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The Efficiency of Stratified Quartile Ranked Set Sampling in Estimating the Population Mean

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Abstract

In this paper, new modified ranked set sampling method is suggested for estimating the population mean, namely, stratified quartile ranked set sampling (SQRSS). The SQRSS is compared with simple random sampling (SRS), stratified simple random sampling (SSRS) and stratified ranked set sampling (SRSS) methods. It is found that the SQRSS estimators are unbiased of the population mean of symmetric distributions. Also, it turns out that the SQRSS is more efficient than its counterparts using SRS, SSRS and SRSS based on the same number of measured units.

Keywords and Phrases: *Simple random sampling, Ranked set sampling, Quartile ranked set sampling, Stratified ranked set sampling.*

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1. Introduction

The problem of estimating the population mean using ranked set sampling is considered by McIntyre (1952). Takahasi and Wakimoto (1968) provided the mathematical theory for RSS. Dell and Clutter (1972) showed that the mean of the RSS is an unbiased estimator of the population mean, whether or not there are errors in ranking. Al-Saleh and Al-Omari (2002) suggested multistage ranked set sampling for estimating the population mean. Muttlak (2003) suggested quartile ranked set sampling (QRSS) to estimate the population mean and showed that QRSS reduces the errors in ranking when compared to RSS. Jemain and Al-Omari (2006) suggested double quartile ranked set sampling method for the estimation of the population mean. Jemain and Al-Omari (2007) have further suggested multistage quartile ranked set sampling. The RSS can be described as follows:

Step 1: Randomly select n^2 units from the target population.

Step 2: Allocate the n^2 selected units as randomly as possible into n sets, each of size n.

Step 3: Without yet knowing any values for the variable of interest, rank the units within each set with respect to variable of interest. This may be based on personal professional judgment or done based on a concomitant variable correlated with the variable of interest.

Step 4: Choose a sample for actual quantification by including the smallest ranked unit in the first set, the second smallest ranked unit in the second set, the process is continued in this way until the largest ranked unit is selected from the last set.

Step 5: Repeat Steps 1 through 4 for r cycles to obtain a sample of size rn for actual quantification.

For more details about RSS see, Al-Omari and Jaber (2008), Bouza (2002), Al-Nasser (2007), Al-Saleh and Al-Hadrami (2003), Jemain et al. (2008), Ohyama et al. (2008), and Al-Omari and Jaber (2010).

This paper is organized as follows: In Section 2, different sampling methods considered in this study are summarized in details. Estimation of the population mean is given in Section 3. A simulation study is conducted in Section 4. Finally, conclusions are provided in Section 5.

2. Sampling Methods

In stratified sampling method, the population of N units is divided into L non overlapping subpopulations each of N_1, $N_2, \cdots, N_L$ units, respectively, and $N_1 + N_2 + \cdots + N_L = N$. These subpopulations are called strata. For full benefit from stratification, the size of the *h*th subpopulation, denoted by N_h for $h = 1, 2, ..., L$, must be known. Then the samples are drawn independently from each stratum, producing samples sizes denoted by $n_1, n_2, ..., n_L$, such that the total sample size is $n = \sum_{h=1}^{L} n_h$. If a simple random sample is taken from each stratum, the whole procedure is known as stratified simple random sampling (SSRS).

The QRSS method is carried out by selecting *n* random samples each of size *n* units from the population of interest and ranking the units in each sample with respect to a variable of interest. If the sample size *n* is even, select for measurement from the first $n/2$ samples the $q_1(n+1)$ th smallest ranked unit and from the second $n/2$ samples the $q_3(n+1)$ th smallest ranked unit. If the sample size *n* is odd, select for measurement from the first $(n-1)/2$ samples the $q_1(n+1)$ th smallest ranked unit, from the last $(n-1)/2$ samples the $q_3(n+1)$ th smallest ranked unit and from the remaining sample the median ranked unit. The cycle can be repeated *r* times if needed to get a sample of size *nr* units. Note that we always take the nearest integer of $q_1(n+1)$ th and $q_3(n+1)$ th where $q_1 = 0.25$ and $q_3 = 0.75$.

If the quartile ranked set sampling method is used to select the sample units from each stratum then the whole procedure is called a stratified quartile ranked set sampling (SQRSS). To illustrate the SQRSS method, let us consider the following example for even sample size.

