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Some Properties of LP-Sasakian Manifolds Admitting a Quarter Symmetric Non Metric Connection *

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Abstract

The object of the present paper is to study weakly symmetric, weakly Ricci symmetric, generalized recurrent and generalized Ricci recurrent LP-Sasakian manifolds admitting a quarter symmetric non metric connection ∇.

Keywords and Phrases: *Quarter-symmetric non metric connection, LP-Sasakian manifolds , Weakly symmetric, Weakly Ricci symmetric, Generalized recurrent, Generalized Ricci recurrent.*

1. Introduction

The idea of metric connection with torsion tensor in a Riemannian manifold was introduced by Hayden [19]. Later, Yano [11] studied some properties of semi symmetric metric connection on a Riemannian manifold. The semi symmetric metric connection has important physical application such as the displacement on the earth surface following a fixed point is metric and semi-

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symmetric. Golab [1] introduced and studied quarter symmetric connection in a Riemannian manifold with an affine connection, which generalizes the idea of semi symmetric connection. After Golab, Rastogi ([6], [7]) continued the systematic study of quarter symmetric metric connection. Pandey and Mishra [2], studied quarter symmetric metric connection in a Riemannian, Kahlerian and Sasakian manifolds. It is also studied by many geometers like as Yano et al. [12], De and Biswas [8], jaiswal [20], Mukhopadhya [9], Mondal et al. [3] and many others.

On the other hand Matsumoto [18] introduced the notion of LP-Sasakian manifold.Then Mihai and Rosoca [10] introduced the same notion independently and obtained several results on this manifold. LP-Sasakian manifolds are also studied by De et al. [4], Mihai [10] , Singh [21] and others.

The notion of weakly symmetric and weakly Ricci symmetric Riemannian manifolds were introduced by Tamassay ([15], [16]). Sular [13] investigated some properties of generalized recurrent, weakly symmetric and weakly Ricci symmetric Kenmotsu manifolds admitting a semi symmetric metric connection. In the present paper we discuss a quarter symmetric non metric connection in a LP-Sasakian manifolds. In Section 2, we give a brief introduction of LP-Sasakian manifolds and quarter symmetric non metric connection. In Section 3 and 4 it is shown that there is no weakly symmetric and weakly Ricci symmetric LP-Sasakian manifolds admitting a quarter symmetric non metric connection, unless $a + c + d$ or $\lambda + \mu + \nu$ vanishes everywhere respectively. In the last Section, it is proved that $B + 2\ A = 0$ on generalized recurrent and generalized Ricci recurrent LP-Sasakian manifolds admitting a quarter symmetric non metric connection.

2. Preliminaries

An n-dimensional differentiable manifolds M^n is a Lorentzian Para-Sasakian manifolds(briefly LP-Sasakian manifolds) if it admits a (1,1) tensor field ϕ, contravariant vector field ξ, a covariant vector field η, and a Lorentzian metric g, which satisfy

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$\phi\xi = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X,\xi) = \eta(X), \tag{2.4}$$

$$(D_X\phi)Y = g(X,Y)\xi + \eta(Y)X + 2\,\eta(X)\eta(Y)\xi, \tag{2.5}$$

$$D_X\xi = \phi X, \tag{2.6}$$

$$(a)\ \ \eta(\xi) = -1 \quad (b) \quad \eta(\phi X) = 0, \tag{2.7}$$

$$rank(\phi) = (n-1), \tag{2.8}$$

$$(D_X\eta)(Y) = g(\phi X, Y) = g(\phi Y, X), \tag{2.9}$$

for any vector fields X and Y,where D denotes covariant differentiation with respect to g ([10], [18]).

In an LP-Sasakian manifold M^n with the structure (ϕ, ξ, η, g) following conditions hold:

$$R(X,Y)Z = g(Y,Z)X - g(X,Z)Y, \tag{2.10}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.11}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.12}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.13}$$

$$R(X,\xi)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.14}$$

for any vector fields X, Y, Z, where R and S are the Riemannian curvature tensor and Ricci tensor of the manifolds respectively ([10], [18]).

