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On Fractional Differential Operator Involving $\bar{H}$ -function *

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Abstract

In the present paper we establish an important result involving a fractional differential operator given by Misra [5] for the product of $\bar{H}$-function, general polynomial set and two general class of polynomials. On account of the general nature of the functions and the polynomials occurring in our main results a large number of simple results follow as its special cases. For the sake of illustration, we present here two special cases involving product of general class of polynomials and the generalized Riemann Zeta function.

Keywords and Phrases: *Fractional derivative, General class of multivariable polynomials, Generalized polynomial set, $\bar{H}$-function.*

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1. Introduction

Fractional Differential Operator

Misra [5] has defined the fractional derivative operators in the following manner:

$$D_x^{\alpha}(x^{\mu-1}) = \frac{d^{\alpha}}{dx^{\alpha}}\, x^{\mu-1} = \frac{\Gamma(\mu)}{\Gamma(\mu-\alpha)}\, x^{\mu-\alpha-1}, \ \alpha \neq \mu \tag{1.1}$$

$$D_{k,\alpha,x}(x^{\mu}) = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)}\, x^{\mu+k}, \ \alpha \neq \mu+1 \tag{1.2}$$

$$D_{k,\alpha,x}^{n}(x) = \prod_{r=0}^{n-1}\left[\frac{\Gamma(\mu+1+rk)}{\Gamma(\mu+1+rk-\alpha)}\right] x^{\mu+nk} \tag{1.3}$$

α and k are not necessarily integers

General class of Polynomials of g-variables

The following general class of polynomials of g-variables due to Srivastava and Garg [7, p.686, eq.(1.4)] is defined and denoted as follows:

$$S_N^{M_1,\ldots,M_g}[x_1,\ldots,x_g] = \sum_{k_1,\ldots,k_g=0}^{M\leq N} [(-N)_M\, B(n;k_1,\ldots,k_g)] \prod_{i=1}^{g} \frac{(x_i)^{g_i}}{g_i\,!} \tag{1.4}$$

where $M = \sum_{i=1}^{g} M_i k_i\,(M_i \geq 1,\ i = 1,\ldots,g)$ *and* $B\,(N;\ k_1,\ldots,k_g)$ are arbitrary constant (real or complex).

A Generalized Polynomial Set

Raizada has introduced and studied a generalized polynomial set. The explicit form of this generalized polynomial set [6, p.71, eq.(2.3.4)] is:

$$S_n^{\alpha,\beta,\tau}[x\,;\ r,s,q,A,B,m,k,\ell] = B^{qn}\, x^{\ell(m+n)}\, (1-\tau x^r)^{sn}\, \ell^{m+n}$$

$$\sum_{p=0}^{m+n}\sum_{e=0}^{p}\sum_{\delta=0}^{m+n}\sum_{i=0}^{\delta} \frac{(-1)^{\delta}(-p)_e\,(-\delta)_i(\alpha)_{\delta}}{p\,!\ \delta\,!\ i\,!\ e\,!}\, \frac{(\alpha-qn)_i}{(1-\alpha-\delta)_i} \left(-\frac{\beta}{\tau}-sn\right)_p$$

$$\left(\frac{\ell + k + re}{\ell}\right)_{m+n} \left(-\frac{\tau x^r}{1-\tau\, x^r}\right)^p \left(\frac{Ax}{B}\right)^\delta$$

If $\tau \to 0$ and using following result

$$\underset{|\,b\,| \to \infty}{lt} (b)_n \left(\frac{z}{b}\right)^n = z^n$$

we arrive at the following polynomial set:

$$S_n^{\alpha,\beta,0}[x\,;\, r, q, 1, 0, m, k, \ell] \;=\; x^{qn+\ell(m+n)}\, \ell^{m+n} \sum_{p=0}^{m+n} \sum_{e=0}^{p} \frac{(-p)_e}{p\,!\,e\,!} \left(\frac{\alpha + qn + k + re}{\ell}\right)_{m+n} (\beta x^r)^p \tag{1.5}$$

