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Extended Cesàro Operators from Mixed Norm Spaces to Zygmund Type Spaces*

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Abstract

Let $H(B)$ denote the space of all holomorphic functions on the unit ball B of $\mathbb{C}^n$ and $\Re h(z) = \sum_{j=1}^n z_j \frac{\partial h}{\partial z_j}(z)$ the radial derivative of h. In this paper we study the boundedness and compactness of the extended Cesàro operator

$$T_g(f)(z) = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \quad f \in H(B),\ z \in B,$$

from mixed norm spaces to Zygmund type spaces.

Keywords and Phrases: *Mixed norm space, Zygmund type space, Extended Cesàro operator.*

1. Introduction

Let B be the unit ball of $\mathbb{C}^n$ and dv the normalized Lebesgue measure on B. Let $H(B)$ be the class of all holomorphic functions in B and $\Re f(z) =$

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$\sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z)$ stand for the radial derivative of $f \in H(B)$. It is well known that, if $f = \sum_\alpha a_\alpha z^\alpha \in H(B)$, then

$$\Re f(z) = \sum_\alpha |\alpha| a_\alpha z^\alpha,$$

where α is a multi-index. We write $\Re^2 f$ for $\Re(\Re f)$ (see [25]).

A positive continuous function μ on the interval $[0,1)$ is called normal if there is $\delta \in [0,1)$ and s and t, $0 < s < t$ such that

$$\begin{aligned} &\frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta,1) \text{ and } \lim_{r\to 1} \frac{\mu(r)}{(1-r)^s} = 0; \\ &\frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta,1) \text{ and } \lim_{r\to 1} \frac{\mu(r)}{(1-r)^t} = \infty. \end{aligned} \tag{1}$$

If we say that $\mu : B \to [0,\infty)$ is normal we will assume that $\mu(z) = \mu(|z|), z \in B$ (see, e.g. [4, 21]).

Let ν be a normal function on $[0,1)$. For $0 < p, q < \infty$, the mixed norm space $H(p,q,\nu) = H(p,q,\nu)(B)$ consists of all $f \in H(B)$ such that

$$\|f\|_{H(p,q,\nu)} = \left(\int_0^1 M_q^p(f,r) \frac{\nu^p(r)}{1-r} dr \right)^{1/p} < \infty,$$

where

$$M_q(f,r) = \left(\int_S |f(r\zeta)|^q d\sigma(\zeta) \right)^{1/q}.$$

For $p = q$ and $\varphi(r) = (1-r^2)^{\frac{\alpha+1}{p}}$, the mixed norm space is equivalent to the weighted Bergman space $A^p_\alpha = A^p_\alpha(B)$, which consists of all $f \in H(B)$ such that

$$\|f\|^p_{A^p_\alpha} = \int_B |f(z)|^p (1-|z|^2)^\alpha dv(z) < \infty.$$

Recall that the Bloch space $\mathcal{B} = \mathcal{B}(B)$ is the space of all $f \in H(B)$ for which (see [25])

$$b(f) = \sup_{z\in B} (1-|z|^2)\, |\Re f(z)| < \infty.$$

Under the norm introduced by $\|f\|_{\mathcal{B}} = |f(0)| + b(f)$, $\mathcal{B}$ is a Banach space.

Let $\Lambda = \Lambda(B)$ denote the class of all $f \in H(B)$ such that

$$\sup_{z\in B} (1-|z|^2) |\Re^2 f(z)| < \infty. \tag{2}$$

Denote by $A(B)$ the ball algebra on B. From [25, p. 261] we see that $f \in \Lambda$ if and only if $f \in A(B)$ and there exists a constant $C > 0$ for which

$$|f(\zeta + h) + f(\zeta - h) - 2f(\zeta)| < Ch,$$

for all $\zeta \in \partial B$ and $\zeta \pm h \in \partial B$, the boundary of B. Write

$$\|f\|_\Lambda = |f(0)| + \sup_{z\in B}(1 - |z|^2)|\Re^2 f(z)|. \tag{3}$$

From [25] we see that Λ is a Banach space with the norm $\|\cdot\|_\Lambda$. Λ is called the Zygmund space. Let Λ_0 denote the class of all $f \in H(B)$ such that

$$\lim_{|z|\to 1}(1 - |z|^2)|\Re^2 f(z)| = 0.$$

It is natural to generalize the Zygmund space to a more general form. Let $\mu :: B \to [0, \infty)$ be normal. An $f \in H(B)$ is said to belong to the Zygmund type space, denoted by $\Lambda_\mu = \Lambda_\mu(B)$, if

$$\sup_{z\in B}\mu(z)|\Re^2 f(z)| < \infty. \tag{4}$$

Under the norm

$$\|f\|_{\Lambda_\mu} = |f(0)| + \sup_{z\in B}\mu(z)|\Re^2 f(z)|, \tag{5}$$

Λ_μ becomes a Banach space. Let $\Lambda_{\mu,0}$ denote the subspace of Λ_μ consisting of those $f \in \Lambda_\mu$ such that

$$\lim_{|z|\to 1}\mu(z)|\Re^2 f(z)| = 0, \tag{6}$$

which will be called the little Zygmund type space.