Example: Suppose that we have two strata, i.e. $L=2$ and $h=1,2$. Let $X_{ih(q_1(n_h+1))}$ and $X_{ih(q_3(n_h+1))}$ be the $\left(q_1\left(n_h+1\right)\right)$th and $\left(q_3\left(n_h+1\right)\right)$th order statistics, respectively, of the *i*th sample in the *h*th stratum. Assume that from the first stratum we select a sample of size 6 and from the second stratum we want a sample of size 8. Then the process as shown below:

Stratum 1: Now, select 6 samples each of size 6 as follows:

$$X_{11(1)}, X_{11(2)}, X_{11(3)}, X_{11(4)}, X_{11(5)}, X_{11(6)}$$
$$X_{21(1)}, X_{21(2)}, X_{21(3)}, X_{21(4)}, X_{21(5)}, X_{21(6)}$$
$$X_{31(1)}, X_{31(2)}, X_{31(3)}, X_{31(4)}, X_{31(5)}, X_{31(6)}$$
$$X_{41(1)}, X_{41(2)}, X_{41(3)}, X_{41(4)}, X_{41(5)}, X_{41(6)}$$
$$X_{51(1)}, X_{51(2)}, X_{51(3)}, X_{51(4)}, X_{51(5)}, X_{51(6)}$$
$$X_{61(1)}, X_{61(2)}, X_{61(3)}, X_{61(4)}, X_{61(5)}, X_{61(6)}$$

For *h*=1, select the second order statistics, $X_{i1(q_1(n_1+1))} = X_{i1(2)}$ for *i*=1,2,3, and the 5*th* order statistics $X_{i1(q_3(n_h+1))} = X_{i1(5)}$ for *i* =4,5,6. Thus, from the first stratum we have:

$$X_{11(2)}, X_{21(2)}, X_{31(2)}, X_{41(5)}, X_{51(5)}, X_{61(5)}.$$

Stratum 2: In the second stratum select 8 samples each of size 8 as follows:

$$X_{12(1)}, X_{12(2)}, X_{12(3)}, X_{12(4)}, X_{12(5)}, X_{12(6)}, X_{12(7)}, X_{12(8)}$$
$$X_{22(1)}, X_{22(2)}, X_{22(3)}, X_{22(4)}, X_{22(5)}, X_{22(6)}, X_{22(7)}, X_{22(8)}$$
$$X_{32(1)}, X_{32(2)}, X_{32(3)}, X_{32(4)}, X_{32(5)}, X_{32(6)}, X_{32(7)}, X_{32(8)}$$
$$X_{42(1)}, X_{42(2)}, X_{42(3)}, X_{42(4)}, X_{42(5)}, X_{42(6)}, X_{42(7)}, X_{42(8)}$$
$$X_{52(1)}, X_{52(2)}, X_{52(3)}, X_{52(4)}, X_{52(5)}, X_{52(6)}, X_{52(7)}, X_{52(8)}$$
$$X_{62(1)}, X_{62(2)}, X_{62(3)}, X_{62(4)}, X_{62(5)}, X_{62(6)}, X_{62(7)}, X_{62(8)}$$
$$X_{72(1)}, X_{72(2)}, X_{72(3)}, X_{72(4)}, X_{72(5)}, X_{72(6)}, X_{72(7)}, X_{72(8)}$$

$$X_{82(1)}, X_{82(2)}, X_{82(3)}, X_{82(4)}, X_{82(5)}, X_{82(6)}, X_{82(7)}, X_{82(8)}$$

For $h = 2$, select $X_{i2(q_1(n_2+1))} = X_{i2(2)}$ for i=1,2,3,4 and $X_{i2(q_3(n_2+1))} = X_{i2(7)}$ for i=5,6,7,8. Then we have $X_{12(2)}, X_{22(2)}, X_{32(2)}, X_{42(2)}, X_{52(7)}, X_{62(7)}, X_{72(7)}, X_{82(7)}$.

Therefore, the SQRSS units are $X_{11(2)}, X_{21(2)}, X_{31(2)}, X_{41(5)}, X_{51(5)}, X_{61(5)}$, $X_{12(2)}$, $X_{22(2)}$, $X_{32(2)}$, $X_{42(2)}$, $X_{52(7)}$, $X_{62(7)}$, $X_{72(7)}$, $X_{82(7)}$. The mean of these units is used as an estimator of the population mean.

