, while the true value of these are Q and TC. The results of sensitivity analysis are summarized in Tables 4 and 5. The following inferences can be made.

1. When the consumption rate (a) decreases or increases the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase. Similarly, the ordering quantity (Q) and the present value of total cost (TC) will also decrease or increase as the ordering cost (A) decrease or increase. That is changes in (a) and (A) will lead to the positive changes in (Q) and (TC).
2. The change in the stock-dependent consumption rate (b) leads to a positive change in the present value of the total cost (TC).
3. The change in net discount rate of inflation (R) leads to a negative change in the

present value of total cost (TC).

4. The change in deterioration rate (θ) leads to a negative change on the ordering quantity (Q) and a positive change in the present value of the total cost (TC). That is Q decreases with the increase of (θ).Whereas (TC) increases with the increase of (θ).
5. Increasing the fresh product time (t_d) increases the order quantity (Q) and decreases the total cost (TC).
6. When the backlogging parameter (δ) increases the ordering quantity (Q) and the present value of total cost (TC) increases. That is change in (δ) leads to a positive change in (Q) and (TC).
7. Changes in carrying cost (h) and shortage cost (s) result in a positive change in the present value of total cost (TC) and a negative change in the ordering quantity (Q).
8. When the opportunity cost (π) increases, the ordering quantity (Q) and the present value of total cost (TC) increases.
9. Change in purchase cost (p) leads to positive change in the present value of total cost (TC) and negative change in the ordering quantity (Q).
10. The present value of total cost (TC) is more sensitive to the consumption rate (a), the unit purchase cost (p) and the net discount rate of inflation (R) as compared to other parameters.
11. Tables 4 and 5 imply that the effect of (R) on (TC) is quite significant for 50% over or under estimation of (R). It implies that the effect of inflation and time value of money on present value of total cost is significant.

Table 1

Optimal solution with shortages

m	$k(m)$	T	Q	$TC(m,k)$
1	0.3149	10.0000	8856.40	26186
2	0.3126	5.0000	3872.80	16327
3	0.3103	3.3333	2479.50	13766
4	0.3080	2.5000	1822.70	12600
5	0.3058	2.0000	1440.40	11955
6	0.3035	1.6667	1190.10	11565
7	0.3012	1.4286	1013.60	11318
8	0.2989	1.2500	882.32	11161
9	0.2967	1.1111	780.93	11063
10	0.2944	1.0000	700.24	11006
11	0.2921	0.9091	634.50	10979
12^*	0.2898^*	0.8333^*	579.91^*	10974^*
13	0.2876	0.7692	533.85	10987

Table 2

Optimal solution with shortages

m	k(m)	T	Q	TC(m,k)
1	0.2831	10.0000	6261.80	19956.0
2	0.2714	5.0000	2766.40	12497.0
3	0.2598	3.3333	1766.30	10525.0
4	0.2482	2.5000	1291.40	9644.9
5	0.2366	2.0000	1013.90	9180.0
6	0.2250	1.6667	831.93	8919.2
7	0.2134	1.4286	703.34	8774.2
8	0.2018	1.2500	607.65	8701.3
9^*	0.1902^*	1.1111^*	533.67^*	8676.5^*
10	0.1786	1.0000	474.76	8685.3

Table 3

Effects of changing the parameter θ, R, t_d, δ on the optimal replenishment policy

parameter	parameter value	m	k	T	Q	$TC(m,k)$
θ	0.04	12	0.2950	0.8333	581.52	10969
	0.06	12	0.2924	0.8333	580.71	10972
	0.08	12	0.2898	0.8333	579.91	10974
	0.10	12	0.2874	0.8333	579.13	10977
R	0.10	12	0.3334	0.8333	598.36	16024
	0.15	12	0.3113	0.8333	588.97	13143
	0.20	12	0.2898	0.8333	579.91	10974
	0.25	12	0.2689	0.8333	571.17	9316
t_d	0.0417	12	0.2879	0.8333	579.72	10981
	0.0625	12	0.2888	0.8333	579.78	10978
	0.0833	12	0.2898	0.8333	579.91	10974
	0.1041	12	0.2909	0.8333	580.12	10972
δ	0.28	9	0.1459	1.1111	430.72	10132
	0.42	11	0.2210	0.9091	503.04	10609
	0.56	12	0.2898	0.8333	579.91	10974
	0.70	13	0.3499	0.7692	628.04	11270

Table 4

Sensitivity analysis for Example 1 with respect to various parameters on order quantity and total cost for stock-dependent consumption rate model.