Here we consider a quarter symmetric non metric connection ∇ on LP-Sasakian manifolds

$$\nabla_X Y = D_X Y - \eta(X)\phi Y \tag{2.15}$$

given by Mishra and Pandey [2] which satisfies

$$(\nabla_X g)(Y, Z) = 2\,\eta(X)\,g(\phi Y, Z). \tag{2.16}$$

The curvature tensor $\overline{R}$ with respect to a quarter symmetric non metric connection ∇ and the curvature tensor R with respect to Riemannian connection D in LP-Sasakian manifolds are related as

$$\begin{aligned}\overline{R}(X, Y)Z &= R(X, Y)Z + g(Y, Z)\,\eta(X)\xi - g(X, Z)\,\eta(Y)\xi \\ &+ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X.\end{aligned} \tag{2.17}$$

Contracting (2.17) with respect to X we get

$$\overline{S}(Y, Z) = S(Y, Z) - g(Y, Z) - n\,\eta(Y)\eta(Z), \tag{2.18}$$

where $\overline{S}$ is Ricci tensor of M^n with respect to quarter symmetric non metric connection .

Lemma 1. *In an LP-Sasakian manifold admitting a quarter symmetric non metric connection, we have*

$$\overline{R}(X, Y)\,\xi = 2R(X, Y)\,\xi. \tag{2.19}$$

$$\overline{S}(X, \xi) = 2S(X, \xi) = 2(n-1)\,\eta(X). \tag{2.20}$$

3. Weakly symmetric LP-Sasakian manifolds admitting a quarter symmetric non metric connection ∇

A non flat Riemannian manifold M^n $(n > 3)$ is called weakly symmetric if there exist 1-forms a, b, c, d and the Riemannian curvature tensor R satisfies the condition ([15], [16])

$$\begin{aligned}(D_X R)(Y, Z)U &= a(X)R(Y, Z)U + b(Y)R(X, Z)U + c(Z)R(Y, X)U \\ &+ d(U)R(Y, Z)X + g(R(Y, Z)U, X)\rho,\end{aligned} \tag{3.1}$$

for vector fields X, Y, Z, U, where a, b, c, d and ρ are not simultaneously zero.

Now the weakly symmetric of a non flat Riemannian manifold M^n $(n > 3)$ with respect to a quarter symmetric non metric connection is given as

$$\begin{aligned} (\nabla_X \overline{R})(Y,Z)U &= a(X)\,\overline{R}(Y,Z)U + b(Y)\,\overline{R}(X,Z)U + c(Z)\,\overline{R}(Y,X)U \\ &+ d(U)\,\overline{R}(Y,Z)X + g(\overline{R}(Y,Z)U,X)\,\rho, \end{aligned} \tag{3.2}$$

for vector fields X, Y, Z, U,where a, b, c, d and ρ are not simultaneously zero . Contracting the above equation with respect to Y, we obtain

$$\begin{aligned} (\nabla_X \overline{S})(Z,U) &= a(X)\,\overline{S}(Z,U) + b(\overline{R}(X,Z)U) + c(Z)\,\overline{S}(X,U) \\ &+ d(U)\,\overline{S}(X,Z) + e(\overline{R}(X,U)Z)), \end{aligned} \tag{3.3}$$

where $e(X) = g(X, \rho)$.
Replacing U with ξ in (3.3) we get

$$\begin{aligned} (\nabla_X \overline{S})(Z,\xi) &= a(X)\,\overline{S}(Z,\xi) + b(\overline{R}(X,Z)\xi) + c(Z)\overline{S}(X,\xi) \\ &+ d(\xi)\overline{S}(X,Z) + e(\overline{R}(X,\xi)Z). \end{aligned}$$

By the virtue of (2.19),(2.20),(2.12) and (2.13), the above equation becomes

$$\begin{aligned} (\nabla_X \overline{S})(Z,\xi) &= 2(n-1)\,a(X)\eta(Z) + 2b(X)\,\eta(Z) - 2b(Z)\,\eta(X) \\ &+ d(\xi)\,\{S(X,Z) - g(X,Z) - n\,\eta(X)\eta(Z)\} \\ &+ 2e(X)\,\eta(Z) + 2e(\xi)\,\eta(X)\eta(Z) \\ &+ 2(n-1)\,c(Z)\,\eta(X). \end{aligned} \tag{3.4}$$

Now, we know that

$$(\nabla_X \overline{S})(Z,U) = \nabla_X.\overline{S}(Z,U) - \overline{S}((\nabla_X Z, U) - \overline{S}(\nabla_X Z, U). \tag{3.5}$$

Putting $U = \xi$ and taking account of (2.6) in (3.5), we get

$$\begin{aligned} (\nabla_X \overline{S})(Z,\xi) &= \nabla_X.\overline{S}(Z,\xi) - \overline{S}((\nabla_X Z,\xi) - \overline{S}(\nabla_X Z,\xi) \\ &= (2n-1)\,g(X,\phi Z) - S(X,\phi Z). \end{aligned} \tag{3.6}$$

