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Majorization Properties for Subclass of Analytic p-Valent Functions Defined by the Generalized Hypergeometric Function *

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Abstract

The object of the present paper is to investigate the majorization properties of certain subclass of analytic and p-valent functions defined by the generalized hypergeometric function.

Keywords and Phrases: *Analytic, p-valent, Majorization.*

1. Introduction

Let f and g be analytic in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$. We say that f is majorized by g in U (see [11]) and write

$$f(z) << g(z) \quad (z \in U), \tag{1.1}$$

if there exists a function φ, analytic in U such that

$$|\varphi(z)| < 1 \quad \textit{and} \quad f(z) = \varphi(z)g(z) \quad (z \in U). \tag{1.2}$$

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It may be noted that (1.1) is closely related to the concept of quasi-subordination between analytic functions.

For $f(z)$ and $g(z)$ are analytic in U, we say that $f(z)$ is subordinate to $g(z)$ written symbolically as follows:

$$f \prec g \;\; or \; f(z) \prec g(z),$$

if there exists a Schwarz function $w(z)$, which (by definition) is analytic in U with $w(0) = 0$ and $|w(z)| < 1$ $(z \in U)$, such that $f(z) = g(w(z))$ $(z \in U)$. Further, if the function $g(z)$ is univalent in U, then we have the following equivalent (see [12, p. 4])

$$f(z) \prec g(z) \;\Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

Let $A(p)$ denote the class of functions of the form:

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{k+p} z^{k+p} \quad (p \in \mathbb{N} = \{1, 2, ...\}), \tag{1.3}$$

which are analytic and p-valent in U.

For complex parameters $\alpha_1, ..., \alpha_q$ and $\beta_1, ..., \beta_s$ $(\beta_j \notin \mathbb{Z}_0^- = \{0, -1, -2, ...\};$ $j = 1, 2, ..., s)$, we now define the generalized hypergeometric function ${}_qF_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z)$ by (see, for example, [5] and [19, p. 20])

$${}_qF_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k ... (\alpha_q)_k}{(\beta_1)_k ... (\beta_s)_k} . \frac{z^k}{k!} \tag{1.4}$$

$$(q \leq s + 1; q, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; z \in U),$$

where $(\theta)_\nu$ is the Pochhammer symbol defined, in terms of the Gamma function Γ, by

$$(\theta)_\nu = \frac{\Gamma(\theta + \nu)}{\Gamma(\theta)} = \begin{cases} 1 & (\nu = 0; \theta \in \mathbb{C}^* = \mathbb{C} \backslash \{0\}), \\ \theta(\theta + 1) ... (\theta + \nu - 1) & (\nu \in \mathbb{N}; \theta \in \mathbb{C}). \end{cases} \tag{1.5}$$

Corresponding to the function $h_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z)$, defined by

$$h_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z) = z^p \; {}_qF_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z), \tag{1.6}$$

we consider a linear operator

$$H_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z) : A(p) \to A(p),$$

which is defined by the following Hadamard product (or convolution):

$$H_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z)f(z) = h_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z) * f(z). \tag{1.7}$$

We observe that, for a function $f(z)$ of the form (1.3), we have

$$H_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)f(z) = z^p + \sum_{k=1}^{\infty} \frac{(\alpha_1)_k ... (\alpha_q)_k}{(\beta_1)_k ... (\beta_s)_k} . \frac{a_{k+p}}{k!} z^{k+p}. \tag{1.8}$$

If, for convenience, we write

$$H_{p,q,s}(\alpha_1) = H_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s), \tag{1.9}$$

then one can easily verify from the definition (1.8) that (see [5])

$$z(H_{p,q,s}(\alpha_1)f(z))' = \alpha_1 H_{p,q,s}(\alpha_1 + 1)f(z) - (\alpha_1 - p)H_{p,q,s}(\alpha_1)f(z). \tag{1.10}$$

It should be remarked that the linear operator $H_{p,q,s}(\alpha_1)$ is a generalization of many other linear operators considered earlier. In particular, for $f(z) \in A(p)$ we have the following observations:

(i) $H_{p,2,1}(a, 1; c)f(z) = L_p(a; c)f(z)$ $(a \in \mathbb{R}; c \in \mathbb{R}\backslash\mathbb{Z}_0^-)$, this linear operator studied by Saitoh [18] which yields the operator $L(a, c)f(z)$ introduced by Carlson and Shaffer [3] for $p = 1$;

(ii) $H_{p,2,1}(n + p, 1; 1)f(z) = D^{n+p-1}f(z)$ $(n \in \mathbb{N}; n > -p)$, this linear operator studied by Goel and Sohi [6]. In the case when $p = 1, D^n f(z)$ is the Ruscheweyh derivative [17] of $f(z) \in A(1)$;

