Tamsui Oxford Journal of Mathematical Sciences **25(3)** (2009) 237-257
Aletheia University

Eigenvalue Approach to Three Dimensional Coupled Thermoelasticity in a Rotating Transversely Isotropic Medium*

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Received August 27, 2007, Accepted December 25, 2007.

Abstract

The theory of coupled thermoelasticity in three dimensions is employed to determine the distribution of temperature and stresses in an infinite medium having an instantaneous point heat source at the origin in a rotating medium. Laplace transform along with the double Fourier transforms have been applied in the basic equations of coupled thermoelasticity and finally the resulting equations are written in the form of a vector–matrix differential equation which is then solved by eigenvalue approach. The inversion of the Laplace transform solution is carried out by applying Bellman method and computations have been done by using MATLAB software. Numerical computations of temperature and stresses have been made in space time

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* 2000 Mathematics Subject Classification. 74F,42C20.
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domain and presented graphically and then compared with current results available in the literture.

Keywords and Phrases: *Laplace transform, Double Fourier transform, Coupled thermoelasticity, Eigenvalue, Transversely isotropic.*

1. Introduction

The coupling between the strain and temperature fields was first studied by Duhamel [1] who derived the equations for the distribution of strains in an elastic medium subjected to temperature gradient. Biot [2] justified and derived on the basis of irreversible thermodynamics, the fundamental relations of the equations of thermoelasticity and stated its variational principles. For static problems this coupling vanished and the thermal field becomes independent of the strain field.

Apart from the constitutive relations, the governing equations for displacement and temperature fields, as in the linear dynamical theory of classical thermoelasticity consist of the coupled partial differential equation of motion and the Fourier heat conduction equation. The equation for displacement field is governed by a wave type hyperbolic equation, whereas, the latter equation for the temperature field is a parabolic diffusion type equation. However, the classical thermoelasticity predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances that are coupled together. In view of Lord and Shulman [3], a part of every solution of equations extends to infinity. In view of mathematical difficulty involved in the coupled equations of thermoelasticity Noda et al [4], Furukawa et al. [5], Chandrasekharaiah and Keshavan [6], Choudhury [7] have considered only one dimensional problems.

On the other hand, several authors Ackerman et al [8, 9, 10], Von Gutfeld and Nethercot [11], Taylor et al [12], Jackson and Walker [13] have conducted on different solids and shown that heat pulses do not propagate with infinite speed. In order to overcome this difficulty involved in an infinite speed of thermal disturbances, several authors Norwood and Warren [14], Green and Lindsay [15], Suhubi [16], Dhaliwal and Rokne [17] have made an attempt on different grounds to modify classical equations of thermoelasticity by suggesting a wave type heat conduction equation. An interesting review paper by Chandrasekharaiah [18] contains most of the major results involving many modifications with a list of reference.

The main object of this paper is to make an investigation about the effect of rotation in a three dimensional problem of coupled thermoelasticity to determine the temperature, deformation and stresses, in an infinite transversely isotropic medium due to an instantaneous heat source.

The solution has been achieved in closed form in the Laplace–double Fourier transform domain and finally numerical inversions in space time domain have been made and some of the results are shown graphically.

Nomenclature

A_{ij}	=	elastic Moduli of material
β_j	=	stress temperature coefficient
ρ	=	density of mass
K_x, K_y, K_z	=	coefficients of thermal conductivity in x, y and z directions respectively
c	=	specific heat per unit mass
K_1	=	$\frac{K_y}{K_x}$
K_2	=	$\frac{K_z}{K_x}$
T	=	absolute temperature
τ_{ij}	=	stress components
T_o	=	Reference temperature
p	=	Laplace transform parameter
ξ, η	=	Fourier transform parameter
$\vec{\Omega}$	=	Rotation vector.
$F_i(\xi, \eta, p)$	=	Function of ξ, η, p
$\delta(.)$	=	Dirac–delta function

I. Basic Equations

We consider a transversely isotropic infinite elastic medium in three dimensions which is unstrained and unstressed initially but has a uniform temperature distribution To. The displacement components u, v & w along the x, y and z directions respectively are of the form

$$u = u(x, y, z, t); \quad v = v(x, y, z, t), \quad w = w(x, y, z, t) \tag{1.1}$$

The stress components related to the displacement components are

$$\tau_{xx} = A_{11}\frac{\partial u}{\partial x} + A_{12}\frac{\partial v}{\partial y} + A_{13}\frac{\partial w}{\partial z} - \beta_1 T$$

$$\tau_{yy} = A_{12}\frac{\partial u}{\partial x} + A_{22}\frac{\partial v}{\partial y} + A_{23}\frac{\partial w}{\partial z} - \beta_2 T$$

$$\tau_{zz} = A_{13}\frac{\partial u}{\partial x} + A_{23}\frac{\partial v}{\partial y} + A_{33}\frac{\partial w}{\partial z} - \beta_3 T$$

$$\tau_{yz} = A_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\tau_{zx} = A_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\tau_{xy} = \frac{A_{11} - A_{12}}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{1.2}$$

Now, we consider the rotation $\vec{\Omega} = (\Omega, 0, 0)$.

The equations of motion in a rotating medium are

$$\tau_{ij,j} = \rho\left[\ddot{u}_i + \{\vec{\Omega}\times(\vec{\Omega}\times\vec{u})\} + (2\vec{\Omega}\times\dot{\vec{u}})\right] \tag{1.3}$$