From (3.4) and (3.6), we have

$$\begin{aligned} 2(n-1)\,a(X)\eta(Z) &+ 2\,b(X)\,\eta(Z) - 2\,b(Z)\,\eta(X) \\ &+ d(\xi)\,\{S(X,Z) - g(X,Z) - n\,\eta(X)\eta(Z)\} \\ &+ 2\,e(X)\,\eta(Z) + 2e(\xi)\,\eta(X)\eta(Z) \\ &+ 2(n-1)\,c(Z)\,\eta(X) \\ &= (2n-1)\,g(X,\phi Z) - S(X,\phi Z). \end{aligned}$$

Putting $X = Z = \xi$ and using (2.7) in the above equation, we obtain

$$2(n-1)\,\{a(\xi)+c(\xi)+d(\xi)\}=0,$$

which gives ($n\ >\ 3$)

$$a(\xi)+c(\xi)+d(\xi)=0. \tag{3.7}$$

Again replacing Z with ξ in (3.3) we get

$$\begin{aligned}(\nabla_X\overline{S})(\xi,U) &= a(X)\,\overline{S}(\xi,U)+b(Y)\overline{R}(X,\xi)U)+c\,(\xi)\,\overline{S}(X,U)\\ &+\ d(U)\,\overline{S}(X,\xi)+e(\overline{R}(X,U)\xi))\end{aligned}$$

Now, by the virtue of (2.19),(2.20),(2.12) and (2.13), the above equation becomes

$$\begin{aligned}(\nabla_X\overline{S})(\xi,U) &= 2(n-1)\,a(X)\eta(U)+2b(X)\,\eta(U)-2b(\xi)\,\eta(X)\eta(U)\\ &+\ c(\xi)\,\{S(X,U)-g(X,U)-n\,\eta(X)\eta(U)\}\\ &+\ 2(n-1)\,d(U)\,\eta(X)+2e(X)\,\eta(U)\\ &-\ 2\,e(U)\,\eta(X).\end{aligned} \tag{3.8}$$

On the other hand we get

$$\begin{aligned}(\nabla_X\overline{S})(\xi,U) &= \nabla_X.\overline{S}(\xi,U)-\overline{S}((\nabla_X\xi,U)-\overline{S}(\nabla_X\xi,U)\\ &= 2(n-1)(\nabla_X\eta)(U)-\overline{S}(X,\phi U)\\ &= (2n-1)g(\phi X,U)-S(\phi X,U).\end{aligned} \tag{3.9}$$

Equating the right hand sides of (3.8) and (3.9) we get

$$\begin{aligned}(2n-1)g(\phi X,U) &-\ S(\phi X,U)\\ &=\ 2(n-1)\,a(X)\eta(U)+2b(X)\,\eta(U)-2b(\xi)\,\eta(X)\eta(U)\\ &+\ c(\xi)\,\{S(X,U)-g(X,U)-n\,\eta(X)\eta(U)\}\\ &+\ 2(n-1)\,d(U)\,\eta(X)+2e(X)\,\eta(U)\\ &-\ 2\,e(U)\,\eta(X).\end{aligned} \tag{3.10}$$

Taking $U = \xi$ and taking account of (2.7) and (2.12) the above equation assumes the form

$$\begin{aligned}&-\ 2(n-1)\,a(X)-2b(X)-b(\xi)\,\eta(X)+2(n-1)\,c(\xi)\,\eta(X)\\ &+\ 2(n-1)\,d(\xi)\,\eta(X)-2e(X)-2\,e(\xi)\,\eta(X)=0.\end{aligned} \tag{3.11}$$

Again taking $X = \xi$ in (3.10), we obtain

$$\begin{aligned} 2(n-1)\, a(\xi)\, \eta(U) &+ 2(n-1)\, c(\xi)\, \eta(U) - 2(n-1)\, d(U) \\ &+ 2\, e(U) + 2\, e(\xi)\, \eta(U) = 0. \end{aligned} \tag{3.12}$$

Replacing U with X in (3.12) we get

$$\begin{aligned} 2(n-1)\, a(\xi)\, \eta(X) &+ 2(n-1)\, c(\xi)\, \eta(X) - 2(n-1)\, d(X) \\ &+ 2\, e(X) + 2\, e(\xi)\, \eta(X) = 0. \end{aligned} \tag{3.13}$$

Adding (3.11) and (3.13) and taking account of (3.7) we get

$$\begin{aligned} -2(n-1)\, a(X) &- 2\, b(X) - 2\, b(\xi)\, \eta(X) \\ &+ 2(n-1)\, c(\xi)\, \eta(X) - 2\, (n-1)\, d(X) = 0. \end{aligned} \tag{3.14}$$

