

On Inventory Models with Ramp Type Demand Rate, Partial Backlogging and Weibull Deterioration Rate *

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Abstract

Recently, Skouri *et al.* (2009) proposed two inventory models with general ramp-type demand rate, Weibull deterioration rate, and partial backlogging of unsatisfied demand: (a) Model 1 was starting with no shortages, and Model 2 was starting with shortages. They derived the optimal solutions for both models. Then they ran 2 numerical examples, and concluded that "the total cost for the model starting with shortages (i.e., Model 2) is less than the total cost for the model starting with no shortages (i.e., Model 1). This observation agrees with known, results

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from literature concerning the finite horizon inventory models.” In this note, we will provide two counterexamples to show that the total cost for Model 2 is not always cheaper than the total cost for Model 1. In fact, the inventory models here involve in various demand rate, backloging rate, and deterioration rate with respect to time. Consequently, it seems not to have simple conditions to determine which model is cheaper to operate than the other.

Keywords and Phrases: *Inventory, Ramp-type demand, Deteriorating items, Partial backloging.*

1. Introduction

Skouri *et al.* (2009) developed an economic order quantity model with general ramp-type demand rate, time dependent (Weibull) deterioration rate, and partial backloging rate. They then proposed two distinct replenishment policies: (a) starting with no shortages and ending with shortages (say, Model 1), and (b) starting with shortages and ending with no shortages (say, Model 2). Next, they derived the optimal replenishment policy for each model. Finally, they ran 2 numerical examples, and concluded that “the total cost for the model starting with shortages is less than the total cost for the model starting with no shortages. This observation agrees with known, results from literature concerning the finite horizon inventory models such as Teng *et al.* (1997) and Skouri and Papachristos (2003).” Unfortunately, the inventory model discussed here is completely different from those four models in Teng *et al.* (1997) and Skouri and Papachristos (2003). Consequently, they jumped into a wrong conclusion that Model 2 is less expensive to operate than Model 1.

In this note, we will provide two counterexamples to show that the total cost for Model 2 is not always less than the total cost for Model 1. As a matter of fact, the ramp-type demand is a non-decreasing function of time. From those two counterexamples below, we know that if the inventory related costs (such as the inventory holding cost per unit per unit time c_1 , and the deterioration cost per unit c_3) are relatively less than the shortage related costs (such as the shortage cost per unit per unit time c_2 , and the opportunity cost per unit due to lost sales c_4), then it seems to be obvious by common sense that the model starting with shortages (i.e., Model 2) is cheaper to operate than

the model starting with no shortages (i.e., Model 1), and vice versa. However, the inventory models here involve in time varying demand rates, backlogging rates and deterioration rates. As a result, it seems to be difficult to obtain some simple conditions in which one model is cheaper to operate than the other.

2. Assumptions and Notation

The assumptions proposed here are similar to those in Skouri *et al* (2009).

1. The demand rate $D(t)$ is a ramp-type function of time given by

$$D(t) = \begin{cases} f(t), & t < \mu, \\ f(u), & t \geq \mu, \end{cases}$$

where $f(t)$ is a positive, continuous, increasing function of $t \in (0, T]$

2. The deterioration rate is $\theta(t) = abt^{b-1}$, where $a > 0$, $b > 0$ and $t > 0$. That is, the time to deterioration of the item is distributed as Weibull (a, b) . There is no replacement or repair of deteriorated units during the period T . For $b = 1$, $\theta(t)$ becomes constant, which corresponds to the exponentially decaying case.
3. Shortages are backlogged at the rate of $\beta(x)$, where $\beta'(x) \leq 0$, $0 \leq \beta(x) \leq 1$, $\beta(0) = 1$ and x is the waiting time up to the next replenishment. Moreover it is assumed that $\beta(x)$ satisfies the relation $\beta(x) + T\beta'(x) \geq 0$. The case with $\beta(x) = 1$ (or 0) correspond to complete backlogging (or complete lost sales) model.
4. Replenishment rate is infinite. The order quantity brings the inventory level up to S .

The following notation is used throughout the entire note.

- T the fixed planning horizon time
 t_1 the time when the inventory level turns to 0 in Model 1 or turns to positive in Model 2 (a decision variable)
 S the maximum inventory level at the scheduling period
 c_1 the inventory holding cost per unit per unit time
 c_2 the shortage cost per unit per unit time

- c_3 the cost incurred from the deterioration of one unit
 c_4 the per unit opportunity cost due to the lost sales
 μ the parameter of the ramp type demand function
 $I(t)$ the inventory level at time $t \in (0, T]$
 $TC_i(t_1)$ the total cost during the time interval $[0, T]$ in Model i , $i = 1, 2$

3. Counterexamples

As we know, the ramp-type demand is a non-decreasing function of time. Amazingly, Skouri *et al.* (2009) provided two numerical examples by setting the inventory related costs to be less than the shortage related costs (i.e., $c_1 < c_2$ and $c_3 < c_4$). As a result, they concluded that the model starting with shortages (i.e., Model 2) is always cheaper to operate than the model starting with no shortages (i.e., Model 1), which seems to be obvious by common sense. In this section, we will use the inventory related costs to be higher than the shortage related costs (i.e., $c_1 > c_2$ and $c_3 > c_4$), and then show that Model 2 is not cheaper to operate than Model 1.

3.1 Example 1

The input parameters are: $c_1 = \$15$ per unit per year, $c_2 = \$3$ per unit per year, $c_3 = \$25$ per unit, $c_4 = \$20$ per unit, $\mu = 0.12$ year, $a = 0.001$, $b = 2$, $T = 1$ year, $f(t) = 3e^{4.5t}$ and $\beta(x) = e^{-0.2x}$. The following Equations are adopted from Skouri *et al.* (2009).

By using (19)(or (24)), we obtain the optimal value of t_1 in Model 1 as

$$t_1^* = 0.296912 > \mu.$$

From (25), we get the optimal order quantity as

$$Q^* = 4.7647.$$

Consequently, we know from (13) that the minimum cost for Model 1 is

$$TC_1(t_1^*) = 11.6539.$$

Similarly, by using (45), we have the optimal value of t_1 in Model 2 as

$$t_1^* = 0.710973 > \mu.$$

From (48) and (39), we obtain the optimal order quantity as

$$Q^* = 4.7772,$$

and the minimum cost for Model 2 as

$$TC_2(t_1^*) = 16.3036.$$

3.2 Example 2

Here, we simply switch the inventory related costs with the shortage related costs as in Example 1 in Skouri et al. (2009).

Similar to Example 1, we can easily obtain the optimal solutions for both Models 1 and 2 as shown below:

c_1	c_2	c_3	c_4	μ	t_1^*	Q^*	$TC_i(t_1^*)$	Model i
15	3	20	5	0.12	0.187976	4.6858	7.4583	Model 1
					0.816869	4.7024	17.2375	Model 2

The numerical result reveals that Model 1 is less expensive to operate than Model 2.

4. Conclusion

In this note, we have proven that the inventory model starting with shortages and ending with no shortages is not always less expensive to operate than the other inventory model starting with no shortages and ending with shortages. Since the inventory models discussed here involve in time varying demand rates, backlogging rates and deterioration rates, and many other parameters, it seems to be difficult to obtain some simple conditions to determine which model is less expensive to operate than the other.

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