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Univalence of a General Integral Operator Associated With the Generalized Hypergeometric Function *

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Abstract

By making use of the generalized hypergeometric function, we introduce a new family of integral operators and investigate their univalence properties. Relevant connections of the results, which are presented in this paper, with various other known results are also pointed out.

Keywords and Phrases: *Analytic functions, Univalent functions, Integral operators, Schwarz Lemma.*

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1. Introduction, Definitions and Preliminaries

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad a_n \geq 0, \tag{1}$$

which are analytic in the open disc $\mathcal{U} = \{z \in \mathbb{C} : | z |< 1\}$ and $\mathcal{S}$ be the class of functions $f \in \mathcal{A}$ which are univalent in $\mathcal{U}$.

The Hadamard product of two functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ and $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$ is given by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k\, b_k\, z^k.$$

For complex parameters $\alpha_1, \ldots, \alpha_q$ and $\beta_1, \ldots, \beta_s$ ($\beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^-$; $\mathbb{Z}_0^- = 0, -1, -2, \ldots$; $j = 1, \ldots, s$), we define the generalized hypergeometric function ${}_qF_s(\alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_s; z)$ by

$${}_qF_s(\alpha_1, \alpha_2, \ldots, \alpha_q; \beta_1, \beta_2, \ldots, \beta_s; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \ \ldots (\alpha_q)_k}{(\beta_1)_k \ \ldots (\beta_s)_k} \frac{z^k}{k!},$$

$$(q \leq s + 1;\ q, s \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}; z \in \mathcal{U})$$

where $\mathbb{N}$ denotes the set of all positive integers and $(x)_k$ is the Pochhammer symbol defined, in terms of the Gamma function Γ, by

$$(x)_k = \frac{\Gamma(x+k)}{\Gamma(x)} = \begin{cases} 1 & if\, k = 0 \\ x(x+1)(x+2) \ \ldots (x+k-1) & if\, k \in \mathbb{N} = \{1, 2, , \ldots\}. \end{cases}$$

Corresponding to a function $\mathcal{G}_{q,s}(\alpha_1, \beta_1; z)$ defined by

$$\mathcal{G}_{q,s}(\alpha_1, \beta_1; z) := z\, {}_qF_s(\alpha_1, \alpha_2, \ldots, \alpha_q; \beta_1, \beta_2, \ldots, \beta_s; z). \tag{2}$$

Recently, the authors [17] defined the linear operator $D_\lambda^m(\alpha_1, \beta_1)f : \mathcal{A} \longrightarrow \mathcal{A}$ by

$$D_\lambda^0(\alpha_1, \beta_1)f(z) = f(z) * \mathcal{G}_{q,s}(\alpha_1, \beta_1; z),$$

$$D_\lambda^1(\alpha_1, \beta_1)f(z) = (1-\lambda)(f(z) * \mathcal{G}_{q,s}(\alpha_1, \beta_1; z)) + \lambda\, z(f(z) * \mathcal{G}_{q,s}(\alpha_1, \beta_1; z))', \tag{3}$$

$$D_\lambda^m(\alpha_1,\,\beta_1)f(z)=D_\lambda^1(D_\lambda^{m-1}(\alpha_1,\,\beta_1)f(z)), \tag{4}$$

where ‘ $*$ ‘ denotes the usual Hadamard product of analytic functions.
If $f\in\mathcal{A}$, then from (3) and (4) we may easily deduce that

$$D_\lambda^m(\alpha_1,\,\beta_1)f(z)=z+\sum_{k=2}^{\infty}\big[1+(k-1)\lambda\big]^m\frac{(\alpha_1)_{k-1}\,\ldots\,(\alpha_q)_{k-1}}{(\beta_1)_{k-1}\,\ldots\,(\beta_s)_{k-1}}\,\frac{a_kz^k}{(k-1)!}, \tag{5}$$

where $m\in\mathbb{N}_0=\mathbb{N}\cup\{0\}$ and $\lambda\geq 0$. We remark that, for a choice of the parameter $m=0$, the operator $D_\lambda^0(\alpha_1,\,\beta_1)f$ reduces to the well-known Dziok-Srivastava operator for functions in $\mathcal{A}$ [8] (see also [1, 9]), for $q=2,\,s=1,\,\alpha_1=\beta_1,\,\alpha_2=1$, we get the operator introduced by F.M.Al-Oboudi [2] and for $q=2,\,s=1,\,\alpha_1=\beta_1,\,\alpha_2=1$ and $\lambda=1$, we get the operator introduced by G. Ş. Sălăgean [16]. Also many (well known and new) integral and differential operators can be obtained by specializing the parameters.