Parameter		Percentage of under estimation and over estimation of parameter				
		-50%	-25%	0%	25%	50%
a	Q'/Q	0.7607	0.9056	1	1.1507	1.2790
	TC'/TC	0.5515	0.7786	1	1.2178	1.4330
b	Q'/Q	1.0087	1.0043	1	0.9959	0.9919
	TC'/TC	0.9971	0.9986	1	1.0014	1.0026
R	Q'/Q	1.0318	1.0156	1	0.9849	0.9702
	TC'/TC	1.4601	1.1977	1	0.8489	0.7312
θ	Q'/Q	1.0028	1.0014	1	0.9987	0.9973
	TC'/TC	0.9996	0.9998	1	1.0003	1.0005
t_d	Q'/Q	0.9997	0.9998	1	1.0004	1.0009
	TC'/TC	1.0006	1.0003	1	0.9998	0.9996
A	Q'/Q	0.6975	0.8527	1	1.2075	1.3072
	TC'/TC	0.9289	0.9674	1	1.0293	1.0557
δ	Q'/Q	0.7427	0.8674	1	1.0830	1.2206
	TC'/TC	0.9233	0.9667	1	1.0270	1.0493
p	Q'/Q	1.1888	1.0873	1	0.9244	0.8584
	TC'/TC	0.7036	0.8599	1	1.1263	1.2404
h	Q'/Q	1.1507	1.1191	1	0.9818	0.9667
	TC'/TC	0.9901	0.9958	1	1.0035	1.0064
s	Q'/Q	1.3358	1.1413	1	0.9627	0.9247
	TC'/TC	0.9231	0.9687	1	1.0235	1.0419
π	Q'/Q	0.8197	0.9489	1	1.0517	1.1038
	TC'/TC	0.8762	0.9412	1	1.0530	1.1004

Table 5

Sensitivity analysis for Example 2 with respect to various parameters on order quantity and total cost for stock-dependent consumption rate model.

Parameter		Percentage of under estimation and over estimation of parameter				
		-50%	-25%	0%	25%	50%
a	Q'/Q	0.7795	0.8540	1	1.1120	1.1994
	TC'/TC	0.5728	0.7902	1	1.2045	1.4051
b	Q'/Q	1.0096	1.0047	1	0.9955	0.9913
	TC'/TC	0.9981	0.9991	1	1.0008	1.0016
R	Q'/Q	1.0313	1.0154	1	0.9850	0.9703
	TC'/TC	1.4620	1.1985	1	0.8481	0.7300
θ	Q'/Q	1.0005	1.0003	1	0.9997	0.9995
	TC'/TC	0.9999	1.0000	1	1.0001	1.0002
t_d	Q'/Q	0.9999	0.9999	1	1.0001	1.0002
	TC'/TC	1.0002	1.0001	1	1.0000	0.9999
A	Q'/Q	0.6618	0.7996	1	1.1386	1.3179
	TC'/TC	0.8988	0.9537	1	1.0409	1.0787
δ	Q'/Q	0.5832	0.8396	1	1.0972	1.2710
	TC'/TC	0.8291	0.9235	1	1.0638	1.1180
p	Q'/Q	1.1881	1.0876	1	0.8162	0.7512
	TC'/TC	0.7503	0.8823	1	1.1051	1.1976
h	Q'/Q	1.0519	1.0224	1	0.9823	0.9681
	TC'/TC	0.9940	0.9974	1	1.0021	1.0038
s	Q'/Q	1.3242	1.0581	1	0.9453	0.8945
	TC'/TC	0.8800	0.9494	1	1.0393	1.0712
π	Q'/Q	0.8200	0.9647	1	1.0354	1.0709
	TC'/TC	0.8956	0.9495	1	1.0480	1.0936

4. Conclusion

In this article, a deterministic inventory model has been framed for non-instantaneous deteriorating items with stock-dependent consumption rate over a finite planning horizon. Shortages are allowed and partially backlogged. Further we have considered the effects of inflation and the time value of money in formulating the inventory replenishment policy. We have given an analytic formulation of the problem on the framework described above and have presented an optimal solution procedure to find the optimal replenishment policy. Sensitivity analysis with respect to various parameters has been carried out. Our research results implies that, the effect of inflation and time value of money on present value of total cost is more significant and increasing the fresh product time increases the order quantity and decreases the total cost.

Thus, this model incorporates some realistic features that are likely to be associated with some kinds of inventory. The model is very useful in their retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products which are more likely with the characteristics above.

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Appendix

Theorem *TC* (*m*, *k*) is convex with respect to *k*.