(iii) $H_{p,2,1}(c, \lambda + p; a)f(z) = I_p^{\lambda}(a, c)f(z)(a, c \in \mathbb{N}\backslash\mathbb{Z}_0^-; \lambda > -p)$, the Cho–Kwon–Srivastava operator [4];

(iv) $H_{p,2,1}(1, p + 1; n + p)f(z) = I_{n,p}f(z)(n \in \mathbb{Z}; n > -p)$, the extended Noor integral operator considered by Liu and Noor [10];

(v) $H_{p,2,1}(p + 1, 1; p + 1 - \lambda)f(z) = \Omega_z^{(\lambda,p)}f(z)$ $(-\infty < \lambda < p + 1)$, the extended fractional differintegral operator considered by Patel and Mishra [15].

Now, by making use of the operator $H_{p,q,s}(\alpha_1)$, we define a new subclass of functions $f \in A(p)$ as follows.

Definition 1. *Let* $-1 \leq B < A \leq 1, p \in \mathbb{N}, j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \gamma \in \mathbb{C}^*, |\gamma(A-B) + \alpha_1 B| < |\alpha_1|$ *and* $f \in A(p)$*. Then* $f \in S^j_{p,q,s}(\gamma; \alpha_{1;} A, B)$*, the class of p-valent functions of complex order* γ *in* U*, if and only if*

$$\left\{1 + \frac{1}{\gamma}\left(\frac{z\left(H_{p,q,s}(\alpha_1)f(z)\right)^{(j+1)}}{\left(H_{p,q,s}(\alpha_1)f(z)\right)^{(j)}} - p + j\right)\right\} \prec \frac{1+Az}{1+Bz}, \tag{1.11}$$

Clearly, we have the following relationships:
(i) $S^j_{p,q,s}(\gamma; \alpha_{1;} 1, -1) = S^j_{p,q,s}(\gamma; \alpha_1)$;
(ii) $S^j_{p,1,0}(\gamma; 1; 1, -1) = S^j_p(\gamma)$;
(ii) $S^0_{1,1,0}(\gamma; 1; 1, -1) = S(\gamma)\,(\gamma \in \mathbb{C}^*)$ (see [13]);
(iii) $S^0_{1,1,0}(1-\alpha; 1, 1, -1) = S^*(\alpha)$ $(0 \leq \alpha < 1)$ (see [16]).
Also, we note that:

(i) For $j = 0, q = s+1$, $\alpha_1 = \beta_1 = p, \alpha_i = 1 (i = 2, 3, ..., s+1)$ and $\beta_i = 1 (i = 2, 3, ...s)$, $S^j_{p,q,s}(\gamma; \alpha_1)$ reduces to the class $S_p(\gamma)\,(\gamma \in \mathbb{C}^*)$ of $p-$valently starlike functions of order γ $(\gamma \in \mathbb{C}^*)$ in U, where

$$S_p(\gamma) = \left\{f(z) \in A(p) : \operatorname{Re}\left(1 + \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - p\right)\right) > 0, p \in \mathbb{N}, \gamma \in \mathbb{C}^*\right\};$$

(ii) For $j = 0, q = s+1, \alpha_1 = p+1, \beta_1 = p, \alpha_i = 1 (i = 2, 3, ..., s+1)$ and $\beta_i = 1 (i = 2, 3, ...s)$, $S^j_{p,q,s}(\gamma; \alpha_1)$ we get the class $K_p(\gamma)\,(\gamma \in \mathbb{C}^*)$ of $p-$valently convex functions of order γ $(\gamma \in \mathbb{C}^*)$ in U, where,

$$K_p(\gamma) = \left\{f(z) \in A(p) : \operatorname{Re}\left(1 + \frac{1}{\gamma}\left(1 + \frac{zf''(z)}{f'(z)} - p\right)\right) > 0, p \in \mathbb{N}, \gamma \in \mathbb{C}^*\right\}.$$

We shall need the following lemma.

Lemma 1 [1]. *Let* $\gamma \in \mathbb{C}^*$ *and* $f \in K^j_p(\gamma)$*. Then* $f \in S^j_p(\frac{1}{2}\gamma)$*, that is,*

$$K^j_p(\gamma) \subset S^j_p(\frac{1}{2}\gamma) \quad (\gamma \in \mathbb{C}^*). \tag{1.12}$$

A majorization problem for the class $S(\gamma)(\gamma \in \mathbb{C}^*)$ has been investigated by Altintas et al. [1]. Also, majorization problem for the class $S^* = S^*(0)$ has been investigated by MacGregor [11]. Recently Goyal and Goswami [8] and Goyal et al. [9] generalized these results for classes of multivalent function defined by fractional derivatives operator and Saitoh operator, respectively. In this paper we investigate majorization problem for the class $S^j_{p,q,s}(\gamma; \alpha_{1;} A, B)$ and other related subclasses.