Using the operator $D_\lambda^m(\alpha_1,\,\beta_1)f$, we now introduce the following:
For $n\in\mathbb{N}\cup\{0\}$ and $\gamma_1,\,\gamma_2,\,\ldots,\,\gamma_n,\delta\in\mathbb{C}\setminus\{0,-1,-2,\ldots\}$, we define the integral operator $F_{\gamma_i,\delta}(\lambda,\,m;\alpha_1,\beta_1;z):\mathcal{A}^n\longrightarrow\mathcal{A}^n$ by

$$F_{\gamma_i,\delta}(\lambda,\,m;\alpha_1,\beta_1;z)=\left\{\delta\int_0^z t^{\delta-1}\prod_{i=1}^{n}\left(\frac{D_\lambda^m(\alpha_1,\,\beta_1)f_i(t)}{t}\right)^{\frac{1}{\gamma_i}}dt\right\}^{\frac{1}{\delta}}, \tag{6}$$

where $f_i\in\mathcal{A}$.

Remark 1.1 It is interesting to note that the integral operator $F_{\gamma_i,\delta}(\lambda,\,m;\alpha_1,\beta_1;z)$ generalizes many operators which were introduced and studied recently. Here we list a few of them

1. Let $q=2,\,s=1,\,\alpha_1=\beta_1,\,\alpha_2=1,\,\gamma_i=1/\alpha_i$ and $\delta=1$, then the operator $F_{\gamma_i,\delta}(\lambda,\,m;\alpha_1,\beta_1;z)$ reduces to the integral operator

$$I(f_1,\,\ldots,\,f_m)(z)=\int_0^z\left(\frac{D_\lambda^m f_1(t)}{t}\right)^{\alpha_1}\ldots\left(\frac{D_\lambda^m f_n(t)}{t}\right)^{\alpha_n}dt, \tag{7}$$

   where $D_\lambda^m f$ is the well known Al-Oboudi differential operator. $I(f_1,\,\ldots,\,f_m)$ was introduced and studied recently by S. Bulut in [7].

2. For $m = 0$, $q = 2$, $s = 1$, $\alpha_1 = \beta_1$, $\alpha_2 = 1$, $\gamma_i = 1/(\alpha - 1)$ and $\delta = n(\alpha - 1) + 1$, then the operator $F_{\gamma_i,\delta}(\lambda, m; \alpha_1, \beta_1; z)$ reduces to an integral operator

$$F_{n,\alpha}(z) = \left[(n(\alpha-1)+1)\int_0^z \left(f_1(t)\right)^{\alpha-1} \ldots \left(f_n(t)\right)^{\alpha-1} dt.\right]^{\frac{1}{(n(\alpha-1)+1)}}, \quad (8)$$

studied recently by D. Breaz, N. Breaz and H. M. Srivastava in [5].

3. Let $m = 0$, $q = 2$, $s = 1$, $\alpha_1 = \beta_1$, $\alpha_2 = 1$, $\gamma_i = 1/\alpha_i$ and $\delta = 1$, then the operator $F_{\gamma_i,\delta}(\lambda, m; \alpha_1, \beta_1; z)$ reduces to an operator

$$F_\alpha(z) = \int_0^z \left(\frac{f_1(t)}{t}\right)^{\alpha_1} \ldots \left(\frac{f_n(t)}{t}\right)^{\alpha_n} dt \quad (9)$$

recently introduced and studied by D.Breaz and N.Breaz in [3].

4. Let $m = 0$, $q = 2$, $s = 1$, $\alpha_1 = 2$, $\beta_1 = 1$, $\alpha_2 = 1$, $\gamma_i = 1/\alpha_i$ and $\delta = 1$, then the operator $F_{\gamma_i,\delta}(\lambda, m; \alpha_1, \beta_1; z)$ reduces to an operator

$$G_\alpha(z) = \int_0^z \left(f_1'(t)\right)^{\alpha_1} \ldots \left(f_n'(t)\right)^{\alpha_n} dt, \quad (10)$$

recently introduced and studied by D.Breaz and H. Ö. Güney in [4].

