On The Orthogonal Stability of The Pexiderized Quadratic Equations in Modular Spaces [*]

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Abstract

In this paper, we establish the Hyers–Ulam stability of the orthogonal quadratic functional equation of Pexiderized type $f(x + y) + f(x - y) = 2g(x) + 2h(y)$, $x \perp y$ in which $\perp$ is the orthogonality in the sense of Rätz in modular spaces.

1. Introduction

The purpose of this paper is to prove the stability of orthogonal Pexiderized quadratic functional equation in the spirit of Hyers–Ulam in modular spaces. The theory of modulars on linear spaces and the corresponding theory of modular linear spaces were founded by Nakano [19] and were intensively developed by his mathematical school: Amemiya, Koshi, Shimogaki, Yamamuro [10, 29] and others. Further and the most complete development of these theories are due to Orlicz, Mazur, Musielak, Luxemburg, Turpin [18, 12, 26, 17] and

their collaborators. In the present time the theory of modulars and modular spaces is extensively applied, in particular, in the study of various Orlicz spaces [20] and interpolation theory [11], which in their turn have broad applications [17, 13]. The importance for applications consists in the richness of the structure of modular spaces, that–besides being Banach spaces (or F–spaces in more general setting)– are equipped with modular equivalent of norm or metric notions.

Definition 1.1. Let $\mathcal{X}$ be an arbitrary vector space.
(a) A functional $\rho : \mathcal{X} \to [0, \infty]$ is called a modular if for arbitrary $x, y \in \mathcal{X}$,
(i) $\rho(x) = 0$ if and only if $x = 0$,
(ii) $\rho(\alpha x) = \rho(x)$ for every scaler α with $|\alpha| = 1$,
(iii) $\rho(\alpha x + \beta y) \leq \rho(x) + \rho(y)$ if and only if $\alpha + \beta = 1$ and $\alpha, \beta \geq 0$,
(b) if (iii) is replaced by
(iii)$'$ $\rho(\alpha x + \beta y) \leq \alpha\rho(x) + \beta\rho(y)$ if and only if $\alpha + \beta = 1$ and $\alpha, \beta \geq 0$,
then we say that ρ is a convex modular.

A modular ρ defines a corresponding modular space, i.e., the vector space $\mathcal{X}_\rho$ given by

$$\mathcal{X}_\rho = \{x \in \mathcal{X} : \quad \rho(\lambda x) \to 0 \quad \text{as } \lambda \to 0\}.$$

Let ρ be a convex modular, the modular space $\mathcal{X}_\rho$ can be equipped with a norm called the Luxemburg norm, defined by

$$\|x\|_\rho = \inf\left\{\lambda > 0 : \quad \rho\left(\frac{x}{\lambda}\right) \leq 1\right\}.$$

A function modular is said to satisfy the Δ_2–condition if there exists $\kappa > 0$ such that $\rho(2x) \leq \kappa\rho(x)$ for all $x \in \mathcal{X}_\rho$.

Definition 1.2. Let $\{x_n\}$ and x be in $\mathcal{X}_\rho$. Then
(i) we say that $\{x_n\}$ is ρ–convergent to x and write $x_n \overset{\rho}{\longrightarrow} x$ if and only if $\rho(x_n - x) \to 0$ as $n \to \infty$,
(ii) the sequence $\{x_n\}$, with $x_n \in \mathcal{X}_\rho$, is called ρ–Cauchy if $\rho(x_n - x_m) \to 0$ as $n, m \to \infty$,
(iii) a subset $\mathcal{S}$ of $\mathcal{X}_\rho$ is called ρ–complete if and only if any ρ–Cauchy sequence is ρ–convergent to an element of $\mathcal{S}$.

The modular ρ has the Fatou property if and only if $\rho(x) \leq \liminf_{n\to\infty} \rho(x_n)$ whenever the sequence $\{x_n\}$ is ρ–convergent to x. For further details and proofs, we refer the reader to [17].