3. Estimation of the Population Mean

Let $X_1, X_2, ..., X_n$ be n independent random variables from a probability density function $f(x)$, with mean μ and variance σ^2. The SRS estimator of the population mean based on a sample of size n is given by

$$\overline{X}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} X_i , \tag{1}$$

with variance

$$Var\left(\overline{X}_{SRS}\right) = \frac{\sigma^2}{n}. \tag{2}$$

The RSS estimator of the population mean is given by

$$\overline{X}_{RSS} = \frac{1}{n}\sum_{i=1}^{n} X_{i(i)} , \tag{3}$$

and the variance is

$$Var\left(\overline{X}_{RSS}\right) = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu\right)^2 , \tag{4}$$

where $\mu_{(i)}$ is the mean of the ith order statistics, $X_{(i)}$ for a sample of size n.

The stratified quartile ranked set sampling estimator of the population mean when n_h is even is defined as

$$\overline{X}_{SQRSS1} = \sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(q_1(n_h+1))} + \sum_{i=\frac{n_h+2}{2}}^{n_h} X_{ih(q_3(n_h+1))} \right), \tag{5}$$

where $W_h = \dfrac{N_h}{N}$, N_h is the stratum size and N is the total population size. The variance of SQRSS1 is given by

$$Var\left(\overline{X}_{SQRSS1}\right) = Var\left(\sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(q_1(n_h+1))} + \sum_{i=\frac{n_h+2}{2}}^{n_h} X_{ih(q_3(n_h+1))} \right)\right)$$

$$= \sum_{h=1}^{L} \frac{W_h^2}{n_h^2} \left(\sum_{i=1}^{\frac{n_h}{2}} Var\left(X_{ih(q_1(n_h+1))}\right) + \sum_{i=\frac{n_h+2}{2}}^{n_h} Var\left(X_{ih(q_3(n_h+1))}\right) \right)$$

$$= \sum_{h=1}^{L} \frac{W_h^2}{n_h^2} \left(\sum_{i=1}^{\frac{n_h}{2}} \sigma_{ih(q_1)}^2 + \sum_{i=\frac{n_h+2}{2}}^{n_h} \sigma_{ih(q_3)}^2 \right). \tag{6}$$

When the sample size n_h is odd, the SQRSS estimator is defined as

$$\overline{X}_{SQRSS2} = \sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{ih(q_1(n_h+1))} + \sum_{i=\frac{n_h+3}{2}}^{n_h} X_{ih(q_3(n_h+1))} + X_{\frac{n_h+1}{2}h\left(\frac{n_h+1}{2}\right)} \right), \tag{7}$$

with variance

$$Var\left(\overline{X}_{SQRSS2}\right) = Var\left(\sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{ih(q_1(n_h+1))} + \sum_{i=\frac{n_h+3}{2}}^{n_h} X_{ih(q_3(n_h+1))} + X_{\frac{n_h+1}{2}h\left(\frac{n_h+1}{2}\right)} \right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h^{\,2}}{n_h^2}\left(\sum_{i=1}^{\frac{n_h-1}{2}}Var\left(X_{ih(q_1(n_h+1))}\right)+\sum_{i=\frac{n_h+3}{2}}^{n_h}Var\left(X_{ih(q_3(n_h+1))}\right)+Var\left(X_{\frac{n_h+1}{2}h\left(\frac{n_h+1}{2}\right)}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h^{\,2}}{n_h^2}\left(\sum_{i=1}^{\frac{n_h-1}{2}}\sigma^2_{ih(q_1)}+\sum_{i=\frac{n_h+3}{2}}^{n_h}\sigma^2_{ih(q_3)}+\sigma^2_{h\left(\frac{n_h+1}{2}\right)}\right). \qquad (8)$$

Lemma: $\overline{X}_{SQRSS1}$ *and* $\overline{X}_{SQRSS2}$ *are unbiased estimators of the mean of symmetric distributions*