From (1.3) using (1.2) we get the following equations of motion in terms of displacement in a rotating medium as follows: –

$$A_{11}\frac{\partial^2 u}{\partial x^2} + A_{44}\frac{\partial^2 u}{\partial y^2} + A_{44}\frac{\partial^2 u}{\partial z^2} + \left(A_{12} + A_{44}\right)\frac{\partial^2 v}{\partial x \partial y} + \left(A_{13} + A_{44}\right)\frac{\partial^2 w}{\partial x \partial z} = \rho\frac{\partial^2 u}{\partial t^2} + \beta_1\frac{\partial T}{\partial x}$$

$$A_{44}\frac{\partial^2 v}{\partial x^2} + A_{22}\frac{\partial^2 v}{\partial y^2} + A_{44}\frac{\partial^2 v}{\partial z^2} + \left(A_{12} + A_{44}\right)\frac{\partial^2 u}{\partial x \partial y} + \left(A_{23} + A_{44}\right)\frac{\partial^2 w}{\partial y \partial z}$$

$$= \rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v - 2\Omega\frac{\partial w}{\partial t}\right) + \beta_2\frac{\partial T}{\partial y}$$

and

$$A_{44}\frac{\partial^2 w}{\partial x^2}+A_{44}\frac{\partial^2 w}{\partial y^2}+A_{33}\frac{\partial^2 w}{\partial z^2}+(A_{13}+A_{44})\frac{\partial^2 u}{\partial x\partial z}+(A_{23}+A_{44})\frac{\partial^2 v}{\partial y\partial z}$$

$$=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w+2\Omega\frac{\partial v}{\partial t}\right)+\beta_3\frac{\partial T}{\partial z} \tag{1.4}$$

The heat conduction equation is

$$K_x\frac{\partial^2 T}{\partial x^2}+K_y\frac{\partial^2 T}{\partial y^2}+K_z\frac{\partial^2 T}{\partial z^2}=\rho_C\frac{\partial T}{\partial t}+T_o\frac{\partial}{\partial t}\left(\beta_1\frac{\partial u}{\partial x}+\beta_2\frac{\partial v}{\partial y}+\beta_3\frac{\partial w}{\partial z}\right)-Q(x,y,z,t) \tag{1.5}$$

We consider a point instantaneous heat source located at the origin whose strength Q is the form:

$$Q(x, y, z, t) = qo\ \delta(x)\ \delta(y)\ \delta(z)\ \delta(t) \tag{1.6}$$

where qo is a constant.

II. Method of Solution

Formulation of a Vector–Matrix Differential Equation :

We apply the Laplace–double Fourier transforms as

$$\overline{T}(x,y,z,p)=\int_o^\infty T(x,y,z,t)\exp(-pt)dt$$

$$\overline{T}_1(\xi,y,z,p)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{T}(x,y,z,p)\exp(i_x\xi x)dx$$

$$\overline{T}_2(\xi,\eta,z,p)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\overline{T}_1(\xi,y,z,p)\exp(i_y\eta y)dy \tag{2.1}$$

where $i_x^2=i_y^2=-1$

where p, ξ and η are transform parameters.

Using the transformation in equations (1.4) and (1.5) we get :

$$-\left(\xi^2 A_{11} + \eta^2 A_{44} + \rho p^2\right)\overline{u}_2 + A_{44}\frac{d^2\overline{u}_2}{dz^2} + i_x i_\eta \xi\eta(A_{12} + A_{44})\overline{v}_2$$
$$-i_x\xi(A_{13} + A_{44})\frac{d\overline{w}_2}{dz} + i_x\xi\beta_1\overline{T}_2 = 0 \tag{2.2}$$

$$i_x i_y \xi\eta(A_{12} + A_{44})\overline{u}_2 - \left\{\xi^2 A_{44} + \eta^2 A_{22} + \rho(p^2 - \Omega^2)\right\}\overline{v}_2$$
$$+A_{44}\frac{d^2\overline{v}_2}{dz^2} - i_y\eta(A_{23} + A_{44})\frac{d\overline{w}_2}{dz} + 2\rho p\Omega\overline{w}_2 + i_y\eta\beta_2\overline{T}_2 = 0 \tag{2.3}$$

$$i_x\xi(A_{13} + A_{44})\frac{d\overline{u}_2}{dz} - i_y\eta(A_{23} + A_{44})\frac{d\overline{v}_2}{dz} - \left\{A_{44}\xi^2 + A_{44}\eta^2 + \rho(p^2 - \Omega^2)\right\}\overline{w}_2$$
$$-2\rho p\Omega\overline{v}_2 + A_{33}\frac{d^2\overline{w}_2}{dz^2} - \beta_3\frac{d\overline{T}_2}{dz} = 0 \tag{2.4}$$

and

$$K_2\frac{d^2\overline{T}_2}{dz^2} - \left\{\xi^2 + K_1\eta^2 + \frac{\rho c p}{K_x}\right\}\overline{T}_2$$
$$= \frac{T_o p}{K_x}\left(-i_x\xi\beta_1\overline{u}_2 - i_y\eta\beta_2\overline{v}_2 + \beta_3\frac{d\overline{w}_2}{dz}\right) - \frac{q_o}{2\pi K_x}\delta(z) \tag{2.5}$$

Since at time t = 0, the body is at rest in an undeformed and unstressed state and is maintained at the reference temperature,so the following initial conditions hold.

$$u(x,y,z,o) = \frac{\partial u(x,y,z,0)}{\partial t} = 0, \qquad v(x,y,z,o) = \frac{\partial v(x,y,z,0)}{\partial t} = 0$$
$$w(x,y,z,o) = \frac{\partial w(x,y,z,0)}{\partial t} = 0 \quad \text{and} \quad \frac{\partial T(x,y,z,0)}{\partial t} = 0 \tag{2.6}$$

We have further assumed that $\overline{u}, \overline{v}, \overline{w}$ and $\overline{T}$ as well as their first derivatives with respect to x and y vanish at infinity. Further we assume that = 0 at t = 0.

Equations (2.2) to (2.5) can be written in the form of a vector–matrix differential equation as :

$$\frac{d\underset{\sim}{v}}{dz} = \underset{\sim}{A}\,\underset{\sim}{v} + \underset{\sim}{f}(z) \tag{2.7}$$

where $\underset{\sim}{v} = \left[\bar{u}_2, \bar{v}_2, \bar{w}_2, \bar{T}_2, \bar{u}_2^{/}, \bar{v}_2^{/}, \bar{w}_2^{/}, \bar{T}_2^{/}\right]^T$

and $$\underset{\sim}{f}(z) = \left[0, 0, 0, 0, 0, 0, 0, -\frac{q_o}{2\pi K_x}\delta(z)\right] \tag{2.8}$$

Here the primes indicate differentiation w.r.t. z.

