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Simplex Type Algorithm for Solving Fuzzy Transportation Problem *

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Abstract

In this paper Arsham - Khan's simplex algorithm is adopted to solve fuzzy transportation problem with imprecise render and requirement condition.The algorithm of this approach is presented, and explained briefly with numerical instance to show its efficiency. Keywords: Transportation problem, Arsham- Khan's Algorithm, Fuzzy numbers.

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1. Introduction

The simplex method "is an iterative algebraic procedure for solving linear programming problems" [[7], p. 610]. In the late 1940s, George Dantzig and his contemporaries were faced with monumental problems that arose in the areas of military logistics, management, shipping, and economics. The perplexing part of the problem was that most optimization scenarios could be expressed only in systems of equations and inequalities that contained very large numbers of variables. In 1947 Dantzig devised the simplex method - a way to reduce the number of calculations. This was the advent of linear programming [13]. The simplex algorithm was the forerunner of many computer programs that are used to solve complex optimization problems [2]. These applications are used extensively in a variety of situations. One of the most important applications of the simplex method is the transportation model [13].

The transportation model considers minimum-cost planning problems for shipping a product from some origins to other destinations, such as from factories to warehouses, or from warehouses to supermarkets, with the shipping cost from one location to another being a linear function of the number of units shipped. The transportation model is a special case of the linear programming models, and obviously, it can be solved by the regular simplex method (big - M) or the dual simplex method. However, these algorithms require additional variables, which complicate the formulation, enlarge the tableaux and increase the number of iterations.

This paper presents a simple simplex type algorithm to minimize the fuzzy objective value of the fuzzy transportation problem with fuzzy render and fuzzy requirements parameters. The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problem. To quantitatively deal with imprecise information in making decisions, Bellman and Zadeh [3] and Zadeh [11] acquainted the concept of fuzziness.

Chanas et al. [4] investigated transportation problems with fuzzy render and requirement quantities and solved them using the parametric programming technique in terms of the Bellman-Zadeh criterion. Shan Huo Chen [8] acquainted the concept of function principle, which is used to calculate the fuzzy transportation cost. The Graded Mean Integration Representation Method, used to defuzzify the fuzzy transportation cost, was also acquainted by Shan Huo Chen [9]. The algorithm of the approach is detailed in the paper, and finally a numerical instance is given to illustrate the approach.

2. Fuzzy Preliminaries

L.A. Zadeh advanced the fuzzy theory in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al (1974) in the frame work of the fuzzy decision of Bellman and Zadeh [3]. Now, we present some necessary definitions that from [5].

Definition 2.1. A fuzzy number $\widetilde{a}$ is a Triangular-Fuzzy number denoted by (a_1, a_2, a_3) and it's membership function $\mu_{\widetilde{a}}(x)$ is given below

$$\mu_{\widetilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2; \\ \frac{x-a_3}{a_2-a_3}, & a_2 \le a_3; \\ 0, & \text{otherwise.} \end{cases}$$

3. Fuzzy Transportation Problem (FTP)

A Fuzzy Transportation Problem (FTP) is a linear programming problem of specific structure. As in the transportation problem, let $\widetilde{a_i}$ represents the renders and $\widetilde{e_j}$ represents the requirements respectively. We define the following quantities

$\widetilde{a_i}$ = fuzzy renders at source i ,
$\widetilde{e_j}$ = fuzzy requirements at termini j ,
$\widetilde{c_{ij}}$ = unit cost of transportation from source i to termini j ,
$\widetilde{x_{ij}}$ = number of units transported from source i to termini j.

The objective is to find how much material should be transported from source i to termini j.

Minimize $\mathrm{z} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \widetilde{\mathrm{c}_{\mathrm{ij}}} \widetilde{\mathrm{x}_{\mathrm{ij}}}$

Subject to the constraints

$$\sum_{j=1}^{n} \widetilde{x_{ij}} \le \widetilde{a}_i, \text{ for i} = 1, 2, ...\mathrm{m}$$

$\sum_{i=1}^{m} \widetilde{x_{ij}} \geq \widetilde{e_j}$ for j $= 1, 2, ...$n

$\widetilde{x_{ij}} \geq 0$ for j $= 1, 2, ...$m i $= 1, 2, ...$m

The matrix form of transportation problem is

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c_{ij}} \widetilde{x_{ij}}$$

Subject to the constraints

$$Ax \leq b$$

$$x \geq 0$$

Where $b = (a_1, a_2, ...a_m, e_1, e_2, ...e_n)$ and A is coefficient matrix of transportation problem. However, in the real-world decision problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be of a probabilistic type. In this situation, the decision maker can model the inexactness by means of fuzzy parameters [2].