$\bar{H}$-function

The $\bar{H}$-function was introduced by Inayat Hussain [4] and studied by Bushman and Srivastava [1]. The $\bar{H}$-function will be defined and represented in the following manner:

$$\bar{H}^{M,N}_{P,Q}[z] \;=\; \bar{H}^{M,N}_{P,Q}\left[z \left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N}\,,\,(a_j,\alpha_j)_{N+1,P}\\ (b_j,\beta_j)_{1,M}\,,\,(b_j,\beta_j;\,B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$= \frac{1}{2\pi\omega} \int \bar{\phi}\,(\xi)\, z^{\xi}\, d\xi \quad (z \;\neq\; 0) \tag{1.6}$$

where

$$\bar{\phi}(\xi) \;=\; \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j\xi)\, \prod_{j=1}^{N} \{\Gamma(1 - a_j + \alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q} \{\Gamma(1 - b_j + \beta_j\xi)\}^{B_j}\, \prod_{j=N+1}^{P} \Gamma(a_j - \alpha_j\xi)} \tag{1.7}$$

Here a_j (j = 1,. . . ,P) and b_j (j = 1,. . . ,Q) are complex parameter $\alpha_j = 0$ (j = 1,. . . ,P), $\beta_j = 0$ (j = 1,. . . ,Q) (not all zero simultaneously) and the exponents a_j (j = 1,. . . ,N) and b_j (j = M+1,. . . ,Q) can take on integer values.

The following sufficient conditions for the absolute convergence of the defining integral for the $\bar{H}$-function given by Bushman and Srivastava [1]

$$T \;=\; \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} |\,A_j\alpha_j| \;-\; \sum_{j=M+1}^{Q} |\,B_j\beta_j| \;-\; \sum_{j=N+1}^{P} \alpha_j \;>\; 0 \tag{1.8}$$

and

|arg z |< 1/2 T π

Binomial Expansion

Binomial Expansion is given

$$(ax^{\nu}+b)^{\lambda} = b^{\lambda} \sum_{\ell=0}^{\lambda} \binom{\lambda}{\ell} \left(\frac{ax^{\nu}}{b}\right)^{\ell} \tag{1.9}$$

where $\lambda > 0$

2. Main Result

$$D^{n}_{k,\alpha,x}\{x^{\mu}(ax^{\nu}+b)^{\lambda}\, S_{N}^{M_1,\ldots,M_g}[y_1 x^{u_1}(ax^{\nu}+b)^{\eta_1},\ldots,\, y_g\, x^{u_g}(ax^{\nu}+b)^{\eta_g}]$$

$$S_{N}^{M_1^{'},\ldots,M_f^{'}}[y_1 x^{u_1^{'}}(ax^{\nu}+b)^{\eta_1^{'}},\ldots,\, y_f\, x^{u_f^{'}}(ax^{\nu}+b)^{\eta_f^{'}}]$$

$$S_{n'}^{\alpha,\beta,0}[\, zx^{u''}(ax^{\nu}+b)^{\eta''};r',q,1,0,m,k',\ell]$$

$$\bar{H}^{M,N}_{P,Q}\left[z\;\; x^{\lambda}\;\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$= \sum_{k_1,\ldots,k_g=0}^{M''\le N}\;\sum_{k_1^{'},\ldots,k_f^{'}=0}^{M'''\le N'}\;\sum_{p=0}^{m+n'}\;\sum_{e=0}^{p}\;\sum_{h=0}^{\infty}\theta(k_1,\ldots,k_g;\,k_1^{'},\ldots,k_f^{'},\,p,e)$$

$$b^{\lambda+\eta''R+\sum_{j=1}^{g}\eta_j k_j+\sum_{s=1}^{f}\eta_s^{'}k_s^{'}}\; x^{\mu+\nu h+\,u''R+nk+\sum_{j=1}^{g}\eta_j k_j+\sum_{s=1}^{f}\eta_s^{'}k_s^{'}}$$

$$\frac{(\lambda+\eta''R\;+\;\sum_{j=1}^{g}\eta_j k_j+\;\sum_{s=1}^{f}\eta_s^{'}\,k_s^{'})\,!}{h\,!\,(\lambda-h+\eta''R\;+\sum_{j=1}^{g}\eta_j k_j+\;\sum_{s=1}^{f}\eta_s^{'}k_s^{'})\,!}$$