Suppose that $g \in H(B)$. We consider the extended Cesàro operator (or the Riemann-Stieltjes operator) T_g as follows

$$T_g f(z) = \int_0^1 f(tz)\frac{dg(tz)}{dt} = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \qquad f \in H(B), \quad z \in B. \tag{7}$$

This operator was introduced in [4], and was studied in [1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 22, 23, 24, 26, 27]. For example, the boundedness and compactness of the extended Cesàro operator on Bloch spaces and Bloch type spaces, were studied in [5, 10, 19, 22, 23, 26, 27]. In [16], Li and Stević studied the boundedness and compactness of the extended Cesàro operator on Zygmund spaces.

In this paper, we study the operator T_g from mixed norm spaces to Zygmund type spaces. Some sufficient and necessary conditions for the operator T_g to be bounded or compact are given.

Throughout the paper, constants are denoted by C, they are positive and may not be the same in every occurrence.

2. Auxiliary Results

In order to prove the main results of this paper, we need some lemmas which follows.

Lemma 1. ([4]) *For every $f, g \in H(B)$ it holds $\Re[T_g(f)](z) = f(z)\Re g(z)$.*

Lemma 2. *Assume that $0 < p, q < \infty$, $g \in H(B)$, $\nu : B \to [0, \infty)$ and $\mu : B \to [0, \infty)$ are normal. Then $T_g : H(p, q, \nu) \to \Lambda_\mu$ is compact if and only if $T_g : H(p, q, \nu) \to \Lambda_\mu$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H(p, q, \nu)$ which converges to zero uniformly on compact subsets of B as $k \to \infty$, we have $\|T_g f_k\|_{\Lambda_\mu} \to 0$ as $k \to \infty$.*

Proof. The proof follows by standard arguments similar to those outlined in Proposition 3.11 of [3]. We omit the details.

Lemma 3. *Assume that $\mu : B \to [0, \infty)$ is normal. A closed set K in $\Lambda_{\mu,0}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(z)|\Re^2 f(z)| = 0.$$

Proof. The proof is similar to the proof of Lemma 1 in [18]. We omit the details.

Lemma 4. ([20]) *Assume that $0 < p, q < \infty$ and $\nu : B \to [0, \infty)$ is normal. If $f \in H(p, q, \nu)$, then there is a positive constant C independent of f such that*

$$|f(z)| \le C \frac{\|f\|_{H(p,q,\nu)}}{\nu(z)(1 - |z|^2)^{\frac{n}{q}}}, \qquad z \in B. \tag{8}$$

Similarly to the proof of Lemma 1 of [20] and by using the following well-known asymptotic formula (see, e.g. [4])

$$\int_0^1 M_q^p(f,r)\frac{\nu^p(r)}{1-r}dr \asymp |f(0)|^q + \int_0^1 M_q^p(\Re f,r)\nu^p(r)(1-r)^{p-1}dr$$

we obtain the following lemma.

Lemma 5. *Assume that $0 < p,q < \infty$ and $\nu : B \to [0,\infty)$ is normal. If $f \in H(p,q,\nu)$, then there is a positive constant C independent of f such that*

$$|\Re f(z)| \le C\frac{\|f\|_{H(p,q,\nu)}}{\nu(z)(1-|z|^2)^{\frac{n}{q}+1}}, \qquad z \in B. \tag{9}$$

3. Main Results and Proofs

In this section, we give our main results and proofs.