Apart from the above several well-known and new integral operators will follow as a special case on specializing the parameters.

We now state the following results which we need to establish our results in the sequel.

Theorem 1.2 *[11] If $f \in \mathcal{A}$ satisfies the condition*

$$\left|\frac{z^2 f'(z)}{f^2(z)} - 1\right| \leq 1, \quad \textit{for all } z \in \mathcal{U}, \quad (11)$$

then the function f is univalent in $\mathcal{U}$.

Theorem 1.3 *(Schwartz Lemma) Let $f \in \mathcal{A}$ satisfy the condition $\mid f(z) \mid \leq 1$, for all $z \in \mathcal{U}$. Then*

$$\mid f(z) \mid \leq \mid z \mid, \quad \textit{for all } z \in \mathcal{U},$$

and equality holds only if $f(z) = \epsilon z$, *where* $|\epsilon| = 1$.

Theorem 1.4[12] *Let* δ *be a complex number with* $Re\delta > 0$. *If* $f \in \mathcal{A}$ *satisfies*

$$\frac{1-|z|^{2\delta}}{Re\,\delta}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1 \quad (z \in \mathcal{U}),$$

then the function

$$F_\delta(z) = \left[\delta \int_0^z t^{\delta-1} f'(t)\,dt\right]^{\frac{1}{\delta}} = z + \ldots$$

is analytic and univalent in $\mathcal{U}$.

Pescar [13] has proved the following result

Theorem 1.5 [13]*Let* $\delta \in \mathbb{C}$, $Re\delta > 0$ *and* $c \in \mathbb{C}$ $(|c| \leq 1;\ c \neq -1)$.
If $f \in \mathcal{A}$ *satisfies*

$$\left|c\,|z|^{2\delta} + \left(1-|z|^{2\delta}\right)\frac{zf''(z)}{\delta f'(z)}\right| \leq 1 \quad (z \in \mathcal{U}),$$

then the function

$$F_\delta(z) = \left[\delta \int_0^z t^{\delta-1} f'(t)\,dt\right]^{\frac{1}{\delta}} = z + \ldots$$

is analytic and univalent in $\mathcal{U}$.

Theorem 1.6 [15]*Let* $f \in \mathcal{A}$ *satisfies the condition (11). Also let*

$$\alpha \in \mathbb{R}, \quad \left(\alpha \in \left[1, \frac{3}{2}\right]\right) \quad \text{and } c \in \mathbb{C}.$$

If

$$|c| \leq \frac{3-2\alpha}{\alpha} \quad (c \neq -1) \quad \text{and} \quad |f(z)| \leq 1 \ (z \in \mathcal{U})$$

then the function

$$H_\alpha(z) = \left(\alpha \int_0^z \left[g(t)\right]^{\alpha-1} dt\right)^{\frac{1}{\alpha}}$$

belongs to $\mathcal{S}$.

2. Main Results

Theorem 2.1 *Let each of the functions $D_\lambda^m(\alpha_1,\, \beta_1)f_i \in \mathcal{A}$ $(i \in \{1, 2,\, \ldots,\, n\})$ satisfy the inequality (11). Also for $M \geq 1$, let $\gamma_i,\, \delta$ be complex numbers such that*

$$Re\,\delta \geq \sum_{i=1}^{n} \frac{2M+1}{|\gamma_i|} \quad \textit{and } c \in \mathbb{C}.$$

If

$$|\, c \,| \leq 1 - \frac{1}{Re\delta} \sum_{i=1}^{n} \frac{2M+1}{|\gamma_i|} \tag{12}$$

and

$$|\, D_\lambda^m(\alpha_1,\, \beta_1)f_i(z) \,| \leq M \; (z \in \mathcal{U}; i \in \{1, 2,\, \ldots,\, n\}),$$

then the function $F_{\gamma_i,\delta}(\lambda,\, m; \alpha_1, \beta_1; z)$ defined by (6) is univalent.