Now, taking $X = \xi$ in (3.6) we get

$$\begin{aligned} 2(n-1)\, a(\xi)\, \eta(Z) &+ 2\, b(\xi)\, \eta(Z) + 2\, b(Z) - 2\, (n-1)\, c(Z) \\ &+ 2\, d(\xi)\, \eta(Z) = 0. \end{aligned} \tag{3.15}$$

Replacing Z by X in (3.15) we get

$$\begin{aligned} 2(n-1)\, a(\xi)\, \eta(X) &+ 2\, b(\xi)\, \eta(X) + 2\, b(Z) - 2\, (n-1)\, c(X) \\ &+ 2\, d(\xi)\, \eta(X) = 0. \end{aligned} \tag{3.16}$$

Finally adding (3.14) and (3.16) and taking account of (3.7) we get

$$2(n-1)\{a(X) + c(X) + d(X)\} = 0,$$

which implies that $(n > 3)$

$$a(X) + c(X) + d(X) = 0,$$

for any vector field X. Thus we can state that:

Theorem 1. *There is no weakly symmetric LP-Sasakian manifolds M^n admitting a quarter symmetric non metric connection, unless $a + c + d$ vanishes everywhere.*

4. Weakly Ricci symmetric LP-Sasakian manifolds admitting a quarter symmetric non metric connection

A non flat Riemannian manifold M^n is called weakly Ricci symmetric if there exist 1-forms λ, μ and ν and Ricci tensor S satisfies the condition [16]

$$(D_X S)(Y,Z) = \lambda(X)S(Y,Z) + \mu(Y)S(X,Z) + \nu(Z)S(Y,X), \tag{4.1}$$

for all vector fields X, Y, Z, where λ, μ and ν are not simultaneously zero. We give the following definition: A non flat Riemannian manifold M^n is called weakly Ricci symmetric with respect to a quarter symmetric non metric connection ∇ if there exist 1-forms λ, μ and ν and Ricci tensor $\overline{S}$ satisfies the condition

$$(\nabla_X \overline{S})(Y,Z) = \lambda(X)\overline{S}(Y,Z) + \mu(Y)\overline{S}(X,Z) + \nu(Z)\overline{S}(Y,X), \tag{4.2}$$

for all vector fields X, Y, Z.

Let us assume that M^n be a weakly Ricci symmetric LP-Sasakian manifold admitting a quarter symmetric non metric connection ∇. So the equation (4.2) take place. Taking $Z = \xi$ in (4.2) we get

$$\begin{aligned}(\nabla_X \overline{S})(Y,\xi) &= \lambda(X)\overline{S}(Y,\xi) + \mu(Y)\overline{S}(X,\xi) \\ &+ \nu(\xi)\overline{S}(Y,X). \end{aligned} \tag{4.3}$$

By the virtue of (3.6) the above equation gives

$$\begin{aligned}\lambda(X)\overline{S}(Y,\xi) &+ \mu(Y)\overline{S}(X,\xi) + \nu(\xi)\overline{S}(Y,X) \\ &= (2n-1)\, g(\phi X, \phi Y) - S(\phi X, Y). \end{aligned} \tag{4.4}$$

Setting $X = Y = \xi$ in above equation, we obtain

$$-(2n-1)\{\lambda(\xi) + \mu(\xi) + \nu(\xi)\} = 0,$$

which implies that

$$\lambda(\xi) + \mu(\xi) + \nu(\xi) = 0. \tag{4.5}$$

Putting $X = \xi$ in (4.4) we get

$$\mu(Y) = -\,\mu(\xi)\,\eta(Y). \tag{4.6}$$

Again taking $Y = \xi$ in (4.4), we obtain

$$\lambda(X) = -\,\lambda(\xi)\,\eta(X). \tag{4.7}$$

Since $(\nabla_\xi \overline{S})(\xi, X) = 0$, then from (4.2), it can be shown that

$$\nu(X) = -\,\nu(\xi)\,\eta(X). \tag{4.8}$$

Replacing Y by X in (4.6), we get

$$\mu(X) = -\,\mu(\xi)\,\eta(X). \tag{4.9}$$

Adding (4.7), (4.8) and (4.9), we get

$$\lambda(X) + \mu(X) + \nu(X) = 0, \tag{4.10}$$

for any vector field X on M^n.
This leads to the following;

Theorem 2. *There is no weakly Ricci symmetric LP-Sasakian manifolds M^n admitting a quarter symmetric non metric connection ∇, unless $\lambda + \mu + \nu$ vanishes everywhere.*

5.Generalized recurrent LP-Sasakian manifolds admitting a quarter symmetric non metric connection ∇