Theorem 1. *Assume that $0 < p,q < \infty$, $g \in H(B)$, $\nu : B \to [0,\infty)$ and $\mu : B \to [0,\infty)$ are normal. Then $T_g : H(p,q,\nu) \to \Lambda_\mu$ is bounded if and only if*

$$M_1 := \sup_{z\in B} \frac{\mu(z)|\Re g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}+1}} < \infty \tag{10}$$

and

$$M_2 := \sup_{z\in B} \frac{\mu(z)|\Re^2 g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}}} < \infty. \tag{11}$$

Proof. Assume that (10) and (11) hold. For any $f \in H(p,q,\nu)$, using Lemmas 1, 4 and 5 we have

$$\begin{aligned} &\mu(z)|\Re^2(T_g f)(z)| = \mu(z)|\Re f(z)\Re g(z) + f(z)\Re^2 g(z)| \\ \le\ & C\|f\|_{H(p,q,\nu)}\frac{\mu(z)|\Re g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}+1}} + C\|f\|_{H(p,q,\nu)}\frac{\mu(z)|\Re^2 g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}}}. \end{aligned} \tag{12}$$

On the other hand, we have that $T_g(f)(0) = 0$. On account of the conditions (10) and (11), the boundedness of the operator $T_g : H(p,q,\nu) \to \Lambda_\mu$ follows from (12) by taking the supremum over B.

Conversely, assume that $T_g : H(p,q,\nu) \to \Lambda_\mu$ is bounded. By taking the function $f(z) = 1$, we get $g \in \Lambda_\mu$, i.e.

$$\sup_{z\in B} \mu(z)|\Re^2 g(z)| < \infty. \tag{13}$$

Taking the functions given by $f_j(z) = z_j, \quad j = 1, \dots, n$, we obtain

$$\sup_{z\in B} \mu(z)|z_j \Re g(z) + z_j \Re^2 g(z)| < \infty, \quad j = 1, \dots, n. \tag{14}$$

(14) together with $g \in \Lambda_\mu$ imply

$$\sup_{z\in B} \mu(z)|z_j \Re g(z)| < \infty, \tag{15}$$

for each $j = 1, \dots, n$. From (15) we get

$$\sup_{1/3<|z|<1} \mu(z)|\Re g(z)| \le 3 \sup_{1/3<|z|<1} \sum_{j=1}^{n} \mu(z)|z_j||\Re g(z)| < \infty,$$

from which we obtain

$$\sup_{z\in B} \mu(z)|\Re g(z)| < \infty. \tag{16}$$

For $a \in B$, set

$$f_a(z) = \frac{(1-|a|^2)^{t+1}}{\nu(a)(1-\langle z,a\rangle)^{\frac{n}{q}+t+1}}. \tag{17}$$

From [20] we see that $f_a \in H(p,q,\nu)$ and $\sup_{a\in B} \|f_a\|_{H(p,q,\nu)} < \infty$. By Lemma 1 we have

$$\begin{aligned} \infty > \|T_g f_a\|_{\Lambda_\mu} &= \sup_{z\in B} \mu(z)|\Re^2(T_g f_a)(z)| = \sup_{z\in B} \mu(z)|\Re(f_a \cdot \Re g)(z)| \\ &= \sup_{z\in B} \mu(z)|\Re f_a(z) \Re g(z) + f_a(z) \Re^2 g(z)| \\ &\ge (\frac{n}{q}+t+1)\frac{\mu(a)|a|^2|\Re g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}+1}} - \frac{\mu(a)|\Re^2 g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}}}. \end{aligned} \tag{18}$$

Set

$$h_a(z) = \frac{1}{\frac{n}{q}+t+1}\frac{(1-|a|^2)^{t+1}}{\nu(a)(1-\langle z,a\rangle)^{\frac{n}{q}+t+1}} - \frac{1}{\frac{n}{q}+t+2}\frac{(1-|a|^2)^{t+2}}{\nu(a)(1-\langle z,a\rangle)^{\frac{n}{q}+t+2}}. \quad (19)$$

Then, as the case of f_a, $h_a \in H(p,q,\nu)$ and $\sup_{a\in B}\|h_a\|_{H(p,q,\nu)} < \infty$. Since

$$\Re h_a(a) = 0, \quad h_a(a) = \frac{1}{\frac{n}{q}+t+1}\frac{1}{\frac{n}{q}+t+2}\frac{1}{\nu(a)(1-|a|^2)^{\frac{n}{q}}},$$

we obtain

$$\begin{aligned}
\infty > \|T_g h_a\|_{\Lambda_\mu} &= \sup_{z\in B}\mu(z)|\Re h_a(z)\Re g(z) + h_a(z)\Re^2 g(z)| \\
&\geq \mu(a)|\Re h_a(a)\Re g(a) + h_a(a)\Re^2 g(a)| \\
&= \mu(a)|h_a(a)\Re^2 g(a)| \\
&= \frac{1}{\frac{n}{q}+t+1}\frac{1}{\frac{n}{q}+t+2}\frac{\mu(a)|\Re^2 g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}}}, \qquad (20)
\end{aligned}$$

which means that (11) holds by the arbitrary of $a \in B$.