The matrix $\underset{\sim}{A}$ is

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ C_{51} & C_{52} & 0 & C_{54} & 0 & 0 & C_{57} & 0 \\ C_{61} & C_{62} & C_{63} & C_{64} & 0 & 0 & C_{67} & 0 \\ 0 & C_{72} & C_{73} & 0 & C_{75} & C_{76} & 0 & C_{78} \\ C_{81} & C_{82} & 0 & C_{84} & 0 & 0 & C_{87} & 0 \end{bmatrix} \tag{2.9}$$

where

$$C_{51} = \frac{\xi^2 A_{11} + \eta^2 A_{44} + \rho p^2}{A_{44}}; \qquad C_{52} = -\frac{i_x i_y \xi\eta(A_{12} + A_{44})}{A_{44}}$$

$$C_{54} = -\frac{i_x \xi\beta_1}{A_{44}}; \qquad C_{57} = \frac{i_x \xi(A_{13} + A_{44})}{A_{44}}$$

$$C_{61} = -\frac{i_x i_y \xi\eta(A_{12} + A_{44})}{A_{44}}; \qquad C_{62} = \frac{\xi^2 A_{44} + \eta^2 A_{22} + \rho(p^2 - \Omega^2)}{A_{44}}$$

$$C_{63} = -\frac{2\rho p\Omega}{A_{44}}; \qquad C_{64} = -\frac{i_y \eta\beta_2}{A_{44}}$$

$$C_{67} = \frac{i_y \eta(A_{23} + A_{44})}{A_{44}}; \qquad C_{72} = \frac{2\rho p\Omega}{A_{33}}$$

$$C_{73} = \frac{A_{44}\left(\xi^2 + \eta^2\right) + \rho(p^2 - \Omega^2)}{A_{33}}; \quad C_{75} = \frac{i_x \xi(A_{13} + A_{44})}{A_{33}}$$

$$C_{76} = \frac{i_y \eta (A_{23} + A_{44})}{A_{33}}; \qquad C_{78} = \frac{\beta_3}{A_{33}}$$

$$C_{81} = \frac{-i_x \xi \beta_1 T_o p}{K_z}; \qquad C_{82} = -\frac{i_y \eta \beta_2 T_o p}{K_z}$$

$$C_{84} = \frac{\xi^2 + K_1 \eta^2 + \dfrac{\rho c p}{K_x}}{K_2}; \qquad C_{87} = \frac{\beta_3 T_o p}{K_z} \tag{2.10}$$

Solution of the Vector–Matrix Equation

We find the solution of equation (2.7) by following the method of eigen value approach as in Das and Bhakta [19].(Appendix-I)

The characteristic equation of matrix $\underset{\sim}{A}$ is of the form

$$\lambda^8 - F_1(\xi, \eta, p)\lambda^6 + F_2(\xi, \eta, p)\lambda^4 - F_3(\xi, \eta, p)\lambda^2 + F_4(\xi, \eta, p) = 0. \tag{2.11}$$

The roots of the equation (2.11) are of the form

$$\lambda = \pm \lambda_1, \ \lambda = \pm \lambda_2, \ \lambda = \pm \lambda_3, \ \lambda = \pm \lambda_4 \tag{2.12}$$

which are also the eigenvalues of the matrix $\underset{\sim}{A}$,where

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 = F_1(\xi, \eta, p)$$

$$\lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_1^2\lambda_4^2 + \lambda_2^2\lambda_3^2 + \lambda_2^2\lambda_4^2 + \lambda_3^2\lambda_4^2 = F_2(\xi, \eta, p)$$

$$\lambda_1^2\lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_2^2\lambda_4^2 + \lambda_2^2\lambda_3^2\lambda_4^2 + \lambda_3^2\lambda_4^2\lambda_1^2 = F_2(\xi, \eta, p)$$

$$\lambda_1^2\lambda_2^2\lambda_3^2\lambda_4^2 = F_2(\xi, \eta, p)$$

The right and left eigen vector $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ of the matrix $\underset{\sim}{A}$ corresponding to λ are respectively.

$$\underset{\sim}{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$$

and $\underset{\sim}{Y} = [y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8]$ (2.13)

The components x_i of $\underset{\sim}{X}$ and y_i of $\underset{\sim}{Y}$ (i = 1, 2, …, 8) can be calculated from the relations

$$\underset{\sim}{A}\,\underset{\sim}{X} = \lambda\,\underset{\sim}{X} \qquad \text{and} \qquad \underset{\sim}{Y}\,\underset{\sim}{A} = \lambda\,\underset{\sim}{Y}$$

which are given in the Appendix.II.

Henceforth we will use the following notations :

$$X_i = \begin{cases} [x]_{\lambda=\lambda_{\left(\frac{i+1}{2}\right)}} & \text{for } i = 1, 3, 5, 7 \\ [x]_{\lambda=-\lambda_{\left(\frac{i}{2}\right)}} & \text{for } i = 2, 4, 6, 8 \end{cases} \tag{2.14}$$

and

$$Y_i = \begin{cases} [Y]_{\lambda=\lambda_{\left(\frac{i+1}{2}\right)}} & \text{for } i = 1, 3, 5, 7 \\ [Y]_{\lambda=-\lambda_{\left(\frac{i}{2}\right)}} & \text{for } i = 2, 4, 6, 8 \end{cases} \tag{2.15}$$

Assuming the regularity condition at z = ∞ as in Das et al. [20] the solution of equation (2.6) is of the form

$$\underset{\sim}{v}(z) = \sum_{i=1}^{4} A_{2i}(z)\, X_{2i} \exp(-\lambda_i z) \tag{2.16}$$

where

$$A_{2i} = \frac{-q_o}{2\pi K_x (Y_{2i} X_{2i})} (y_8)_{\lambda=-\lambda_i}; \quad i = 1, 2, 3, 4$$