2. Main Results

Unless otherwise mentioned we shall assume throughout the paper that $-1 \leq B < A \leq 1, \gamma, \alpha_1 \in \mathbb{C}^*, j \in \mathbb{N}_0$ and $p \in \mathbb{N}$.

Theorem 1. *Let the function $f \in A(p)$ and suppose that $g \in S^j_{p,q,s}(\gamma; \alpha_{1;} A, B)$. If $(H_{p,q,s}(\alpha_1)f(z))^{(j)}$ is majorized by $(H_{p,q,s}(\alpha_1)g(z))^{(j)}$ in U, then*

$$\left|(H_{p,q,s}(\alpha_1+1)f(z))^{(j)}\right| \leq \left|(H_{p,q,s}(\alpha_1+1)g(z))^{(j)}\right| \qquad (|z| < r_0), \tag{2.1}$$

where $r_0 = r_0(\gamma, \alpha_1, A, B)$ is the smallest positive root of the equation

$$|\gamma(A-B)+\alpha_1 B|\, r^3 - (2\,|B| + |\alpha_1|)r^2 - (2 + |\gamma(A-B)+\alpha_1 B|)r + |\alpha_1| = 0. \tag{2.2}$$

Proof. Since $g \in S^j_{p,q,s}(\gamma; \alpha_{1;} A, B)$, we find from (1.11) that

$$1 + \frac{1}{\gamma}\left(\frac{z\,(H_{p,q,s}(\alpha_1)g(z))^{(j+1)}}{(H_{p,q,s}(\alpha_1)g(z))^{(j)}} - p + j\right) = \frac{1+Aw(z)}{1+Bw(z)}, \tag{2.3}$$

where w is analytic in U with $w(0) = 0$ and $|w(z)| < 1$ $(z \in U)$. From (2.3), we have

$$\frac{z\,(H_{p,q,s}(\alpha_1)g(z))^{(j+1)}}{(H_{p,q,s}(\alpha_1)g(z))^{(j)}} = \frac{(p-j) + (\gamma(A-B) + (p-j)B)w(z)}{1+Bw(z)}. \tag{2.4}$$

Also from (1.10), we have

$$z\,(H_{p,q,s}(\alpha_1)g(z))^{(j+1)} = \alpha_1\,(H_{p,q,s}(\alpha_1+1)g(z))^{(j)} - (\alpha_1+j-p)\,(H_{p,q,s}(\alpha_1)g(z))^{(j)}. \tag{2.5}$$

From (2.4) and (2.5), we have

$$\left|(H_{p,q,s}(\alpha_1)g(z))^{(j)}\right| \leq \frac{|\alpha_1|\,(1+|B|\,|z|)}{|\alpha_1| - |\gamma(A-B)+\alpha_1 B|\,|z|}\left|(H_{p,q,s}(\alpha_1+1)g(z))^{(j)}\right|. \tag{2.6}$$

Next, since $(H_{p,q,s}(\alpha_1)f(z))^{(j)}$ is majorized by $(H_{p,q,s}(\alpha_1)g(z))^{(j)}$ in U, from (1.2), we have

$$(H_{p,q,s}(\alpha_1)f(z))^{(j)} = \varphi(z)\,(H_{p,q,s}(\alpha_1)g(z))^{(j)}. \tag{2.7}$$

Differentiating (2.7) with respect to z and multiplying by z, we have

$$z\left(H_{p,q,s}(\alpha_1)f(z)\right)^{(j+1)} = z\varphi^{'}(z)\left(H_{p,q,s}(\alpha_1)g(z)\right)^{(j)} + z\varphi(z)\left(H_{p,q,s}(\alpha_1)g(z)\right)^{(j+1)}, \tag{2.8}$$

using (2.5) in (2.8), we have

$$\left(H_{p,q,s}(\alpha_1+1)f(z)\right)^{(j)} = \frac{z\varphi^{'}(z)}{\alpha_1}\left(H_{p,q,s}(\alpha_1)g(z)\right)^{(j)} + \varphi(z)\left(H_{p,q,s}(\alpha_1+1)g(z)\right)^{(j)}. \tag{2.9}$$