Proof. From the definition of the operator $D_\lambda^m(\alpha_1,\, \beta_1)f$, it can be easily seen that

$$\frac{D_\lambda^m(\alpha_1,\, \beta_1)f(z)}{z} \neq 0 \quad (z \in \mathcal{U})$$

and moreover for $z = 0$, we have

$$\left(\frac{D_\lambda^m(\alpha_1,\, \beta_1)f_1(z)}{z}\right)^{\frac{1}{\gamma_1}} \cdots \left(\frac{D_\lambda^m(\alpha_1,\, \beta_1)f_n(z)}{z}\right)^{\frac{1}{\gamma_n}} = 1.$$

We define the function

$$h(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{D_\lambda^m(\alpha_1,\, \beta_1)f_i(t)}{t}\right)^{\frac{1}{\gamma_i}} dt,$$

so that, obviously

$$h'(z) = \prod_{i=1}^{n} \left(\frac{D_\lambda^m(\alpha_1,\, \beta_1)f_i(z)}{z}\right)^{\frac{1}{\gamma_i}}$$

and

$$\frac{zh''(z)}{h'(z)} = \sum_{i=1}^{n} \frac{1}{\gamma_i}\left(\frac{z(D_\lambda^m(\alpha_1,\, \beta_1)f_i(z))'}{D_\lambda^m(\alpha_1,\, \beta_1)f_i(z)} - 1\right). \tag{13}$$

So, from (13) we have

$$\begin{aligned} &\left| c|z|^{2\delta} + (1-|z|^{2\delta})\frac{zh''(z)}{\delta h'(z)} \right| \\ = &\left| c|z|^{2\delta} + (1-|z|^{2\delta})\frac{1}{\delta}\sum_{i=1}^{n}\frac{1}{\gamma_i}\left(\frac{z(D_\lambda^m(\alpha_1,\,\beta_1)f_i(z))'}{D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)} - 1\right) \right| \\ \leq &\ |c| + \frac{1}{|\delta|}\sum_{i=1}^{n}\frac{1}{|\gamma_i|}\left(\left|\frac{z^2(D_\lambda^m(\alpha_1,\,\beta_1)f_i(z))'}{\left[D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)\right]^2}\right|\frac{|D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)|}{|z|} + 1\right). \end{aligned}$$

Since $|\ D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)\ | \leq M\ (z \in \mathcal{U}; i \in \{1, 2,\, \ldots,\, n\})$, on applying the Theorem 1 we have,

$$|\ D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)\ | \leq M\ |\ z\ | \quad (z \in \mathcal{U}; i \in \{1, 2,\, \ldots,\, n\}).$$

By using the inequality (11), we obtain

$$\left| c|z|^{2\delta} + (1-|z|^{2\delta})\frac{zh''(z)}{\delta h'(z)} \right| \leq |c| + \frac{1}{Re\,\delta}\sum_{i=1}^{n}\frac{2M+1}{|\ \gamma_i\ |} \quad (z \in \mathcal{U}),$$

which, in the light of the hypothesis (12), yields

$$\left| c|z|^{2\delta} + (1-|z|^{2\delta})\frac{zh''(z)}{\delta h'(z)} \right| \leq 1 \quad (z \in \mathcal{U}).$$

Finally applying Theorem 1.3, we conclude that the function $F_{\gamma_i,\delta}(\lambda,\, m; \alpha_1, \beta_1; z)$ defined by (6) is univalent. This completes the proof of Theorem 2.1.

Upon setting $m = 0,\ q = 2,\ s = 1,\ \alpha_1 = \beta_1,\ \alpha_2 = 1,\ \gamma_i = 1/(\alpha - 1)$ and $\delta = n(\alpha - 1) + 1$ (α a real number), we obtain

Corollary 2.2 *[5]Let $M \geq 1$ and suppose that each of the functions $f_i \in \mathcal{A}$ (i=1, ..., n) satisfies the inequality (11). Also let*

$$\alpha \in \mathbb{R}, \quad \left(\alpha \in \left[1, \frac{(2M+1)n}{(2M+1)n-1}\right]\right) \quad \textit{and } c \in \mathbb{C}.$$

If

$$|\ c\ | \leq 1 + \left(\frac{1-\alpha}{\alpha}\right)(2M+1)n \quad \textit{and} \quad |\ f_i(z)\ | \leq M\ (z \in \mathcal{U};\ i \in \{1, \ldots, n\}),$$

then the function $F_{n,\alpha}(z)$ defined by (8) is in the univalent function class $\mathcal{S}$.

Remark 2.3 Corollary 2 provides an extension of Theorem 1 due to Pescar [15], if we let $n = 1$.