From equation (2.16) we can find the expression of $\overline{u}_2(\xi, \eta, z, p), \overline{v}_2(\xi, \eta, z, p)$, $\overline{w}_2(\xi, \eta, z, p)$ and $\overline{T}_2(\xi, \eta, z, p)$ as follows :

$$\overline{u}_2(\xi, \eta, z, p) = \sum_{i=1}^{4} A_{2i}(z)\, x_{1i} \exp(-\lambda_i z) \tag{2.17}$$

$$\bar{v}_2(\xi,\eta,z,p)=\sum_{i=1}^{4} A_{2i}(z)\,x_{2i}\exp(-\lambda_i z) \tag{2.18}$$

$$\bar{w}_2(\xi,\eta,z,p)=\sum_{i=1}^{4} A_{2i}(z)\,x_{3i}\exp(-\lambda_i z) \tag{2.19}$$

$$\bar{T}_2(\xi,\eta,z,p)=\sum_{i=1}^{4} A_{2i}(z)\,x_{4i}\exp(-\lambda_i z) \tag{2.20}$$

where $x_{ij}=(x_i)_{\lambda=-\lambda_j}$ and $y_{ij}=(y_i)_{\lambda=-\lambda_j}$

Where forms of x_i and y_i ($i=1,2,...8$) are given in Appendix II.Also the expressions for the stresses $(\bar{\tau}_{xx})_2,(\bar{\tau}_{yy})_2,(\bar{\tau}_{zz})_2,(\bar{\tau}_{yz})_2,(\bar{\tau}_{zx})_2$ and $(\bar{\tau}_{xy})_2$ in the Laplace–double Fourier transform domain can be obtained from (1.2) applying the transforms as given in equations (2.1) using the expressions (2.17) – (2.20). Using the inverse Fourier transforms in the resulting expressions for temperature and stresses, we will get their integral representations of the temperature and stresses in the Laplace transform domain as

$$[\bar{T},\bar{\tau}_{xx},\bar{\tau}_{yy},\bar{\tau}_{zz}](x,y,z,p)$$

$$=\int_0^\infty\int_0^\infty \left[\bar{T}_2,(\bar{\tau}_{xx})_2,(\bar{\tau}_{yy})_2,(\bar{\tau}_{zz})_2\right]\cos\xi x\cos\eta y\,d\xi d\eta$$

$$\bar{\tau}_{xy}(x,y,z,p)=\int_0^\infty\int_0^\infty (\bar{\tau}_{xy})_2\sin\xi x\sin\eta y\,d\xi d\eta$$

$$\bar{\tau}_{zx}(x,y,z,p)=\int_0^\infty\int_0^\infty (\bar{\tau}_{zx})_2\sin\xi x\cos\eta y\,d\xi d\eta$$

$$\bar{\tau}_{yz}(x,y,z,p)=\int_0^\infty\int_0^\infty (\bar{\tau}_{yz})_2\cos\xi x\sin\eta y\,d\xi d\eta \tag{2.21}$$

The evaluation of these infinite integrals when all the terms are written out in full form become very unwidely and moreover we have to perform the inverse Laplace transform to these expressions in order to find the temperature and stresses in space time domain.

III. Numerical Solution

The Laplace–double Fourier inversion of the expressions for temperature and stresses in space–time domain are very complex and we prefer to develop an efficient computer programme for the inversion of these integral transforms. For the inversion of Laplace transform we follow the method of Bellman et al [21] and choose seven values of the time $t = t_i$; $i = 1, 2, 3, 4, 5, 6, 7$ as the time range at which the temperature and stresses are to be determined where t_i are the roots of the Legendre polynomial of degree seven. Simultaneous calculations for the inversion of double Fourier transforms were done by evaluating the infinite integrals (2.20) numerically by seven point Gaussian quadrature formula for several prescribed values of x, y and z.

The following data for the material Cobalt (considered as transversely isotropic) in SI units have been used vide Dhaliwal and Singh [22].

$$
\begin{aligned}
A_{11} &= 3.071 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{12} &= 1.650 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{13} &= 1.027 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{22} &= 3.071 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{23} &= 1.027 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{33} &= 3.581 \times 10^{11}\ \mathrm{Nm}^{-2}\\
A_{44} &= 1.027 \times 10^{11}\ \mathrm{Nm}^{-2}\\
\beta_1 &= \beta_2 = 7.04 \times 10^{6}\ \mathrm{Nm}^{-2}\ \mathrm{deg}^{-1}\\
\beta_3 &= 6.90 \times 10^{6}\ \mathrm{Nm}^{-2}\ \mathrm{deg}^{-1}\\
K_x &= K_y = K_z = 0.69 \times 10^{2}\ \mathrm{Wm}^{-1}\ \mathrm{deg}^{-1}\\
\rho &= 8.836 \times 10^{3}\ \mathrm{kg\ m}^{-3}\\
c &= 4.27 \times 10^{2}\ \mathrm{J\ kg}^{-1}\ \mathrm{deg}^{-1}\\
T_o &= 298^{o}\ \mathrm{K}
\end{aligned}
\tag{3.1}
$$

2. Concluding Remarks

In order to study the stress characteristic we have drawn seven graphs of the stress τ_{xx}, τ_{yy}, τ_{zz}, τ_{xy}, τ_{yz}, τ_{zx} and temperature for different values of the space variables at times

$t = 0.025775, 0.138382, 0.352509, 0.693147, 1.21376, 2.04612, 3.67119$. It is observed that

1. The characteristic of the stresses τ_{xx} and τ_{yy} for the material considered [as in (3.1)] are almost the same is respect of wave propagation.
2. For fixed values of y and z as x increases, the amplitude of τ_{xx}, τ_{zz}, τ_{xy}, τ_{yz}, τ_{zx} and T gradually decreases with same wave length as t increases.
3. For fixed values of x and z as y increases the amplitudes of τ_{xx}, τ_{zz}, τ_{xy}, τ_{yz}, τ_{zx} and T gradually decreases with same wave length, as t increases.
4. For fixed values of x, y and t

   i) τ_{xx}, τ_{yz}, τ_{zx}, τ_{xy} and T gradually decreases as z increases from 0.0001 to 1.

   ii) The amplitudes of τ_{zz} initially increases as z increases from 0.0001 to 0.001 then decreases as z increases from 0.001 to 1.
5. It has also been observed that when x and y assume relatively larger values than z then stress τ_{xy} assumes only positive values for time t_i.
6. For fixed values of x, y, z and t the absolute values of τ_{xx}, τ_{zz}, τ_{xy}, τ_{yz}, τ_{zx} and T increases as Ω varies from 0 to 10^4.