The matrix form of fuzzy transportation problem is

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c_{ij}} \widetilde{x_{ij}}$$

Subject to the constraints

$$Ax = \widetilde{b}$$

$$x \geq 0$$

where $\widetilde{b} = (\widetilde{a_1}, \widetilde{a_2}, ...\widetilde{a_m}, \widetilde{e_1}, \widetilde{e_2}, ...\widetilde{e_n})$ and A is the coefficient matrix of FTP. It can be demonstrated that, matrix A is a unimodular one. It means that, determinant of each square sub matrix of A is $\pm$ or zero.

4. Arsham-Kahn Algorithm

Using coefficient matrix characteristic, Arsham and Kahn represented a simple simplex algorithm in two phases for the transportation problem. In the first phase, a feasible or an infeasible basic solution is obtained. Then in the second phase it is led to feasibility and turns into a basic optimal solution. These two phases utilize the row-column matrix by Gauss-Jordan. This algorithm also defines the redundant constraint. It is noteworthy that, Arsham-Kahn algorithm consequently eliminates non-basic columns from the simplex table (in order to decrease the computation).

5. Numerical Instances

1. A company has two factories O_1, O_2 and two retail stores D_1, D_2. The production quantities per month O_1 and O_2 are $(150, 201, 246)$ and $(50, 99, 154)$ tons respectively. The demands per month for D_1 and D_2 are $(100, 150, 200)$ and $(100, 150, 200)$ tons respectively. The transportation cost per ton $\widetilde{c_{ij}}$, $i = 1, 2; j = 1, 2$ is the following $\widetilde{c_{11}} = (15, 19, 29), \widetilde{c_{12}} = (22, 31, 34)$, $\widetilde{c_{21}} = (8, 10, 12), \widetilde{c_{22}} = (30, 39, 54)$.

Step 1: Row-Column Reduction

(-14,0,14)	(-26,0,26)	(150,201,246)
(-4,0,4)	(-1,17,53)	(50,99,154)
(100,150,200)	(100,150,200)	

Step 2: Using Arsham - Kh an's Simplex Tableau

	Var	$\widetilde{x_{11}}$	$\widetilde{x_{12}}$	$\widetilde{x_{21}}$	$\widetilde{x_{22}}$	RHS
O_1	?	(1,1,1)	(1,1,1)	$*$	$*$	(150,201,246)
O_2	?	$*$	$*$	(1,1,1)	(1,1,1)	(50,99,154)
D_1	?	(1,1,1)	$*$	(1,1,1)	$*$	(100,150,200)
D_2	?	$*$	(1,1,1)	$*$	(1,1,1)	(100,150,200)
		(-14,0,14)	(-26,0,26)	(-4,0,4)	(-1,17,53)	
O_1	?	(1,1,1)	(1,1,1)	$*$	$*$	(150,201,246)
O_2	?	$*$	$*$	(1,1,1)	(1,1,1)	(50,99,154)
D_2	?	$*$	(1,1,1)	$*$	(1,1,1)	(100,150,200)
		(-14,0,14)	(-26,0,26)	(-4,0,4)	(-1,17,53)	
O_1	$\widetilde{x_{11}}$	$*$	(1,1,1)	$*$	$*$	(150,201,246)
O_2	$\widetilde{x_{21}}$	$*$	$*$	$*$	(1,1,1)	(50,99,154)
D_2	?	$*$	(1,1,1)	$*$	(1,1,1)	(100,150,200)
		$*$	(-26,0,26)	$*$	(-1,17,53)	
O_1	$\widetilde{x_{11}}$	$*$	$*$	$*$	(-1,-1,-1)	(-76,51,172)
O_2	$\widetilde{x_2}1$	$*$	$*$	$*$	(1,1,1)	(50,99,154)
D_2	$\widetilde{x_{12}}$	$*$	$*$	$*$	(1,1,1)	(74,150,226)
		$*$	$*$	$*$	(-1,17,53)	

Right hand side values are non-negative, therefore Arsham- Khan's simplex tableau is now optimum.