Next we set $m = 0$, $q = 2$, $s = 1$, $\alpha_1 = \beta_1$, $\alpha_2 = 1$, $\delta = 1$ and $\gamma_i = \frac{1}{\alpha_i}$ (α_i, a real number) , we obtain the following.

Corollary 2.4 *Let $M \geq 1$ and suppose that each of the functions $f_i \in \mathcal{A}$ (i=1, ..., n) satisfies the inequality (11). Also let*

$$\alpha \in \mathbb{R}, \quad \left(\alpha \in \left[1, \frac{(2M+1)n}{(2M+1)n-1}\right]\right) \quad \text{and } c \in \mathbb{C}.$$

If

$$|\, c \,| \leq 1 + \alpha(2M+1)n \quad \text{and} \quad |\, f_i(z) \,| \leq M \ (z \in \mathcal{U};\, i \in \{1, \ldots, n\}),$$

then the function $F_\alpha(z)$ defined by (9) is univalent.

Remark 2.5 Many other interesting corollaries and results can be obtained by specializing the parameters in Theorem 2.1, for example see [5, 6, 10].

We now prove another univalence criterion for univalence of the integral operator $F_{\gamma_i,\delta}(\lambda,\, m; \alpha_1, \beta_1; z)$ in the unit disc $\mathcal{U}$.

Theorem 2.6 *Let each of the functions $D_\lambda^m(\alpha_1,\, \beta_1) f_i \in \mathcal{S}$ $(i \in \{1, 2,\, \ldots,\, n\})$ and δ be a complex number with $Re\delta > 0$. If*

$$\frac{1}{|\gamma_1|} + \frac{1}{|\gamma_2|} + \ \ldots \ + \frac{1}{|\gamma_n|} \leq 1/4,$$

then the function $F_{\gamma_i,\delta}(\lambda,\, m; \alpha_1, \beta_1; z)$ is in the class $\mathcal{S}$.

Proof. Let $h(z)$ be defined as in Theorem (2.1). From (13) we have

$$\frac{zh''(z)}{h'(z)} = \sum_{i=1}^{n} \frac{1}{\gamma_i} \left(\frac{z(D_\lambda^m(\alpha_1,\, \beta_1) f_i(z))'}{D_\lambda^m(\alpha_1,\, \beta_1) f_i(z)} - 1 \right).$$

and

$$\frac{1-|z|^{2Re\delta}}{\mathrm{Re}\,(\delta)}\left|\frac{zh''(z)}{h'(z)}\right| = \frac{1-|z|^{2Re\,\delta}}{\mathrm{Re}\,(\delta)}\sum_{i=1}^{n}\frac{1}{|\gamma_i|}\left|\frac{z(D_\lambda^m(\alpha_1,\,\beta_1)f_i(z))'}{D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)}-1\right|$$
$$\leq \frac{1-|z|^{2Re\,\delta}}{\mathrm{Re}\,(\delta)}\sum_{i=1}^{n}\frac{1}{|\gamma_i|}\left(\left|\frac{z(D_\lambda^m(\alpha_1,\,\beta_1)f_i(z))'}{D_\lambda^m(\alpha_1,\,\beta_1)f_i(z)}\right|+1\right)$$
$$\leq \frac{1-|z|^{2Re\,\delta}}{\mathrm{Re}\,(\delta)}\sum_{i=1}^{n}\frac{1}{|\gamma_i|}\left(\frac{1+|z|}{1-|z|}+1\right)$$
$$= \frac{1-|z|^{2Re\,\delta}}{1-|z|}\frac{2}{\mathrm{Re}\,(\delta)}\sum_{i=1}^{n}\frac{1}{|\gamma_i|}.$$

Here, we note that

$$\frac{1-|z|^{2Re\,\delta}}{1-|z|}\leq\begin{cases}1 & if\; 0<\mathrm{Re}\,(\delta)<\frac{1}{2}\\ 2\,\mathrm{Re}\,(\delta) & if\; \frac{1}{2}\leq\mathrm{Re}\,(\delta)<\infty.\end{cases} \tag{14}$$

Using the inequality (14) and the hypothesis of the theorem, we obtain

$$\frac{1-|z|^{2Re\delta}}{\mathrm{Re}\,(\delta)}\left|\frac{zh''(z)}{h'(z)}\right|\leq 1. \tag{15}$$

Hence, by applying Theorem 1, we conclude that the function $F_{\gamma_i,\delta}(\lambda,\, m;\alpha_1,\beta_1;z)$ defined by (6) is univalent. This completes the proof of Theorem 2.