Thus, by noting that $\varphi(z)$ satisfies the inequality (see [14]),

$$\left|\varphi^{'}(z)\right| \leq \frac{1-|\varphi(z)|^2}{1-|z|^2} \quad (z \in U), \tag{2.10}$$

and making use of (2.6) and (2.10) in (2.9), we have

$$\left|\left(H_{p,q,s}(\alpha_1+1)f(z)\right)^{(j)}\right| \leq$$
$$\left(|\varphi(z)| + \frac{1-|\varphi(z)|^2}{1-|z|^2}.\frac{(1+|B|\,|z|)\,|z|}{|\alpha_1| - |\gamma(A-B)+\alpha_1 B\,|z||}\right)\left|\left(H_{p,q,s}(\alpha_1+1)g(z)\right)^{(j)}\right|, \tag{2.11}$$

which upon setting

$$|z| = r \quad \text{and} \quad |\varphi(z)| = \rho \quad (0 \leq \rho \leq 1),$$

leads us to the inequality

$$\left|\left(H_{p,q,s}(\alpha_1+1)f(z)\right)^{(j)}\right| \leq$$
$$\frac{\Psi(\rho)}{(1-r^2)(|\alpha_1| - |\gamma(A-B)+\alpha_1 B|\,r)}\left|\left(H_{p,q,s}(\alpha_1+1)g(z)\right)^{(j)}\right|,$$

where

$$\begin{aligned}\Psi(\rho) &= -r\,(1+|B|\,r)\,\rho^2 + (1-r^2)(|\alpha_1| - |\gamma(A-B)+\alpha_1 B|\,r)\rho \\ &\quad +r\,(1+|B|\,r)\,,\end{aligned} \tag{2.12}$$

takes its maximum value at $\rho = 1$, with $r_0 = r_0(\gamma, \alpha_1, A, B)$, where $r_0(\gamma, \alpha_1, A, B)$ is the smallest positive root of (2.2), then the function $\Phi(\rho)$ defined by

$$\begin{aligned}\Phi(\rho) &= -\sigma\,(1+|B|\,\sigma)\,\rho^2 + (1-\sigma^2)\,[|\alpha_1| - |\gamma(A-B)+\alpha_1 B|\,\sigma]\,\rho \\ &\quad +\sigma\,(1+|B|\,\sigma) \qquad (2.13)\end{aligned}$$

is an increasing function on the interval $0 \leq \rho \leq 1$, so that

$$\begin{aligned}\Phi(\rho) &\leq \Phi(1) = (1-\sigma^2)(|\alpha_1| - |\gamma(A-B)+\alpha_1 B|\,\sigma) \\ &\quad (0 \leq \rho \leq 1;\ 0 \leq \sigma \leq r_0(p,\gamma,\alpha_1,A,B)). \qquad (2.14)\end{aligned}$$

Hence upon setting $\rho = 1$ in (2.13), we conclude that (2.1) holds true for $|z| \leq r_0 = r_0(\gamma, \alpha_1, A, B)$, where $r_0(\gamma, \alpha_1, A, B)$ is the smallest positive root of (2.2). This completes the proof of Theorem 1.

Putting $A = 1$ and $B = -1$ in Theorem 1, we obtain the following corollary.

Corollary 1. *Let the function $f \in A(p)$ and suppose that $g \in S^j_{p,q,s}(\gamma;\alpha_1)$. If $(H_{p,q,s}(\alpha_1)f(z))^{(j)}$ is majorized by $(H_{p,q,s}(\alpha_1)g(z))^{(j)}$ in U, then*

$$\left|(H_{p,q,s}(\alpha_1+1)f(z))^{(j)}\right| \leq \left|(H_{p,q,s}(\alpha_1+1)g(z))^{(j)}\right| \qquad (|z| < r_0),$$

where $r_0 = r_0(\gamma;\alpha_1)$ is given by

$$r_0 = r_0(\gamma;\alpha_1) = \frac{k - \sqrt{k^2 - 4\,|2\gamma-\alpha_1|\,|\alpha_1|}}{2\,|2\gamma-\alpha_1|},$$

where $(k = 2 + |\alpha_1| + |2\gamma-\alpha_1|\,, \gamma, \alpha_1 \in \mathbb{C}^*)$.

Putting $q = s+1$, $\alpha_1 = \beta_1 = p, \alpha_i = 1 (i = 2, 3, ..., s+1)$ and $\beta_i = 1 (i = 2, 3, ...s)$ in Corollary 1, we obtain the following corollary.