We have drawn the temperature and stresses for $x = y = 1$ and $z = 0.0001$ for values of time mentioned earlier. They are presented graphically with the help of a computer using cubic spline formation. It is found that the amplitudes of stresses and temperature increases with greater wave lengths as t increases.

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Appendix I

Solution of the vector matrix differential equation

Consider a vector matrix differential

$$\frac{dv}{dx} = Av + f(x) \tag{4.1}$$

with the condition

$$v(x_0) = C \tag{4.2}$$

where A is an n $\times$ n constant real matrix, C is given constant real n vector and f is real n vector function.

Let

$$v = X \exp(\lambda x) \tag{4.3}$$

be the solution of the homogeneous equation

$$\frac{dv}{dx} = Av \tag{4.4}$$

where λ is a scalar and X is an n vector independent of x. Substituting Eq.(4.3), we get ,

$$(AX - \lambda X)e^{\lambda x} = 0 \Rightarrow AX = \lambda X$$

This may be interpreted that λ is an eigenvalue of the matrix A and X the corresponding right eigenvector.Let $\lambda_1, \lambda_2, \lambda_3, ... \lambda_n$ be n distinct eigenvalues of the matrix A and $X_1, X_2, X_3, ... X_n$ be the corresponding right eigenvector of the matrix A. Then the vectors $X_1, X_2, X_3, ... X_n$ are linearly independent and so they form a basis of the space Γ^n,where Γ denotes the field of complex numbers. We can find the scalers $b_1, b_2, b_3, \ldots b_n$ such that

$$C = b_1X_1 + b_2X_2 + b_3X_3 \ldots b_nX_n$$

Choose $c_i = b_i e^{-\lambda_i x_0}$, (i= 1,2,3, ….n)

Let ,

$$u(x) = \sum_{i=1}^{n} c_i X_i e^{\lambda_i x} \tag{4.5}$$

Thus $u(x)$ is the solution of the eq(4.4) and

$$u(x_0) = \sum_{i=1}^{n} c_i X_i e^{\lambda_i x_0} = \sum_{i} b_i X_i = \text{C}$$

Now , let

$$w(x) = \sum_{i=1}^{n} a_i(x) X_i e^{\lambda_i x} \tag{4.6}$$

be the solution of Eq.(4.1), $a_1(x), a_2(x), a_3(x),,, a_n(x)$ are scalar function of x such that $a_i(x_0)=0$,

Differentiating Eq(4.6) with respect to ,x, we get

$$w'(x) = \sum_{i=1}^{n} a'_i(x) X_i e^{\lambda_i x} + \sum_{i=1}^{n} a_i(x) \lambda_i X_i e^{\lambda_i x} \tag{4.7}$$

Substituting Eq.(4.6) and (4.7) in Eq(4.1), we have

$$\sum_{i=1}^{n} a'_i(x) X_i e^{\lambda_i x} + \sum_{i=1}^{n} a_i(x) \lambda_i X_i e^{\lambda_i x} = \sum_{i=1}^{n} a_i(x) A X_i e^{\lambda_i x} + f(x)$$

or, $$\sum_{i=1}^{n} a'_i(x) X_i e^{\lambda_i x} = \sum_{i=1}^{n} a_i(x)[A X_i - \lambda_i X_i] e^{\lambda_i x} + f(x) = f(x) \tag{4.8}$$

multiplying Eq.(4.8) by $Y_j e^{-\lambda_j x}$ (where $Y_1, Y_2, Y_3, \ldots . Y_n$ are left eigenvector corresponding to the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots . \lambda_n$), we get

$$\sum_{i=1}^{n} a'_i(x) Y_j e^{(\lambda_i - \lambda_j)x} = Y_j f(x) e^{-\lambda_j x}$$

or, $a_j^{/}(x)\, Y_j\, X_j = Y_j\, f(x) e^{-\lambda_j x}$, $[Y_j\, X_j = 0$ for $i \neq j]$,

$$a_j^{/}(x) = \frac{1}{Y_j X_j} Y_j\, f(x) e^{-\lambda_j x}$$

$$a_j(x) = \int_{x_0}^{x} (Y_j X_j)^{-1} Y_j\, f(x) e^{-\lambda_j s} ds \tag{4.9}$$

$[a_j(x_o) = 0,$ for $j = 1,2,3......n]$

Now take

$$v(x) = u(x) + w(x) \tag{4.10}$$

Differentiatimg we get,

$$v^{/}(x) = u^{/}(x) + w^{/}(x) = Au(x) + Aw(x) + f(x) = A[u(x) + v(x)] + f(x)$$
$$= Av(x) + f(x)$$

$v^{/}(x_0) = u^{/}(x_0) + w^{/}(x_0) = C$.

Hence , $v(x) = u(x) + w(x)$ is the unique solution of the differential eq.(4.1), satisfying the condition (4.2).