Proof: If n_h is even, we have

$$E\left(\overline{X}_{SQRSS1}\right)=E\left(\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}}X_{ih(q_1(n_h+1))}+\sum_{i=\frac{n_h+2}{2}}^{n}X_{ih(q_3(n_h+1))}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}}E\left(X_{ih(q_1(n_h+1))}\right)+\sum_{i=\frac{n_h+2}{2}}^{n_h}E\left(X_{ih(q_3(n_h+1))}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}}\mu_{h(q_1)}+\sum_{i=\frac{n_h+2}{2}}^{n_h}\mu_{h(q_3)}\right)$$

where $\mu_{h(q_1)}$ and $\mu_{h(q_3)}$ are the means of the order statistics correspond to the first and third quartiles, respectively. Since the distribution is symmetric about μ, then $\mu_{h(q_1)}+\mu_{h(q_3)}=2\mu_h$. Therefore, we have

$$E\left(\overline{X}_{SQRSS1}\right)=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\frac{n_h}{2}\mu_{h(q_1)}+\frac{n_h}{2}\mu_{h(q_3)}\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\frac{n_h}{2}\left(\mu_{h(q_1)}+\mu_{h(q_3)}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\frac{n_h}{2}\left(2\mu_h\right)\right)$$

$$=\sum_{h=1}^{L}W_h\mu_h=\mu\,.$$

If n_h is odd, then

$$E\left(\overline{X}_{SQRSS\,2}\right)=E\left(\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}X_{ih(q_1(n_h+1))}+\sum_{i=\frac{n_h+3}{2}}^{n_h}X_{ih(q_3(n_h+1))}+X_{\frac{n_h+1}{2}h\left(\frac{n_h+1}{2}\right)}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}E\left(X_{ih(q_1(n_h+1))}\right)+\sum_{i=\frac{n_h+3}{2}}^{n_h}E\left(X_{ih(q_3(n_h+1))}\right)+E\left(X_{\frac{n_h+1}{2}h\left(\frac{n_h+1}{2}\right)}\right)\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}}\mu_{h(q_1)}+\sum_{i=\frac{n_h+3}{2}}^{n_h}\mu_{h(q_3)}+\mu_{h\left(\frac{n_h+1}{2}\right)}\right),$$

where $\mu_{h(q_1)}$ is the mean of the first quartile for the first $\left(\frac{n_h-1}{2}\right)$ samples in the *h*th stratum, $\mu_{h(q_3)}$ is the mean of the third quartile for the last $\left(\frac{n_h-1}{2}\right)$ samples in the *h*th stratum, and μ_h is the mean for the stratum *h*. Since the distribution is symmetric about μ, then $\mu_{h(q_1)}+\mu_{h(q_3)}=2\mu_h$. Therefore,

$$E\left(\overline{X}_{SQRSS\,2}\right)=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\left(\frac{n_h-1}{2}\right)\mu_{h(q_1)}+\left(\frac{n_h-1}{2}\right)\mu_{h(q_3)}+\mu_{h\left(\frac{n_h+1}{2}\right)}\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\left(\frac{n_h-1}{2}\right)\left(\mu_{h(q_1)}+\mu_{h(q_3)}\right)+\mu_{h\left(\frac{n_h+1}{2}\right)}\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\frac{n_h-1}{2}\left(2\mu_h\right)+\mu_h\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\left(n_h-1\right)\mu_h+\mu_h\right)$$

$$=\sum_{h=1}^{L}\frac{W_h}{n_h}\left(n_h\mu_h\right)$$

$$=\sum_{h=1}^{L}W_h\mu_h=\mu\,.$$

4. Simulation Study

In this section a simulation study is conducted to investigate the performance of SQRSS in estimating the population mean. Symmetric and asymmetric distributions are considered for $n=7,\ 12,14,15,18$ by assuming that the population is partitioned into two or three strata. Using 100000 replications, estimates of the means, variances and mean square errors were computed. For each distribution we assumed that the distribution of each stratum follows that distribution. When the underlying distribution is symmetric, the efficiency of SQRSS relative to $SRS, SSRS, SRSS$ is given by

$$eff\left(\overline{X}_{SQRSS},\overline{X}_{SSRS}\right)=\frac{Var\left(\overline{X}_{SSRS}\right)}{Var\left(\overline{X}_{SQRSS}\right)},\quad eff\left(\overline{X}_{SQRSS},\overline{X}_{SRSS}\right)=\frac{Var\left(\overline{X}_{SRSS}\right)}{Var\left(\overline{X}_{SQRSS}\right)},$$