Step3:

(15,19,29) $\widetilde{x_{11}}$=(-50,51,146)	(22,31,34) $\widetilde{x_{12}}$=(100,150,200)	(150,201,246)
(8,10,12) $\widetilde{x_{21}}$=(50,99,154)	(30,39,54) $\widetilde{x_{22}}$=0	(50,99,154)
(100,150,200)	(100,150,200)	

The optimum fuzzy transportation cost is (1150, 6609, 12882)

Defuzzified fuzzy transportation cost is 6745.

2. Cost Matrix for the 3 x 2 problem is given beneath,

	B_1	B_2	Supply
A_1	(1,3,5)	(3,5,13)	(300,399,504)
a_2	92,3,10)	(3,4,1)	(250,301,346)
A_3	(5,6,13)	(1,3,5)	(300,399,504)
Demand	(400,448,508)	(300,351,396)	

This matrix needs to be balanced with a dummy destination,

(1,3,5) $\widetilde{x_{11}}$	(3,5,13) $\widetilde{x_{12}}$	(0,0,0) $\widetilde{x_{13}}$	(300,399,504)
(2,3,10) $\widetilde{X_{21}}$	(3,4,11) $\widetilde{X_{22}}$	(0,0,0) $\widetilde{X_{23}}$	(250,301,346)
(5,6,13) $\widetilde{X_{31}}$	(1,3,5) $\widetilde{X_{32}}$	(0,0,0) $\widetilde{X_{33}}$	(300,399,504)
(400,448,508)	(300,351,396)	(150,300,450)	

Step 1: Row-Column Reduction

(-4,0,4) $\widetilde{X_{11}}$	(-2,2,12) $\widetilde{X_{12}}$	(0,0,0) $\widetilde{X_{13}}$	(300,399,504)
(-3,0,9) $\widetilde{X_{21}}$	(-2,1,10) $\widetilde{X_{22}}$	(0,0,0) $\widetilde{X_{23}}$	(250,301,346)
(0,3,12) $\widetilde{X_{31}}$	(-4,0,4) $\widetilde{X_{32}}$	(0,0,0) $\widetilde{X_{33}}$	(300,399,504)
(400,448,508)	(300,351,396)	(150,300,450)	

Step2: Arsham - Khan's Simplex Tableau
Phase - 1: Basic Variable Phase

	Var	$\widetilde{x_{11}}$	$\widetilde{x_{12}}$	$\widetilde{x_{13}}$	$\widetilde{x_{21}}$	$\widetilde{x_{22}}$	$\widetilde{x_{23}}$	$\widetilde{x_{31}}$	$\widetilde{x_{32}}$	$\widetilde{x_{33}}$	RHS
A_1	?	(1,1,1)	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	$*$	$*$	(300,399,504)
A_2	?	$*$	$*$	$*$	(1,1,1)	(1,1,1)	(1,1,1)	$*$	$*$	$*$	(250,301,346)
A_3	?	$*$	$*$	$*$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	(1,1,1)	(300,399,504)
B_1	?	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(400,448,508)
B_2	?	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	(300,351,396)
B_3	?	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	(150,300,450)
		(-4,0,4)	(-2,2,12)	(0,0,0)	(-3,0,9)	(-2,1,10)	(0,0,0)	(0,3,12)	(-4,0,4)	(0,0,0)	
A_1	?	(1,1,1)	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	$*$	$*$	(300,399,504)
A_2	?	$*$	$*$	$*$	(1,1,1)	(1,1,1)	(1,1,1)	$*$	$*$	$*$	(250,301,346)
A_3	?	$*$	$*$	$*$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	(1,1,1)	(300,399,504)
B_1	?	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(400,448,508)
B_2	?	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	(300,351,396)
		(-4,0,4)	(-2,2,12)	(0,0,0)	(-3,0,9)	(-2,1,10)	(0,0,0)	(0,3,12)	(-4,0,4)	(0,0,0)	
A_1	$\widetilde{x_{13}}$	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	$*$	$*$	$*$	(300,399,504)
A_2	$\widetilde{x_{23}}$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	(250,301,346)
A_3	$\widetilde{x_{33}}$	$*$	$*$	$*$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	$*$	(300,399,504)
B_1	?	[(1,1,1)]	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(400,448,508)
B_2	?	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	(300,351,396)
		(-4,0,4)	(-2,2,12)	$*$	(-3,0,9)	(-2,1,10)	$*$	(0,3,12)	(-4,0,4)	$*$	
A_1	$\widetilde{x_{13}}$	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	$*$	$*$	$*$	(300,399,504)
A_2	$\widetilde{x_{23}}$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	(250,301,346)
A_3	$\widetilde{x_{33}}$	$*$	(-1,-1,-1)	$*$	$*$	(-1,-1,-1)	$*$	(1,1,1)	$*$	$*$	(300,399,504)
B_1	?	[(1,1,1)]	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(400,448,508)
B_2	$\widetilde{x_{32}}$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	$*$	$*$	(300,351,396)
		(-4,0,4)	(-2,2,12)	$*$	(-3,0,9)	(-2,1,10)	$*$	(0,3,12)	$*$	$*$	
A_1	$\widetilde{x_{13}}$	$*$	(1,1,1)	$*$	(-1,-1,-1)	$*$	$*$	(-1,-1,-1)	$*$	$*$	(-208,-49,104)
A_2	$\widetilde{x_{23}}$	$*$	$*$	$*$	(1,1,1)	(1,1,1)	$*$	$*$	$*$	$*$	(250,301,346)
A_3	$\widetilde{x_{33}}$	$*$	(-1,-1,-1)	$*$	$*$	(-1,-1,-1)	$*$	(1,1,1)	$*$	$*$	(-96,48,204)
B_1	$\widetilde{x_{11}}$	$*$	$*$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(400,448,508)
B_2	$\widetilde{x_{32}}$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	$*$	$*$	(300,351,396)
		$*$	(-2,2,12)	$*$	(-3,0,9)	(-2,1,10)	$*$	(0,3,12)	$*$	$*$	