Appendix II

We take,

$a_1 = (C_{51} - \lambda^2)\, C_{64} - C_{61}\, C_{54}$

$a_2 = C_{52}\, C_{64} - (C_{62} - \lambda^2)\, C_{54}$

$a_3 = C_{57}\, C_{64}\, \lambda - (C_{63} + \lambda\, C_{67})\, C_{54}$

$b_1 = \lambda(C_{61}\, C_{78} - C_{75}\, C_{64})$

$b_2 = (C_{62} - \lambda^2)\, \lambda\, C_{78} - (C_{72} + \lambda\, C_{76})\, C_{64}$

$b_3 = (C_{63} + \lambda\, C_{67})\, \lambda\, C_{78} - (C_{73} - \lambda^2)\, C_{64}$

$c_1 = \lambda\, C_{75}\, (C_{84} - \lambda^2) - \lambda\, C_{78}\, C_{81}$

$$c_2 = (C_{72} + \lambda C_{76})(C_{84} - \lambda^2) - C_{82} C_{78} \lambda$$

$$c_3 = (C_{73} - \lambda^2)(C_{84} - \lambda^2) - \lambda^2 C_{78} C_{87}$$

$$f_5 = (C_{51} - \lambda^2) C_{82} - C_{52} C_{81}$$

$$f_6 = C_{82} C_{61} - (C_{62} - \lambda^2) C_{81}$$

$$f_7 = C_{75} C_{82} \lambda - (C_{72} + C_{76} \lambda) C_{81}$$

$$g_5 = \lambda(C_{52} C_{87} - C_{57} C_{87})$$

$$g_6 = (C_{62} - \lambda^2) C_{87} \lambda - (C_{63} + C_{67}\lambda) C_{82}$$

$$g_7 = (C_{72} + C_{76} \lambda) C_{87} \lambda - (C_{73} - \lambda^2) C_{82}$$

$$h_5 = (C_{84} - \lambda^2) C_{57} \lambda - C_{54} C_{87} \lambda$$

$$h_6 = (C_{65} + C_{67} \lambda)(C_{84} - \lambda^2) - C_{64} C_{87} \lambda$$

$$h_7 = (C_{73} - \lambda^2)(C_{84} - \lambda^2) - C_{78} C_{87} \lambda^2$$

The components of $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ are as follows :–

$$x_1 = b_2 c_3 - b_3 c_2$$

$$x_2 = b_3 c_1 - b_1 c_3$$

$$x_3 = b_1 c_2 - b_2 c_1$$

$$x_4 = \frac{-1}{C_{54}} [(C_{51} - \lambda^2) x_1 + C_{52} x_2 + C_{57} \lambda x_3]$$

$$x_5 = \lambda x_1$$

$$x_6 = \lambda x_2$$

$$x_7 = \lambda x_3$$

$$x_8 = \lambda x_4$$

$$y_1 = \lambda y_5 - C_{75} y_5$$

$$y_2 = \lambda y_6 - C_{76} y_7$$

$$y_3 = - C_{57} y_5 - C_{67} y_6 + \lambda y_7 - C_{87} y_8$$

$$y_4 = - C_{78} y_7 + \lambda y_8$$

$$y_5 = g_6 h_7 - g_7 h_6$$

$$y_6 = g_7h_5 - g_5h_7$$

$$y_7 = g_5h_6 - g_6h_5$$

$$y_8 = -\frac{1}{C_{81}}\,[(C_{51} - \lambda^2)y_5 + C_{61}y_6 + C_{75}\,\lambda y_7]$$