$$(a/b)^{h}\;\bar{H}^{M,\,N+n}_{P+n,Q+n}\left[zx^{\lambda}\left|\begin{matrix}(-\mu-tk-\nu h-u''R-\sum_{j=1}^{g}u_j k_j-\sum_{s=1}^{f}u_s^{'}k_s^{'}\;\lambda\,;1)_{t=0,n-1}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(\alpha-\mu-tk-\nu h-u''R-\sum_{j=1}^{g}u_j k_j-\sum_{s=1}^{f}u_s^{'}k_s^{'},\lambda\,;1)_{t=0,n-1}\end{matrix}\right] \tag{2.1}$$

where

$$\theta\,(k_1,...,k_g;\,k_1^{'},...,k_f^{'},p,e)\;=\;\frac{(-N)_{M''}}{k_1!\,...\,k_g!}\,\frac{(-N')_{M'''}}{k_1^{'}\,!\,...\,k_f^{'}\,!}\,E\,(N;\;k_1,...,k_g)$$

$$F\,(N';k_1^{'},...,k_f^{'})\,y_1^{k_1},...,y_g^{k_g};\,y_1^{k_1^{'}},...,y_f^{k_f^{'}}(\ell)^{m+n'}\,\frac{(-p)_e}{p\,!\,e\,!}\,\left(\frac{\alpha+qn'+k'+r'e}{\ell}\right)_{m+n'}\beta^p z^R$$

$$M''\;=\;M_1k_1+...+\;M_gk_g$$

$$M'''=\;m_1^{'}k_1^{'}+...+\;M_f^{'}\;k_f^{'}$$

$$R\;=\;qn'\;+\;r'p\;+\;\ell\,(m+n')$$

and provided that the following conditions are satisfied:
(i) The quantities $u_j,\,\eta_j,\,u_s^{'},\,\eta_{s'}^{'},\,u'',\eta''\,(j\;=\;1,...,g;\;s\;=\;1,...,f)$are all positive.
(ii) $\nu\,;\,M_j,\,M_s^{'}(j\;=\;1,...,g\;;\;s\;=\;1,...,f)$ are arbitrary positive integers and the coefficients $E(N;k_1,...,k_g)$, $F(N';k_1^{'},...,k_f^{'})$, $(N,N';k_1^{'},...,k_g^{'},\,k_1^{'},...,k_f^{'}\;\geq 0)$ are arbitrary constants, real or complex.
(iii) $T\;\;=\;\;\sum_{j=1}^{M}\beta_j+\sum_{j=1}^{N}|\,A_j\alpha_j|-\sum_{j=M+1}^{Q}|\,B_j\beta_j|-\sum_{j=N+1}^{P}\alpha_j\;\;>\;\;0$ (2.2)
(iv) $|\,arg\;z\,|\;\;<\;\frac{1}{2}\;T\,\pi$
(v) $Re\;(\mu)\;\;>\;\;0,$
(vi) $\alpha<0$
(vii) $\lambda+\eta''\;q\;n'+\eta''r'p+\eta''\ell(m+n')\;\;+\;\;\sum_{j=1}^{g}\eta_jk_j+\sum_{s=1}^{f}\eta_s^{'}k_s^{'}\;>\;0$

Proof of Our Main Result

To establish (2.1), we first use series representation (1.4) and (1.5) for $S_N^{M_1,...,M_g}[x_1,...,x_g]$ and $S_n^{\alpha,\beta,0}$respectively and contour integral representation (1.6) for $\bar{H}$-function occurring in the left hand side of (2.1), it takes the following form (say Δ)

$$\Delta\;\;=\;\;D_{k,\alpha,x}^{n}\{x^{\mu}(ax^{\nu}+b)^{\lambda}\sum_{k_1,...,k_g=0}^{M_1k_1+...+M_gk_g\leq N}\frac{(-N)_{M_1k_1+...+M_gk_g}}{k_1\,!\,...\,k_g\,!}$$

$$E\,(N\,;\;k_1,...,k_g)\,y_1^{k_1},...,y_g^{k_g}\;x^{u_1k_1+...+u_gk_g}(a\,x^{\nu}+b)^{\eta_1k_1+...+\eta_gk_g}$$