Let $n=1,\ m=0,\ q=2,\ s=1,\ \alpha_1=\beta_1,\ \alpha_2=1,\ \gamma_1=1/\delta$ in Theorem 2. Then, we have the following

Corollary 2.7 *[14] Let $f\in\mathcal{S}$ and $\delta=a+bi$ be a complex number and $\delta\in(0,4]$. If*

$$a^4+a^2b^2-4\geq 0,\ a\in\left(0,\frac{1}{2}\right)\quad and\quad a^2+b^2-16\geq 0,\ a\in\left[\frac{1}{2},4\right]$$

then the function

$$F_\delta(z)=\left[\delta\int_0^z t^{\delta-1}\left(\frac{f(t)}{t}\right)^{\frac{1}{\delta}}dt\right]^{\frac{1}{\delta}} \tag{16}$$

is in the class $\mathcal{S}$.

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References

[1] R.Aghalary, S. B. Joshi, R. N. Mohapatra, and V. Ravichandran, Subordinations for analytic functions defined by the Dziok-Srivastava linear operator, *Appl. Math. Comput.* **187** (2007), no. 1, 13–19.

[2] F. M. Al-Oboudi, On univalent functions defined by a generalized Sălăgean operator, *Int. J. Math. Math. Sci.* **2004**, no. 25-28, 1429–1436.

[3] D. Breaz and N. Breaz, Two integral operators, *Studia Univ. Babeş-Bolyai Math.* **47** (2002), no. 3, 13–19.

[4] D. Breaz and H. Ö. Güney, The integral operator on the classes $\mathcal{S}_\alpha^*(b)$ and $\mathcal{C}_\alpha(b)$, *J. Math. Inequal.* **2** (2008), no. 1, 97–100.

[5] D. Breaz, N. Breaz, and H. M. Srivastava, An extension of the univalent condition for a family of operators, *Appl. Math. Lett.* **22** (2009), no. 1, 41–44.

[6] D. Breaz and H. Ö. Güney, On the univalence criterion of a general integral operator, *J. Inequal. Appl.* **2008**, Article ID 702715, 8.

[7] S. Bulut, Sufficient conditions for univalence of an integral operator defined by Al-Oboudi differential operator, *J. Inequal. Appl.* **2008**, Art. ID 957042, 5.

[8] J. Dziok and H. M. Srivastava, Classes of analytic functions associated with the generalized hypergeometric function, *Appl. Math. Comput.*, **103**(1999), 1-13.

[9] J. Dziok and H. M. Srivastava, Certain subclasses of analytic functions associated with the generalized hypergeometric function, *Integral Transforms Spec. Funct.* **14** (2003), no. 1, 7–18.

[10] G. I. Oros, G. Oros, and D. Breaz, Sufficient conditions for univalence of an integral operator, *J. Inequal. Appl.* **2008**, Art. ID 127645, 7.

[11] S. Ozaki and M. Nunokawa, The Schwarzian derivative and univalent functions, *Proc. Amer. Math. Soc.,* **33** (1972), 392–394.

[12] N. N. Pascu, On a univalence criterion (II) , Itinerant seminar on functional equations, approximation and convexity, Cluj- Napoca, *Preprint Nr.* **6**(1985), 153–154.

[13] V. Pescar, A new generalization of Ahlfors's and Becker's criterion of univalence, *Bull. Malaysian Math. Soc.* (2) **19** (1996), no. 2, 53–54.

[14] V. Pescar, New univalence criterion for certain integral operators, *Studia Univ. Babeş-Bolyai Math.* **46** (2001), no. 2, 107–109.

[15] V. Pescar, On the univalence of some integral operators, *J. Indian Acad. Math.* **27** (2005), no. 2, 239–243.

[16] G. Ş. Sălăgean, Subclasses of univalent functions, in *Complex analysis—fifth Romanian-Finnish seminar, Part 1 (Bucharest, 1981)*, 362–372, Lecture Notes in Math., 1013, Springer, Berlin.

[17] C. Selvaraj and K. R. Karthikeyan, Differential sandwich theorems for certain subclasses of analytic functions, *Math. Commun.* **13** (2008), no. 2, 311–319.