$$eff\left(\bar{X}_{SQRSS}, \bar{X}_{SRS}\right) = \frac{Var\left(\bar{X}_{SRS}\right)}{Var\left(\bar{X}_{SQRSS}\right)},$$

respectively, and if the distribution is asymmetric the efficiency is defined as

$$eff\left(\bar{X}_{SQRSS}, \bar{X}_{SSRS}\right) = \frac{MSE\left(\bar{X}_{SSRS}\right)}{MSE\left(\bar{X}_{SQRSS}\right)}, \; eff\left(\bar{X}_{SQRSS}, \bar{X}_{SRSS}\right) = \frac{MSE\left(\bar{X}_{SRSS}\right)}{MSE\left(\bar{X}_{SQRSS}\right)},$$

$$eff\left(\bar{X}_{SQRSS}, \bar{X}_{SRS}\right) = \frac{MSE\left(\bar{X}_{SRS}\right)}{MSE\left(\bar{X}_{SQRSS}\right)},$$

where MSE is the mean square error (MSE) which is defined as

$$MSE\left(\bar{X}\right) = Var\left(\bar{X}\right) + \left[Bias\left(\bar{X}\right)\right]^2.$$

Table 1: The efficiency of SQRSS1 relative to SRSS, SSRS and SRS for $n=14$ with $n_1=8$ and $n_2=6$.

	$eff\left(\overline{X}_{SQRSS1},\overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SQRSS1},\overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SQRSS1},\overline{X}_{SRS}\right)$
Uniform (0,1)	1.2951	1.1961	1.1765
Normal (0,1)	1.5421	1.7392	1.7081
Student T (3)	2.5941	3.0907	2.9233
Geometric (0.5)	2.3171	1.8348	1.8045
Exponential (1)	1.4827	2.9393	2.8866
Gamma (1,2)	2.8220	2.8187	2.7522
Beta (1,2)	2.0800	1.6000	1.4815
Beta (5,2)	1.5714	1.4615	1.3846
LogNormal (0,1)	2.4629	2.8177	2.7685
Weibull (1,2)	2.4512	2.4762	2.4286

Table 2: The efficiency of SQRSS2 relative to SRSS, SSRS and SRS for $n=7$ with $n_1=4$ and $n_2=3$.

	$eff\left(\overline{X}_{SQRSS2},\overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SQRSS2},\overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SQRSS2},\overline{X}_{SRS}\right)$
Uniform (0,1)	1.4044	1.9680	1.9520
Normal (0,1)	2.2923	1.3206	1.2979
Student T (3)	3.2733	4.1918	4.0163
Geometric (0.5)	3.1237	3.0990	3.0437
Exponential (1)	4.6853	4.5361	4.4577
Gamma (1,2)	4.5464	4.9583	4.8654
Beta (1,2)	2.6986	1.1096	1.0959
Beta (5,2)	1.2593	1.3704	1.3704
LogNormal (0,1)	1.0519	4.1557	4.0867
Weibull (1,2)	1.5090	1.2724	1.2480

Table 3: The efficiency of SQRSS1 relative to SRSS, SSRS and SRS for $n=12$ with $n_1=5$ and $n_2=7$.

	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SRS}\right)$
Uniform (0,1)	2.0526	1.9726	1.9452
Normal (0,1)	3.0702	4.4249	4.3212
Student T (3)	3.7740	2.3829	2.3589
Geometric (0.5)	5.9230	5.5405	5.3883
Exponential (1)	5.1000	7.2101	6.9916
Gamma (1,2)	5.9486	5.5614	5.4599
Beta (1,2)	1.5625	1.4688	1.4375
Beta (5,2)	2.0000	1.6923	1.6154
LogNormal (0,1)	6.2195	8.3230	8.1325
Weibull (1,2)	1.8829	2.0674	2.0112

Table 4: The efficiency of SQRSS1 relative to SRSS, SSRS and SRS for $n=18$ with $n_1=4$, $n_2=6$ and $n_3=8$.