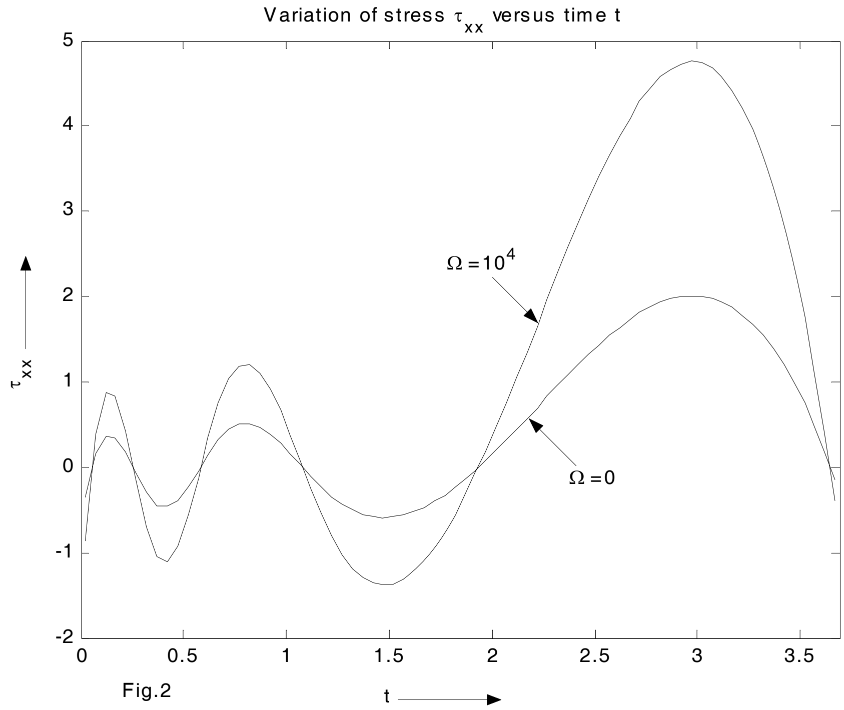


Fig.2

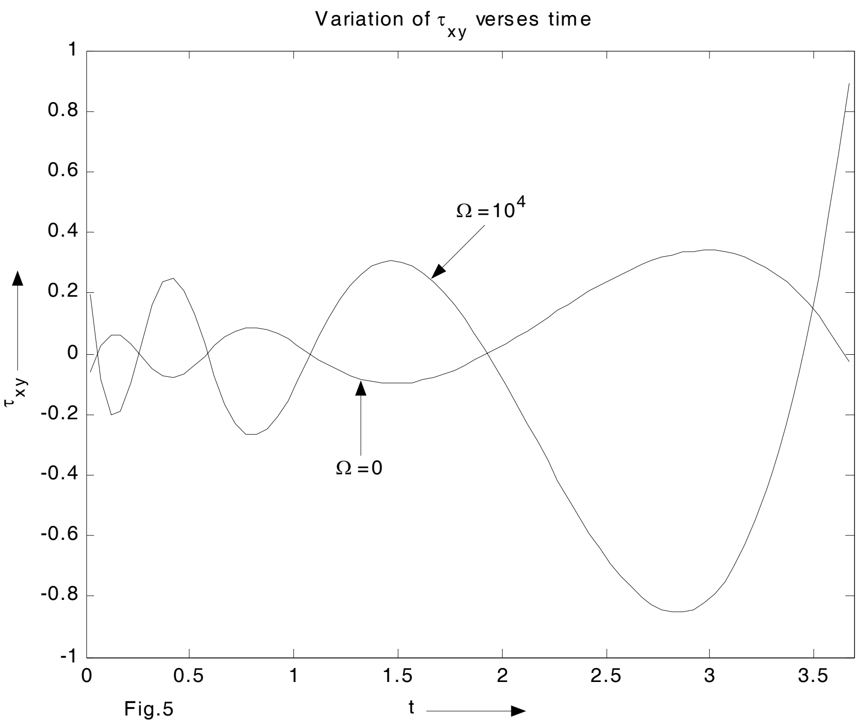

Variation of τ_{xy} verses time
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
τ_{xy}
$\Omega = 10^4$
$\Omega = 0$
0
0.5
1
1.5
2
2.5
3
3.5
t

Fig.5

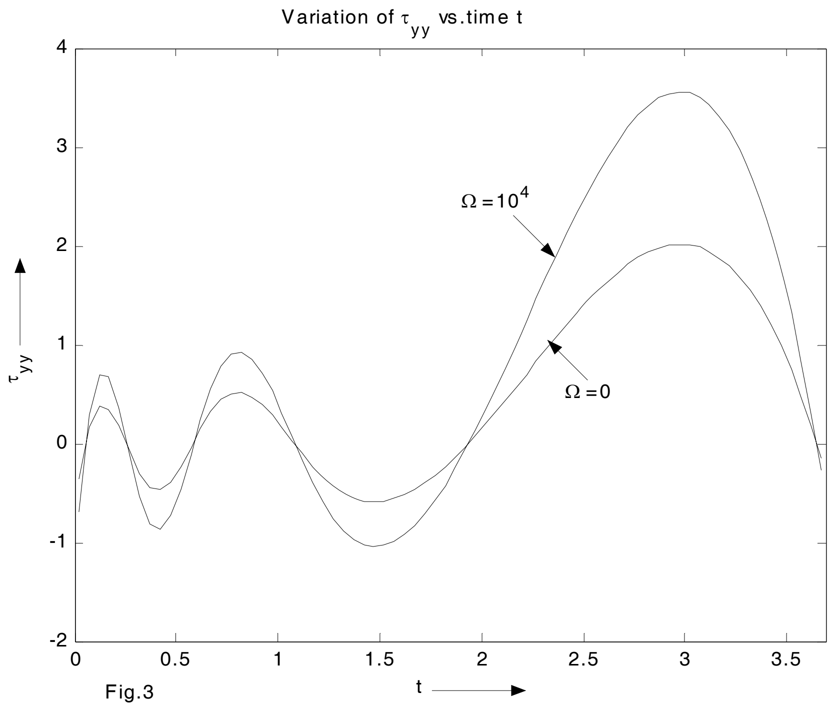
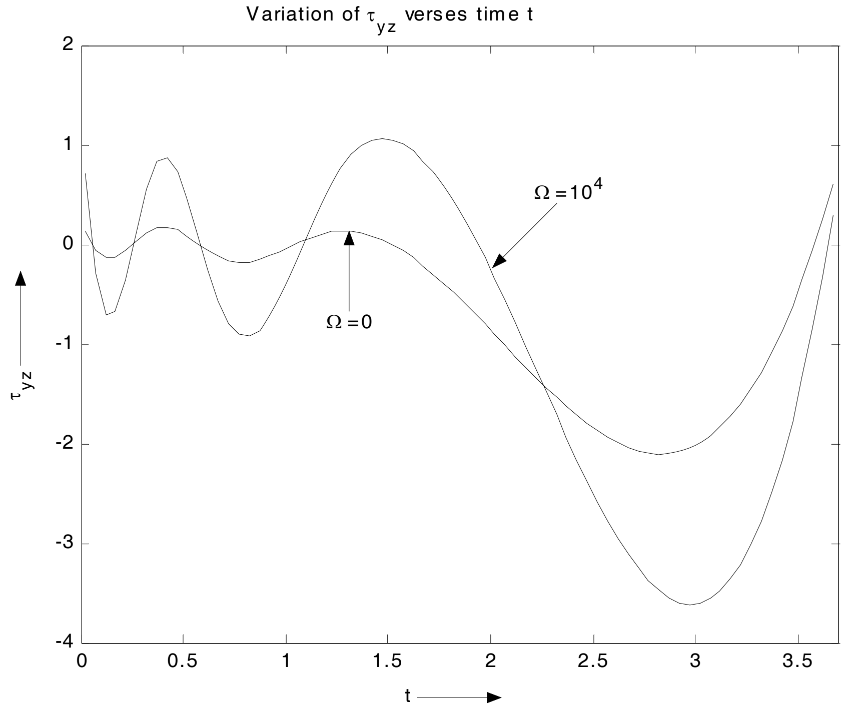

Variation of τ_{yy} vs.time t
4
3
2
1
0
-1
-2
τ_{yy}
$\Omega = 10^4$
$\Omega = 0$
0
0.5
1
1.5
2
2.5
3
3.5
t

Fig.3


Variation of τ_{yz} verses time t
$\Omega = 10^4$
$\Omega = 0$
τ_{yz}
t
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2
2.5
3
3.5