The present value of total cost for the planning time Horizon *H* is given by

$$TC(m,k) = Ga\left\{\frac{\mathrm{A}}{a} + \frac{h}{\theta+b}\left\{\frac{\theta+\mathrm{b}}{\mathrm{bR}}\left[e^{-Rt_d}-1\right] + \frac{1}{b+R}\left(e^{bt_d}-e^{-Rt_d}\right)\left[e^{(\theta+b)(kH/m-t_d)}+\frac{\theta}{b}\right]\right\}\right.$$

$$+\frac{h}{\theta+b}\left\{\frac{1}{R}\left[e^{-RkH/m}-e^{-Rt_d}\right]+\frac{1}{\theta+b+R}\left[e^{(\theta+b)(kH/m-t_d)-Rt_d}-e^{-RkH/m}\right]\right\}$$

$$+\frac{s\delta}{R^2}\left\{e^{-RkH/m}-e^{-RH/m}\left[RH/m\left(1-k\right)+1\right]\right\}$$

$$+\frac{\pi(1-\delta)}{R}\left[e^{-RkH/m}-e^{-RH/m}\right]$$

$$+ p\left\{\frac{1}{\theta+b}\left[e^{(\theta+b)(kH/m-t_d)}-1\right]e^{bt_d}+\frac{1}{b}\left[e^{bt_d}-1\right]\right.$$
$$\left.+ e^{-RH/m}\delta(H/m)(1-k)\right\}\right\} + Ae^{-RH}$$

Therefore we have

$$\frac{dTC\,(m,k)}{dk}=\frac{1-e^{-RH}}{1-e^{-RH/m}}\,a\,(H/m)\left\{\frac{h}{b+R}\left(e^{bt_d}-e^{-Rt_d}\right)e^{(\theta+b)(kH/m-t_d)}\right.$$
$$+\frac{h}{(\theta+b+R)}\left[e^{(\theta+b)(kH/m-t_d)-Rt_d}-e^{-RkH/m}\right]$$
$$\left.+\frac{\delta s}{R}\left(e^{-RH/m}-e^{-RkH/m}\right)-\pi(1-\delta)e^{-RkH/m}+p\left[e^{(\theta+b)k(H/m)-\theta t_d}-\delta\, e^{-RH/m}\right]\right\}$$

Therefore we have

$$\frac{d^2TC(m,k)}{dk^2}=\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)a\,(H/m)^2\left\{\frac{h(\theta+b)}{b+R}\left(e^{bt_d}-e^{-Rt_d}\right)e^{(\theta+b)(kH/m-t_d)}\right.$$
$$+\frac{h}{\theta+b+R}\left\{(\theta+b)e^{(\theta+b)(kH/m-t_d)-Rt_d}+R\,e^{-RkH/m}\right\}$$
$$\left.+s\delta\, e^{-RkH/m}+\pi R(1-\delta)e^{-RkH/m}\right\}\geq 0.$$

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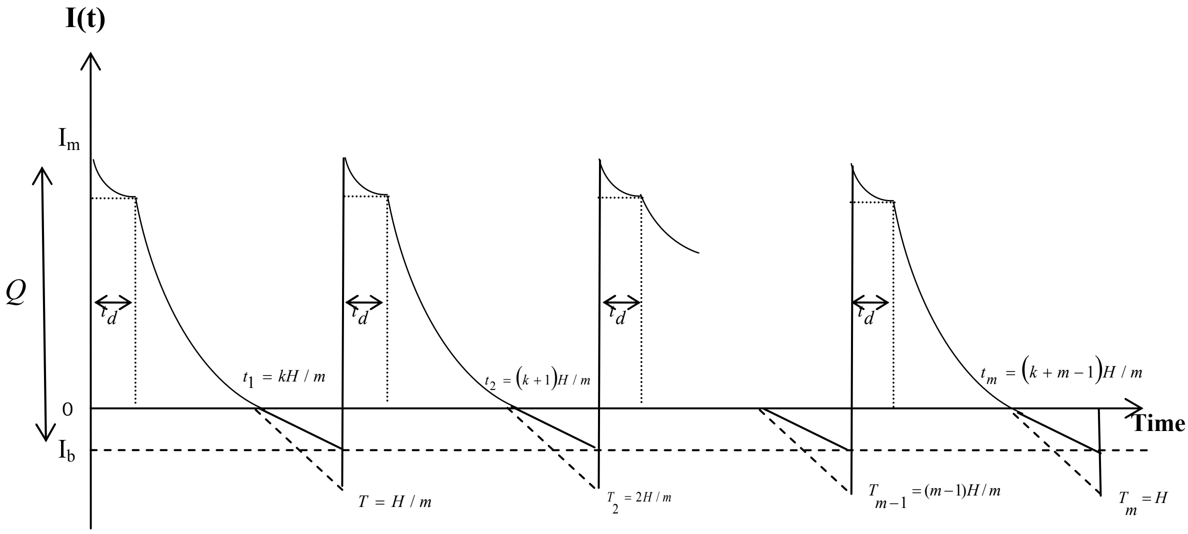

Fig.1 Graphical representation of inventory system