From (18) and (20) we obtain

$$\sup_{a\in B}\frac{\mu(a)|a|^2|\Re g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}+1}} < \infty, \qquad (21)$$

which implies that

$$\sup_{|a|>\frac{1}{3}}\frac{\mu(a)|\Re g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}+1}} < 9\sup_{|a|>\frac{1}{3}}\frac{\mu(a)|a|^2|\Re g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}+1}} < \infty. \qquad (22)$$

From (16),

$$\sup_{|a|\leq\frac{1}{3}}\frac{\mu(a)|\Re g(a)|}{\nu(a)(1-|a|^2)^{\frac{n}{q}+1}} < C\sup_{|a|\leq\frac{1}{3}}\mu(a)|\Re g(a)| < \infty. \qquad (23)$$

Combining (22) with (23) we get (10). The proof of this theorem is completed. □

Theorem 2. *Assume that* $0 < p, q < \infty$, $g \in H(B)$, $\nu : B \to [0, \infty)$ *and* $\mu : B \to [0, \infty)$ *are normal. Then* $T_g : H(p, q, \nu) \to \Lambda_{\mu,0}$ *is bounded if and only if* $T_g : H(p, q, \nu) \to \Lambda_\mu$ *is bounded,* $g \in \Lambda_{\mu,0}$ *and*

$$\lim_{|z|\to 1} \mu(z)|\Re g(z)| = 0. \tag{24}$$

Proof. Assume $T_g : H(p, q, \nu) \to \Lambda_{\mu,0}$ is bounded. Then it is clear that $T_g : H(p, q, \nu) \to \Lambda_\mu$ is bounded. Taking $f = 1$, from the boundedness of $T_g : H(p, q, \nu) \to \Lambda_{\mu,0}$ it follows that $g \in \Lambda_{\mu,0}$. By taking the functions given by $f_j(z) = z_j, \quad j = 1, \ldots, n$, and similarly to the proof of the Theorem 6 of [17], we see that (24) holds .

Conversely, suppose that $T_g : H(p, q, \nu) \to \Lambda_\mu$ is bounded, $g \in \Lambda_{\mu,0}$ and (24) holds. For any polynomial $p(z)$,

$$\mu(z)|\Re^2(T_g p)(z)| \le \mu(z)|\Re g(z)||\Re p(z)| + \mu(z)|\Re^2 g(z)||p(z)|. \tag{25}$$

From $g \in \Lambda_{\mu,0}$ and (24), we have that $T_g p \in \Lambda_{\mu,0}$. Since the set of all polynomials is dense in $H_{p,q,\nu}$ we have that for each $f \in H_{p,q,\nu}$, there exist a sequence of polynomials $(p_k)_{k\in\mathbb{N}}$ such that $\|f - p_k\|_{H_{p,q,\nu}} \to 0$, as $k \to \infty$. From the boundedness of the operator $T_g : H_{p,q,\nu} \to \Lambda_\mu$, we have

$$\|T_g f - T_g p_k\|_{\Lambda_\mu} \le \|T_g\|_{H_{p,q,\nu}\to\Lambda_\mu} \|f - p_k\|_{H_{p,q,\nu}}.$$

From this and since $\Lambda_{\mu,0}$ is closed, we obtain

$$T_g f = \lim_{k\to\infty} T_g p_k \in \Lambda_{\mu,0},$$

which together with the boundedness of $T_g : H_{p,q,\nu} \to \Lambda_\mu$ implies the boundednes of $T_g : H_{p,q,\nu} \to \Lambda_{\mu,0}$. The proof of this theorem is completed. □

Theorem 3. *Assume that* $0 < p, q < \infty$, $g \in H(B)$, $\nu : B \to [0, \infty)$ *and* $\mu : B \to [0, \infty)$ *are normal. Then the following statements are equivalent.*

(a) $T_g : H(p, q, \nu) \to \Lambda_\mu$ *is compact;*

(b) $T_g : H(p, q, \nu) \to \Lambda_{\mu,0}$ *is compact;*

(c)

$$\lim_{|z|\to 1} \frac{\mu(z)|\Re g(z)|}{\nu(z)(1 - |z|^2)^{\frac{n}{q}+1}} = 0 \tag{26}$$

and

$$\lim_{|z|\to 1} \frac{\mu(z)|\Re^2 g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}}} = 0. \tag{27}$$