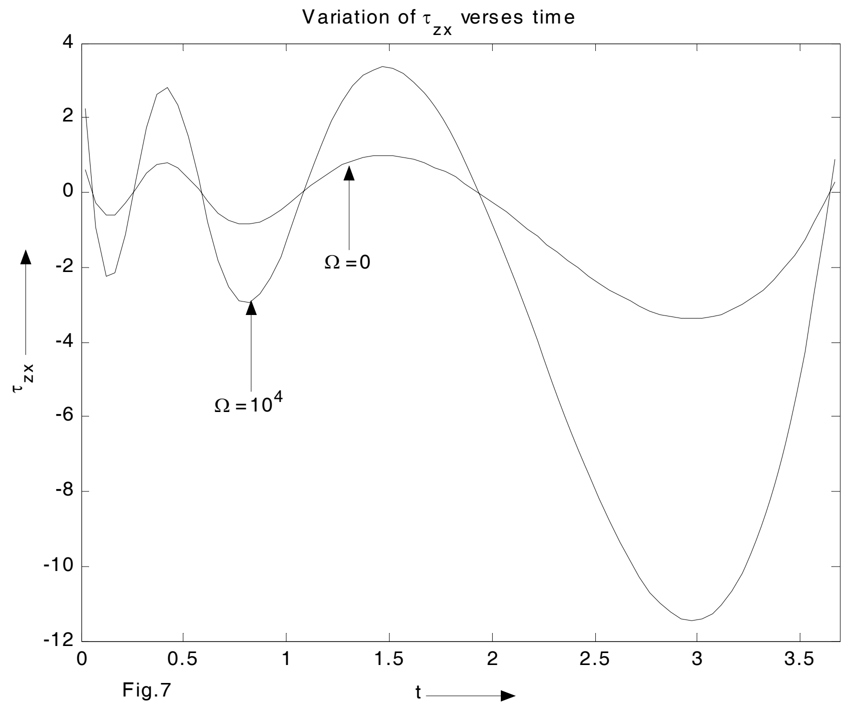

Variation of τ_{zx} verses time
$\Omega = 0$
$\Omega = 10^4$
τ_{zx}
t
4
2
0
-2
-4
-6
-8
-10
-12
0
0.5
1
1.5
2
2.5
3
3.5

Fig.7

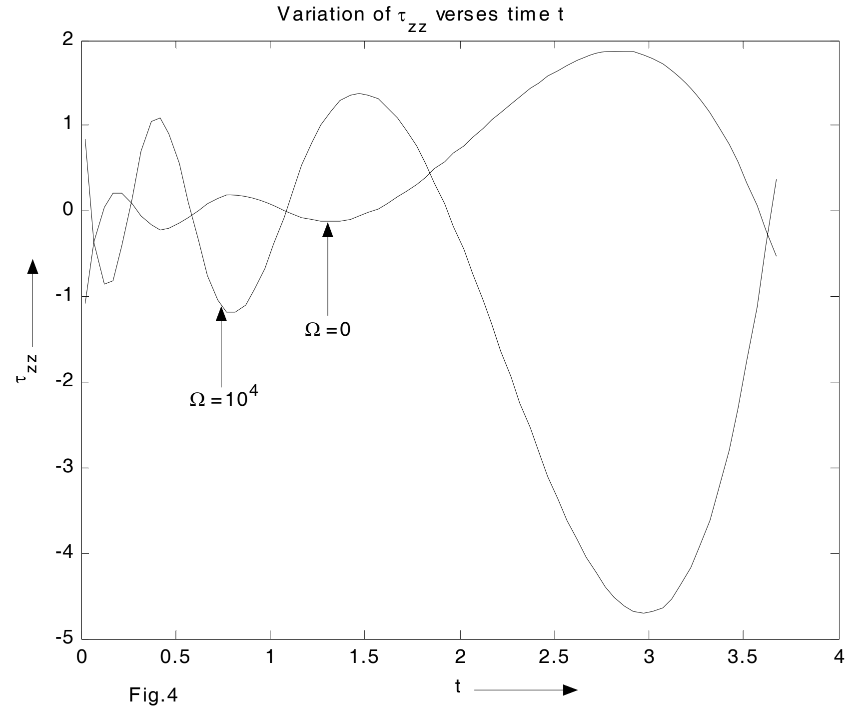

Variation of τ_{zz} verses time t
2
1
0
-1
-2
-3
-4
-5
τ_{zz}
0
0.5
1
1.5
2
2.5
3
3.5
4
t
$\Omega = 0$
$\Omega = 10^4$

Fig.4

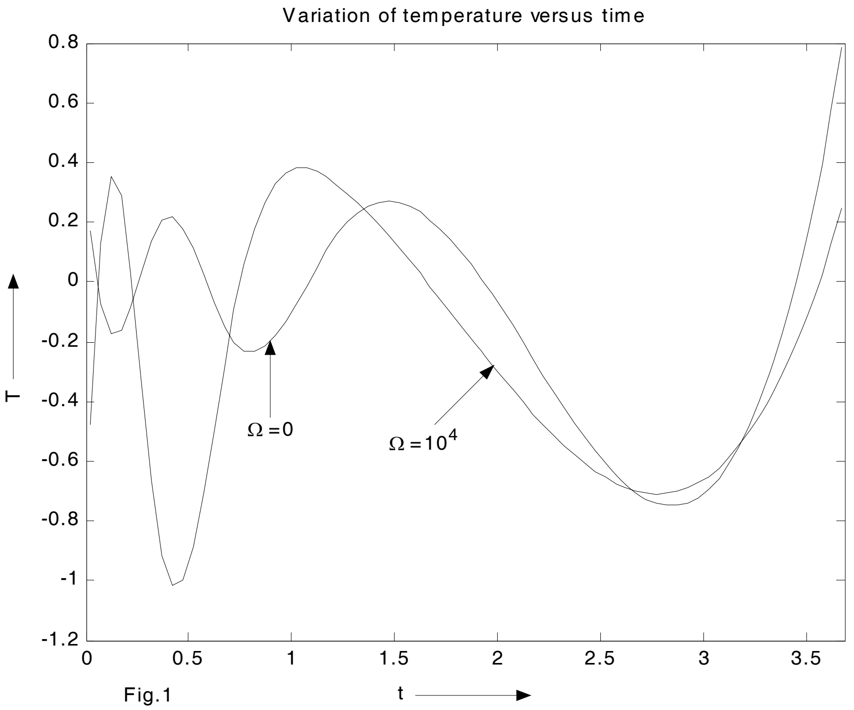

Variation of temperature versus time
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
T
0
0.5
1
1.5
2
2.5
3
3.5
t
$\Omega = 0$
$\Omega = 10^4$

Fig.1