$$\sum_{k_1^{'},...,k_f^{'}=0}^{M_1'k_1'+...+M_f'k_f'\le N'}\frac{(-N')_{M_1'k_1'+...+M_f'k_f'}}{k_1^{'}\,!...\,k_f^{'}\,!}\,F\,(N',k_1^{'},...,k_f^{'})\;y_1^{k_1^{'}},...,\;y_f^{k_f^{'}}\;x^{u_1^{'}k_1^{'}+...+u_f^{'}k_f^{'}}$$

$$(ax^{\nu}+b)^{\eta_1^{'}k_1^{'}+...+\eta_f^{'}k_f^{'}}[z\;x^{u''}(ax^{\nu}+b)^{\eta''}]^{qn'+\ell(m+n')}\;\ell^{(m+n')}$$

$$\sum_{p=0}^{m+n'}\sum_{e=0}^{p}\frac{(-p)_e}{p\,!\,e\,!}\left(\frac{\alpha+qn'+k'+r'e}{\ell}\right)_{m+n'}$$

$$\{\beta\,z^{r'}\;x^{u''r'}\,(ax^{\nu}+b)^{\eta''r'}\}^{p}\left\{\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\bar{\phi}(\xi)\,z^{\xi}\,x^{\lambda\xi}\,d\xi\right\}\qquad(2.3)$$

Interchanging the order of summation and integration (which is permissible under the condition stated)

$$\Delta=\sum_{k_1,...,k_g=0}^{M_1k_1+...+M_gk_g\le N}\;\sum_{k_1^{'},...,k_f^{'}=0}^{M_1'k_1'+...+M_f'k_f'\le N'}\;\sum_{p=0}^{m+n'}\sum_{e=0}^{p}\frac{(-N)_{M_1k_1+...+M_gk_g}}{k_1\,!...\,k_g\,!}$$

$$\frac{(-N')_{M_1'k_1'+...+M_f'k_f'}}{k_1^{'}\,!...\,k_f^{'}\,!}E(N;k_1,...,k_g)\;F(N':k_1^{'},...,k_g^{'})\;y_1^{k_1},...,y_g^{k_g}\;y_1^{k_1^{'}},...,y_f^{k_f^{'}}$$
$$\frac{(-p)_e}{p\,!\,e\,!}\;\ell^{(m+n')}\;\left(\frac{\alpha+qn'+k'+r'e}{\ell}\right)_{m+n'}$$

$$\beta^{p}\,z^{qn'+r'p+\ell(m+n')}\,\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\bar{\phi}\,(\xi)\,z^{\xi}$$

$$D_{k,\alpha,x}^{n}\{\,x^{\mu+u''qn'+u''r'p+\,\ell u''(m+n')\,+\,\sum_{j=1}^{g}u_jk_j+\sum_{s=1}^{f}u_s^{'}k_s^{'}+\lambda\xi}$$

$$(ax^{\nu}+b)^{\lambda+\eta''\,q\,n'+\eta''r'p\,+\,\eta''\ell(m+n')+\sum_{j=1}^{g}\eta_jk_j+\sum_{s=1}^{f}\eta_s^{'}k_s^{'}}\}\,d\xi\qquad(2.4)$$

Now using the Binomial Expansion given by (1.9), we get with the help of equation (1.3)

$$\Delta = \sum_{k_1,...,k_g=0}^{M_1k_1+...+M_gk_g\leq N} \sum_{k_1',...,k_f'=0}^{M_1'k_1'+...+M_f'k_f'\leq N'} \sum_{p=0}^{m+n'} \sum_{e=0}^{p} \sum_{h=0}^{\infty} \frac{(-N)_{M_1k_1+...+M_gk_g}}{k_1!...k_g!}$$

$$\frac{(-N')_{M_1'k_1'+...+M_f'k_f'}}{k_1'!...k_f'!} E\,(N\,;\,k_1+...+k_g)\,(N'\,;\,k_1',...,k_f')$$