Proof. $(c) \Rightarrow (b)$. Assume that (c) holds. Then, for any $f \in H(p,q,\nu)$, it follows from Lemmas 4 and 5 that

$$\begin{aligned} &\mu(z)|\Re^2(T_g f)(z)| \\ \leq\ & C\|f\|_{H(p,q,\nu)} \frac{\mu(z)|\Re g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}+1}} + C\|f\|_{H(p,q,\nu)} \frac{\mu(z)|\Re^2 g(z)|}{\nu(z)(1-|z|^2)^{\frac{n}{q}}}. \end{aligned}$$

Employing Lemma 3 and condition (c), we see that $T_g : H(p,q,\nu) \to \Lambda_{\mu,0}$ is compact.

$(b) \Rightarrow (a)$. This implication is clear.

$(a) \Rightarrow (c)$. Assume that $T_g : H(p,q,\nu) \to \Lambda_\mu$ is compact. Let $(z_k)_{k\in\mathbb{N}}$ be a sequence in B such that $\lim_{k\to\infty} |z_k| = 1$. Set

$$f_k(z) = \frac{(1-|z_k|^2)^{t+1}}{\nu(z_k)(1-\langle z, z_k\rangle)^{\frac{n}{q}+t+1}}. \tag{28}$$

Then $f_k \in H(p,q,\nu)$, $\sup_k \|f_k\|_{H(p,q,\nu)} < \infty$ and $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$. By Lemma 2,

$$\lim_{k\to\infty} \|T_g f_k\|_{\Lambda_\mu} = 0. \tag{29}$$

On the other hand, we have

$$\begin{aligned} 0 \ \leftarrow\ & \|T_g f_k\|_{\Lambda_\mu} = \sup_{z\in B} \mu(z)|\Re f_k(z)\Re g(z) + f_k(z)\Re^2 g(z)| \\ \geq\ & \mu(z_k)|\Re f_k(z_k)\Re g(z_k) + f_k(z_k)\Re^2 g(z_k)| \\ \geq\ & \left|(\frac{n}{q}+t+1)\frac{\mu(z_k)|z_k|^2|\Re g(z_k)|}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}+1}} - \frac{\mu(z_k)|\Re^2 g(z_k)|}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}}}\right|, \end{aligned} \tag{30}$$

as $k \to \infty$. Now set

$$h_k(z) = \frac{1}{\frac{n}{q}+t+1} \frac{(1-|z_k|^2)^{t+1}}{\nu(z_k)(1-\langle z, z_k\rangle)^{\frac{n}{q}+t+1}} - \frac{1}{\frac{n}{q}+t+2} \frac{(1-|z_k|^2)^{t+2}}{\nu(z_k)(1-\langle z, z_k\rangle)^{\frac{n}{q}+t+2}}.$$

Then $h_k \in H(p,q,\nu)$, $\sup_k \|h_k\|_{H(p,q,\nu)} < \infty$ and $h_k \to 0$ uniformly on compact subsets of B as $k \to \infty$. By Lemma 2, it hold $\lim_{k\to\infty} \|T_g h_k\|_{\Lambda_\mu} = 0$. Since

$$\Re h_k(z_k) = 0 \quad \text{and} \quad h_k(z_k) = \frac{1}{\frac{n}{q}+t+1}\frac{1}{\frac{n}{q}+t+2}\frac{1}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}}},$$

we have

$$\begin{aligned} 0 \leftarrow \|T_g h_k\|_{\Lambda_\mu} &\geq \sup_{z\in B} \mu(z)|\Re h_k(z)\Re g(z) + h_k(z)\Re^2 g(z)| \\ &\geq \mu(z_k)|\Re h_k(z_k)\Re g(z_k) + h_k(z_k)\Re^2 g(z_k)| \\ &\geq \frac{1}{\frac{n}{q}+t+1}\frac{1}{\frac{n}{q}+t+2}\frac{\mu(z_k)|\Re^2 g(z_k)|}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}}}, \end{aligned} \tag{31}$$

as $k \to \infty$, which means that (27) holds. It follows from (30) and (31) that

$$\lim_{k\to\infty}(\frac{n}{q}+t+1)\frac{\mu(z_k)|z_k|^2|\Re g(z_k)|}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}+1}} = \lim_{k\to\infty}\frac{\mu(z_k)|\Re^2 g(z_k)|}{\nu(z_k)(1-|z_k|^2)^{\frac{n}{q}}}, \tag{32}$$

if one of the limits exists. From (27) and (32), (26) follows. The proof of this theorem is completed. □

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