Since there is no open rows Basic variable set is complete.

Phase - 2: Feasibility phase

	Var	$\widetilde{x_{11}}$	$\widetilde{x_{12}}$	$\widetilde{x_{13}}$	$\widetilde{x_{21}}$	$\widetilde{x_{22}}$	$\widetilde{x_{23}}$	$\widetilde{x_{31}}$	$\widetilde{x_{32}}$	$\widetilde{x_{33}}$	RHS
A_1	$\widetilde{x_{21}}$	$*$	(-1,-1,-1)	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	(-104,49,208)
A_2	$\widetilde{x_{23}}$	$*$	(1,1,1)	$*$	(0,0,0)	(1,1,1)	$*$	(-1,-1,-1)	$*$	$*$	(42,252,450)
A_3	$\widetilde{x_{33}}$	$*$	(-1,1,-1)	$*$	$*$	(-1,-1,-1)	$*$	(1,1,1)	$*$	$*$	(-96,48,204)
B_1	$\widetilde{x_{11}}$	$*$	(1,1,1)	$*$	(0,0,0)	$*$	$*$	(1,1,1)	$*$	$*$	(192,399,612)
B_2	$\widetilde{x_{32}}$	$*$	(1,1,1)	$*$	$*$	(1,1,1)	$*$	$*$	$*$	$*$	(300,351,396)
		$*$	(-1,3,13)	$*$	(0,0,0)	(-2,1,10)	$*$	(-1,2,11)	$*$	$*$	

Right hand side is non-negative therefore feasibility iteration tableau is now optimum.

Step 3 :

(1,3,5) $\widetilde{x_{11}}$=(192,399,612)	(3,5,13) $\widetilde{x_{12}}$= (0,0,0)	(0,0,0) $\widetilde{x_{13}}$=(0,0,0)	(300,399,504)
(2,3,10) $\widetilde{x_{21}}$=(-104,49,208)	(3,4,11) $\widetilde{x_{22}}$=(0,0,0)	(0,0,0) $\widetilde{x_{23}}$=(42,252,450)	(250,301,346)
(5,6,13) $\widetilde{x_{31}}$=(0,0,0)	(1,3,5) $\widetilde{x_{32}}$=(300,351,396)	(0,0,0) $\widetilde{x_{33}}$=(-96,48,204)	(300,399,504)
(400,448,508)	(300,351,396)	(150,300,450)	

The optimum fuzzy transportation cost = (-548, 2397, 7120)

Defuzzified fuzzy transportation cost is 2693

6. Conclusions

In this paper we presented a simple simplex type algorithm but general framework to solve the fuzzy transportation problem manually in an effective and efficient manner without using any artificial variables.

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