	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SRSS}\right)$	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SSRS}\right)$	$eff\left(\bar{X}_{SQRSS1},\bar{X}_{SRS}\right)$
Uniform (0,1)	1.9037	3.0625	2.8750
Normal (0,1)	2.4148	4.5581	4.3023
Student T (3)	3.0018	3.1649	2.9785
Geometric (0.5)	2.6504	3.0281	2.8414
Exponential (1)	4.4286	6.3913	6.0217
Gamma (1,2)	2.1230	3.4107	3.2293
Beta (1,2)	2.0000	3.4178	3.5317
Beta (5,2)	1.5346	3.8884	3.6292
LogNormal (0,1)	1.8972	3.1877	3.0023
Weibull (1,2)	1.9744	3.2308	3.0513

Table 5: The efficiency of SQRSS2 relative to SRSS, SSRS and SRS for $n = 15$ with $n_1 = 3$, $n_2 = 5$ and $n_3 = 7$.

	$eff\left(\overline{X}_{SQRSS2}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SQRSS2}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SQRSS2}, \overline{X}_{SRS}\right)$
Uniform (0,1)	1.1053	3.1579	2.9474
Normal (0,1)	1.2982	4.9792	4.6250
Student T (3)	2.5238	3.1970	3.0256
Geometric (0.5)	2.7542	6.2294	5.7835
Exponential (1)	3.3857	4.9247	4.5479
Gamma (1,2)	1.8189	3.6919	3.4249
Beta (1,2)	2.6345	5.7726	5.3396
Beta (5,2)	1.3050	3.8128	3.6009
Log Normal (0,1)	4.1032	5.4317	4.8524
Weibull (1,2)	2.8636	4.5294	4.2059

Table 6: The bias values of SQRSS1 for $n = 12$, 14, 18.

	$n = 14$	$n = 12$	$n = 18$	$n = 18$
	$n_1 = 8$, $n_2 = 6$	$n_1 = 5, n_2 = 7$	$n_1 = 10, n_2 = 8$	$n_1 = 4$, $n_2 = 6, n_3 = 8$
Geometric (0.5)	0.0168	0.0168	0.0086	0.0061
Exponential (1)	0.0308	0.0308	0.0297	0.0059
Gamma (1,2)	0.0599	0.0761	0.1124	0.0312
Beta (1,2)	0.0052	0.0052	0.0048	0.0008
Beta (5,2)	0.0347	0.0372	0.0253	0.0092
LogNormal (0,1)	0.0799	0.0852	0.0644	0.0158
Weibull (1,2)	0.0385	0.0458	0.0349	0.0118

Table 7: The bias values of SQRSS2 for $n = 7$, 15

	$n = 7$	$n = 15$
	$n_1 = 4$, $n_2 = 3$	$n_1 = 3, n_2 = 5$, $n_3 = 7$
Geometric (0.5)	0.0170	0.0085
Exponential (1)	0.0309	0.0062
Gamma (1,2)	0.0899	0.0369
Beta (1,2)	0.0054	0.0009
Beta (5,2)	0.0705	0.0094
Log Normal (0,1)	0.0974	0.0118
Weibull (1,2)	0.0770	0.0122

Based on Tables 1-7, we can conclude that:

1. A gain in efficiency is attained using SQRSS method for estimating the population mean of the variable of interest. For example, for $n=18$ with $n_1=4$, $n_2=6$ and $n_3=8$, the efficiency of SQRSS1 with respect to SRSS is 1.9037 for estimating the mean of the uniform distribution.
2. SQRSS is more efficient than SRSS, SSRS and SRS based on the same number of measured units. For example, when $n=12$, the efficiency value of SQRSS1 with respect to SRSS, SSRS and SRS are 3.0702, 4.4249 and 4.3212, respectively, for estimating the mean of the normal distribution.
3. The suggested estimators are more efficient when the underlying distribution is symmetric as compared to some asymmetric distributions.
4. As the number of strata increases, the bias values are decreases. For example, when $n=18$ for three strata the bias of SQRSS is 0.0008 while the bias is 0.0048 for two strata for estimating the mean of $B(1,2)$.

5. Conclusions

In this paper, SQRSS is suggested for estimating the population mean. It is found that SQRSS estimators are unbiased of the population mean of symmetric distribution. Also, the SQRSS is more efficient than SRSS, SSRS and SRS. The SQRSS is recommended for estimating the population mean of symmetric and asymmetric distribution when the bias is small.

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