Remark 1.3. *If $x \in \mathcal{X}_\rho$ then $\rho(ax)$ is a nondecreasing function of $a \geq 0$. Suppose $0 < a < b$, then property (iii) of Definition 1.1 with $y = 0$ shows that*

$$\rho(ax) = \rho\left(\frac{a}{b}bx\right) \leq \rho(bx).$$

Moreover, if ρ is convex modular on $\mathcal{X}$ and $|\alpha| \leq 1$ then, $\rho(\alpha x) \leq |\alpha|\rho(x)$ and also $\rho(x) \leq \frac{1}{2}\rho(2x)$ for all $x \in \mathcal{X}$.

The stability problem for functional equations first was planed in 1940 by Ulam [27]:

Let G_1 be a group and G_2 be a metric group with the metric $d(\cdot, \cdot)$. Does, for any $\epsilon > 0$, there exists $\delta > 0$ such that, for any mapping $f : G_1 \to G_2$ which satisfies $d(f(xy), f(x)f(y)) \leqslant \delta$ for all $x, y \in G_1$, there exists a homomorphism $h : G_1 \to G_2$ so that, for any $x \in G_1$, we have $d(f(x), h(x)) \leqslant \epsilon$?

In 1941, Hyers [7] answered to the Ulam's question when G_1 and G_2 are Banach spaces. Subsequently, the result of Hyers was generalized by Aoki [2] for additive mappings and Rassias [22] for linear mappings by considering an unbounded Cauchy difference. The paper of Rassias [22] has provided a lot of influences in the development of the Hyers-Ulam-Rassias stability of functional equations (see [15]). During the last decades several stability problems of functional equations have been investigated by a number of mathematicians in various spaces, such as fuzzy normed spaces, non–Archimedean normed spaces and random normed spaces; see [4, 8, 9, 14, 3, 21, 30] and reference therein. Recently, the author present a fixed point method to prove generalized Hyers–Ulam stability of the generalized Jensen functional equation $f(rx + sy) = rg(x) + sh(x)$ in modular spaces [24].

There are several orthogonality notions on a real normed spaces as Birkhoff–James, semi–inner product, Carlsson, Singer, Roberts, Pythagorean, isosceles and Diminnie (see, e.g., [1]). Let us recall the orthogonality space in the sense of Rätz; cf. [23].

Suppose $\mathcal{L}$ is a real vector space with dim $\mathcal{L} \geq 2$ and $\perp$ is a binary relation on $\mathcal{L}$ with the following properties:
(i) totality of $\perp$ for zero: $x \perp 0$, $0 \perp x$ for all $\mathcal{L}$;
(ii) independence: if $x, y \in \mathcal{L} - \{0\}, x \perp y$, then, x, y are linearly independent;
(iii) homogeneity: if $x, y \in \mathcal{L}, x \perp y$, then $\alpha x \perp \beta y$ for all $\alpha, \beta \in \mathbb{R}$;
(iv) the Thalesian property: if $\mathcal{P}$ is a 2–dimensional subspace of $\mathcal{L}, x \in \mathcal{P}$ and $\lambda \in \mathbb{R}_+$, then there exists $y_0 \in \mathcal{P}$ such that $x \perp y_0$ and $x + y_0 \perp \lambda x - y_0$.

The pair $(\mathcal{L}, \perp)$ is called an orthogonality space. By an orthogonality normed space, we mean an orthogonality space having a normed structure. Some interesting examples of orthogonality spaces are:

- The trivial orthogonality on a vector space $\mathcal{L}$ defined by (i), and for nonzero elements $x, y \in \mathcal{L}, x \perp y$ iff x, y are linearly independent.

- The ordinary orthogonality on an inner product space $(\mathcal{L}, \langle . \rangle)$ given by $x \perp y$ iff $\langle x, y \rangle = 0$.

- The Birkhoff–James orthogonality on a normed space $(\mathcal{L}, \|.\|)$ defined by $x \perp y$ iff $\|x\| \le \|x + \lambda y\|$ for all $\lambda \in \mathbb{R}$.