Corollary 2 [1, Theorem 1]. *Let the function $f \in A(p)$ and suppose that $g \in S^j_p(\gamma)$. If $f^{(j)}(z)$ is majorized by $g^{(j)}(z)$ in U, then*

$$\left|f^{(j+1)}(z)\right| \leq \left|g^{(j+1)}(z)\right| \qquad (|z| < r_0),$$

where $r_0 = r_0(\gamma, p, j)$ *is given by*

$$r_0 = r_0(\gamma, p, j) = \frac{k - \sqrt{k^2 - 4p\,|2\gamma - p + j|}}{2\,|2\gamma - p + j|},$$

where $(k = 2 + p - j + |2\gamma - p + j|\,, p \in \mathbb{N}, j \in \mathbb{N}_0, \gamma \in \mathbb{C}^*)$.

Putting $j = 0$ in Corollary 2, we obtain the following corollary.

Corollary 3. *Let the function* $f \in A(p)$ *and suppose that* $g \in S_p(\gamma)$. *If* $f(z)$ *is majorized by* $g(z)$ *in* U, *then*

$$|f'(z)| \le |g'(z)| \qquad (|z| < r_0)\,,$$

where $r_0 = r_0(\gamma, p)$ *is given by*

$$r_0 = r_0(\gamma, p) = \frac{k - \sqrt{k^2 - 4p\,|2\gamma - p|}}{2\,|2\gamma - p|},$$

where $(k = 2 + p + |2\gamma - p|\,, p \in \mathbb{N}, \gamma \in \mathbb{C}^*)$.

Putting $j = 0, q = s + 1, \alpha_1 = p + 1, \beta_1 = p, \alpha_i = 1(i = 2, 3, ..., s + 1)$ and $\beta_i = 1(i = 2, 3, ...s)$, in Corollary 1, with the aid of Lemma 1 (with $j = 0$), we obtain the following corollary.

Corollary 4. *Let the function* $f \in A(p)$ *and suppose that* $g \in K_p(\gamma)$. *If* $f(z)$ *is majorized by* $g(z)$ *in* U, *then*

$$|f'(z)| \le |g'(z)| \qquad (|z| < r_0)\,,$$

where $r_0 = r_0(\gamma, p)$ *is given by*

$$r_0 = r_0(\gamma, p) = \frac{k - \sqrt{k^2 - 4p\,|\gamma - p|}}{2\,|\gamma - p|},$$

where $(k = 2 + p + |\gamma - p|\,, p \in \mathbb{N}, \gamma \in \mathbb{C}^*)$.

Putting $A = 1, B = -1, p = 1, j = 0$ and $q = 2, s = 1, \alpha_1 = \alpha_2 = \beta_1 = 1$, in Theorem 1, we obtain the following corollary.

Corollary 5 [2, Theorem 1]. *Let the function $f \in A$ and suppose that $g \in S(\gamma)$. If $f(z)$ is majorized by $g(z)$ in U, then*

$$|f'(z)| \leq |g'(z)| \qquad (|z| < r_0),$$

where $r_0 = r_0(\gamma)$ is given by

$$r_0 = r_0(\gamma) = \frac{k - \sqrt{k^2 - 4\,|2\gamma - 1|}}{2\,|2\gamma - 1|},$$

where $(k = 3 + |2\gamma - 1|, \gamma \in \mathbb{C}^*)$.

Letting $\gamma \to 1$ in Corollary 5, we obtain the following corollary.

Corollary 6 [11]. *Let the function $f \in A$ and suppose that $g \in S^*$. If $f(z)$ is majorized by $g(z)$ in U, then*

$$|f'(z)| \leq |g'(z)| \qquad (|z| < r_0),$$

where r_0 is given by

$$r_0 = 2 - \sqrt{3}.$$

Remarks. (i) Putting $q = 2, s = 1, \alpha_1 = p + 1, \alpha_2 = 1$ and $\beta_1 = p + 1 - \lambda$, in Theorem 1 we obtain the result obtained by Goswami and Wang [7, Theorem 1];

(ii) Putting $A = 1, B = -1, q = 2, s = 1, \alpha_1 = p + 1, \alpha_2 = 1$ and $\beta_1 = p + 1 - \lambda$, in Corollary 1 we obtain the result obtained by Goyal and Goswami [8, Theorem 1];

(iii) Putting $q = 2, s = 1, \alpha_1 \in \mathbb{R}, \alpha_2 = 1$ and $\beta_1 \in \mathbb{R}\backslash\mathbb{Z}_0^-$, in Theorem 1 we obtain the result obtained by Goyal et al. [9, Theorem 1];

(iv) Also by specializing the parameters p,$\alpha_i (i = 1, 2, ..., q)$ and $\beta_j (j = 1, 2, ..., s)$, we obtain various results corresponding to various operators defined in the introduction.

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