$$y_1^{k_1},...,y_g^{k_g}\,y_1^{k_1'},...,\,y_f^{k_f'}\,\frac{(-p)_e}{p\,!\,e\,!}\,(\ell)^{m+n'}\left(\frac{\alpha+qn'+k'+r'e}{\ell}\right)_{m+n'}$$

$$\beta^p\,z^{qn'+r'p+\ell(m+n')}\,b^{\lambda+\eta'\,'\{qn'+r'p+\ell(m+n')\}+\sum_{j=1}^{g}\eta_jk_j+\sum_{s=1}^{f}\eta_s'k_s'}$$

$$\frac{(\lambda+\eta'\,'\{\,qn'+r'p+\ell(m+n')\;+\;\sum_{j=1}^{g}\eta_jk_j+\;\sum_{s=1}^{f}\eta_s'k_s'\})\,!}{h\,!\,(\lambda-h+\eta'\,'\{\,qn'+r'p\;+\;\ell(m+n')\;+\;\sum_{j=1}^{g}\eta_jk_j+\;\sum_{s=1}^{f}\eta_s'k_s'\})\,!}\left(\frac{a}{b}\right)^h$$

$$x^{\mu+\nu h+u'\,'\{qn'+r'p+\ell(m+n')\}\,+\,nk\,+\,\sum_{j=1}^{g}u_jk_j+\sum_{s=1}^{f}u_s'k_s'\,\{\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\bar{\phi}(\xi)\,(x^\lambda z)^\xi}$$

$$\left[\prod_{t=0}^{n-1}\frac{\Gamma(1+tk+\mu+\nu h+u'\,'\{qn'+r'p+\ell(m+n')\}+\sum_{j=1}^{g}u_jk_j+\sum_{s=1}^{f}u_s'k_s'+\lambda\xi)}{\Gamma(1+\mu+\nu h+tk-\alpha+u'\,'\{qn'+r'p+\ell(m+n')\}+\sum_{j=1}^{g}u_jk_j+\sum_{s=1}^{f}u_s'k_s'+\lambda\xi)}\right]\}d\xi$$

Further interpreting the above result with the help of (1.7), we get required result (2.1) after a little simplification.

3. Special Cases

(i) If in the result (2.1) we take

$$M_j = M\;(j=\;1,...,g),\;E\,(N\,;\,k_1,...,k_g)\;=\;E_{N,k}$$

and

$$M_s' = M'\;(s\;=\;1,...,f),\;F\,(N';k_1',...,k_f')\;=\;F_{N',k'}$$

then

$$S_N^{M_1,\ldots,M_g}[x] \;\rightarrow\; S_N^{M}[x] \; and \; S_{N'}^{M_1',\ldots,M_f'}[x] \;\rightarrow\; S_{N'}^{M'}[x]$$

and we arrive at the following result:

$$D_{k,\alpha,x}^{n}\{x^{\mu}(ax^{\nu}+b)^{\lambda}\, S_N^{M}[yx^{u}(ax^{\nu}+b)^{\eta}]\, S_{N'}^{M'}[yx^{u'}(ax^{\nu}+b)^{\eta'}]\}$$

$$S_{n'}^{\alpha,\beta,0}[z\, x^{u''}(ax^{\nu}+b)^{\eta''}\, :\, r',\; q,1,0,m,k',\ell]$$

$$\bar{H}_{P,Q}^{M,N}\left[z\, x^{\lambda}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},\,(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},\,(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$= \sum_{k=0}^{[N/M]}\sum_{k'=0}^{[N'/M']}\sum_{p=0}^{m+n'}\sum_{e=0}^{p}\sum_{h=0}^{\infty}\frac{(-N)_{M,k}}{k\,!}\,\frac{(-N')_{M',k'}}{k'\,!}\,E_{N,k}\,F_{N',k'}\,y^{k}\,y^{k'}$$

$$\frac{(-p)_e}{p\,!\,e\,!}\,(\ell)^{m+n'}\left(\frac{\alpha+qn'+k'+r'e}{\ell}\right)_{m+n'}\beta^{p}\,z^{R}\,b^{\lambda+\eta''R+\eta k+\eta' k'}$$