A non flat n-dimensional differentiable manifold M^n is called generalized recurrent [5] if curvature tensor R satisfies the condition

$$(D_X R)(Y, Z)U = A(X)\,R(Y, Z)U + B(X)\,\{g(Z, U)Y - g(Y, U)Z\}, \tag{5.1}$$

where $A,\ B$ are two 1-forms, $(B \neq 0)$ defined by

$$A(X) = g(X, \rho_1), \qquad B(X) = g(X, \rho_2) \tag{5.2}$$

and ρ_1, ρ_2 are vector fields related with 1-forms A, B respectively. Now, we give the following definition. A non flat n-dimensional differentiable manifold M^n is called generalized recurrent with respect to a quarter symmetric non metric connection ∇ if curvature tensor $\overline{R}$ satisfies the condition

$$(\nabla_X \overline{R})(Y,Z)U = A(X)\ \overline{R}(Y,Z)U + B(X)\ \{g(Z,U)Y - g(Y,U)Z\}. \quad (5.3)$$

Let the manifold M^n is generalized recurrent LP-Sasakian manifold admitting a quarter symmetric non metric connection ∇. Then from above equation holds. Setting $Y = Z = \xi$ in (5.3) we get

$$\begin{aligned}(\nabla_X \overline{R})(\xi,Z)\xi &= A(X)\ \overline{R}(\xi,Z)\xi + B(X)\ \{\eta(Z)\xi + Z\}.\\ &= [B(X) + 2\ A(X)]\{\eta(Z)\xi + Z\}. \end{aligned} \quad (5.4)$$

On the other hand it is obvious that

$$\begin{aligned}(\nabla_X \overline{R})(\xi,Z)\xi &= \nabla_X.\overline{R}(\xi,Z)\xi - \overline{R}(\nabla_X\xi,Z)\xi\\ &- \overline{R}(\xi,\nabla_X Z)\xi - \overline{R}(\xi,Z)\nabla_X\xi. \end{aligned} \quad (5.5)$$

In view of (2.19), (2.13) and (2.6) the above equation gives

$$(\nabla_X \overline{R})(\xi,Z)\xi = 0. \quad (5.6)$$

Hence we obtain

$$[B(X) + 2\ A(X)]\{\eta(Z)\xi + Z\} = 0, \quad (5.7)$$

which implies that $B(X) + 2\ A(X) = 0$ for any vector field X. This leads to the following:

Theorem 3. *If a generalized recurrent LP-Sasakian manifolds M^n admits a quarter symmetric non metric connection ∇, then $B + 2\ A = 0$ holds on M^n.*

A non flat n-dimensional differentiable manifold M^n is called generalized Ricci recurrent [5] if its Ricci tensor S satisfies the condition

$$(D_X S)(Y,Z) = A(X)\ S(Y,Z) + (n-1)B(X)\ g(Y,Z), \quad (5.8)$$

where A, B are are given by [5.2]. Analogous to above definition a non flat n-dimensional differentiable manifold M^n is called generalized Ricci recurrent with respect to a quarter symmetric non metric connection ∇ if its Ricci tensor $\overline{S}$ satisfies the condition

$$(\nabla_X\overline{S})(Y,Z) = A(X)\,\overline{S}(Y,Z) + (n-1)B(X)\,g(Y,Z). \tag{5.9}$$

Putting $Z=\xi$ in above equation we obtain

$$\begin{aligned}(\nabla_X\overline{S})(Y,\xi) &= A(X)\,\overline{S}(Y,\xi) + (n-1)B(X)\,g(Y,\xi)\\ &= (n-1)[B(X)+2\,A(X)]\,\eta(Y).\end{aligned} \tag{5.10}$$

On the other hand by virtue of (3.6) we have

$$(\nabla_X\overline{S})(Y,\xi) = (2n-1)\,g(\phi X,Y) - S(\phi X,Y). \tag{5.11}$$

Comparing equations (5.10) and (5.11), we obtain

$$\begin{aligned}(n-1)[B(X) &+ 2\,A(X)]\,\eta(Y)\\ &= (2n-1)\,g(\phi X,Y) - S(\phi X,Y).\end{aligned} \tag{5.12}$$

Taking $Y=\xi$ in above equation we get

$$(n-1)[B(X) + 2\,A(X)] = 0,$$

which implies that

$$B(X) + 2\,A(X) = 0, \tag{5.13}$$

for all vector field X. Thus we can state that:

Theorem 4. *Let M^n be a generalized Ricci recurrent LP-Sasakian manifolds admitting a quarter symmetric non metric connection ∇. Then $B+2\,A=0$ holds on M^n.*

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