$$(a/b)^{h}\;\, x^{\mu+\nu h+u''R+\eta k+\eta' k'+nk}\,\frac{(\lambda+\eta''R+\eta k+\eta' k')\,!}{h\,!\,(\lambda-h+\eta''R+\eta k+\eta' k')\,!}$$

$$\bar{H}_{P+n,Q+n}^{M,\,N+n}\left[z\, x^{\lambda}\left|\begin{matrix}(-\mu-tk-\nu h-u''R-uk-u'k',\,\lambda\,;\,1)_{t=0,n-1},\,(a_j,\alpha_j;A_j)_{1,N}(a_j,\alpha_j)_{N+1,p}\\(b_j,\beta_j)_{1,M},\,(b_j,\beta_j;B_j)_{M+1,Q},\,(\alpha-\mu-tk-\nu h-u''R-uk-u'k',\,\lambda\,;\,1)_{t=0,n-1}\end{matrix}\right.\right] \quad (3.1)$$

(ii) If we reduce the $\bar{H}$-function occurring in the left hand side of (2.1) to Generalized Riemann Zeta function [2, p.27, 1.11, eq.(1); 3, p.314-315, eq.(1.6) and (1.7)] $\phi(z,\; p',\eta)$given by

$$\phi\,(z,p',\eta) \;=\; \sum_{r=0}^{\infty}\frac{z^{r}}{(\eta+r)^{p'}} \;=\; \bar{H}_{2,2}^{1,2}\left[-z\;\left|\begin{matrix}(0,1,1),\,(1-\eta,1;\,p')\\(0,1),(-\eta,1;\,p')\end{matrix}\right.\right],$$

we arrive at the following result

$$D_{k,\alpha,x}^{n}\{x^{\mu}\,(ax^{\nu}+b)^{\lambda}\, S_N^{M_1,\ldots,M_g}[y_1x^{u_1}(ax^{\nu}+b)^{\eta_1},\ldots,y_g\, x^{u_g}(ax^{\nu}+b)^{\eta_g}]$$

$$S_N^{M_1',...,M_f'}[y_1 x^{u_1'}(ax^\nu+b)^{\eta_1'},...,y_f' x^{u_f'}(ax^\nu+b)^{\eta_f'}]$$

$$S_{n'}^{\alpha,\beta,0}[z\ u''(ax^\nu+b)^{\eta''}\ :\ r',q,1,0,m,k',\ell]\ \phi(z\ (x^\lambda,p'\eta)$$

$$=\sum_{k_1,...,k_g=0}^{M''\le N}\ \sum_{k_1',...,k_f'=0}^{M'''\le N'}\ \sum_{p=0}^{m+n'}\ \sum_{e=0}^{p}\ \sum_{h=0}^{\infty}\theta(k_1,...,k_g;\ k_1',...,k_f',\ p,e)$$

$$b^{\lambda+\eta''R+\sum_{j=1}^g \eta_j k_j+\sum_{s=1}^f \eta_s' k_s'}\ x^{\mu+\nu h+u''R+\ \sum_{j=1}^g u_j k_j+\sum_{s=1}^f u_s' k_s'+nk}$$

$$\frac{(\lambda+\eta'\,'R\ +\ \sum_{j=1}^g \eta_j k_j\ +\ \sum_{s=1}^f \eta_s' k_s')\,!}{h\,!\,(\lambda-h+\eta'\,'R\ +\ \sum_{j=1}^g \eta_j k_j\ +\ \sum_{s=1}^f \eta_s' k_s')\,!}\,(a/b)^h$$

$$\bar{H}_{P+n+2,Q+n+2}^{M+1,N+n+2}\left[-zx^\lambda\ \middle|\begin{matrix}(-\mu-tk-\nu h-u'\,'R-\sum_{j=1}^g u_j k_j-\sum_{s=1}^f u_s' k_s',\lambda;1)_{t=0,n-1},(0,1;1),(1-\eta,1,p')\\(0,1),(-\eta,1,p'),\ (\alpha-\mu-tk-\nu h-u''R-\sum_{j=1}^g u_j k_j-\sum_{s=1}^f u_s' k_s',\lambda;1)_{t=0,n-1}\end{matrix}\right] \tag{3.2}$$

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