Let $\mathcal{L}$ be an orthogonality space and $(\mathcal{X}, +)$ be an Abelian group. A mapping $f : \mathcal{L} \to \mathcal{X}$ is said to be (orthogonally) quadratic if it satisfies

$$f(x + y) + f(x - y) = 2f(x) + 2f(y) \tag{1.1}$$

for all $x, y \in \mathcal{L}$ (with $x \perp y$). The orthogonally quadratic functional equation (1.1), was first investigated by Vajzović [28] when $\mathcal{L}$ is a Hilbert space, $\mathcal{X}$ is equal to $\mathbb{C}$, f is continuous and $\perp$ means the Hilbert space orthogonality. Later Drlijević, Fochi and Szabó generalized this result [5, 6, 25].

One of the significant conditional equation is the so–called orthogonally quadratic functional equation of Pexideized type,

$$f(x + y) + f(x - y) = 2g(x) + 2h(y) \quad (x \perp y). \tag{1.2}$$

Moslehian in [16] obtained the Hyers–Ulam stability of this Pexiderized equation.

In the present paper, we establish the stability of orthogonal Pexiderized quadratic functional equation (1.2) in the spirit of Hyers–Ulam in modular spaces. Therefore, we generalized the main theorem of [16].

2. Orthogonal stability of Pexiderized quadratic functional equation

The author prove the generalized Hyers–Ulam–Rassias stability of a generalized Jensen functional equation in modular spaces [24]. Applying some ideas

from [16], we deal with the conditional stability problem for equation (1.2) in modular spaces.

Throughout this paper, we assume that the convex modular ρ has the Fatou property such that satisfies the Δ_2–condition with $0 < \kappa \leq 2$. In addition, we assume that $(\mathcal{L}, \perp)$ denotes an orthogonality space.

Theorem 2.1. *Suppose $\perp$ is symmetric on $\mathcal{L}$ and $\mathcal{X}_\rho$ is ρ–complete modular space. Let $f, g, h : \mathcal{L} \to \mathcal{X}_\rho$ be mappings fulfilling*

$$\rho\left(f(x+y) + f(x-y) - 2g(x) - 2h(y)\right) \leq \varepsilon \tag{2.1}$$

for all $x, y \in \mathcal{L}$ with $x \perp y$, $\varepsilon > 0$ and $f(0) = g(0) = h(0) = 0$. Assume that f is odd. Then there exist unique additive mapping $T : \mathcal{L} \to \mathcal{X}_\rho$ and unique quadratic mapping $Q : \mathcal{L} \to \mathcal{X}_\rho$ such that

$$\rho(f(x) - T(x) - Q(x)) \leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon$$

$$\rho(g(x) - T(x) - Q(x)) \leq \frac{\kappa}{4}\left[1 + \kappa^2(1 + \kappa)\right]\varepsilon$$

for all $x \in \mathcal{L}$.

Proof. Put $x = 0$ in equation (2.1). We can do this because of (i). Then

$$\rho(h(y)) \leq \frac{1}{2}\rho\left(2h(y)\right) \leq \frac{\varepsilon}{2} \tag{2.2}$$

for all $y \in \mathcal{L}$. Similarly, by putting $y = 0$ in equation (2.1) we obtain

$$\rho(f(x) - g(x)) \leq \frac{1}{2}\rho\left(2f(x) - 2g(x)\right) \leq \frac{\varepsilon}{2} \tag{2.3}$$

for all $x \in \mathcal{L}$. Hence

$$\begin{aligned}
\rho(f(x+y) + f(x-y) - 2f(x)) &\leq \frac{1}{2}\rho(2[f(x+y) + f(x-y) - 2g(x) - 2h(y)]) \\
&\quad + \frac{1}{2}\rho(2[2f(x) - 2g(x) + 2h(y)]) \\
&\leq \frac{\kappa}{2}\varepsilon + \frac{\kappa^2}{2}\left[\frac{1}{2}\rho(2f(x) - 2g(x)) + \frac{1}{2}\rho(2h(y))\right] \\
&\leq \left(\frac{\kappa}{2} + \frac{\kappa^2}{2}\right)\varepsilon
\end{aligned} \tag{2.4}$$

for all $x, y \in \mathcal{L}$ with $x \perp y$. Fix $x \in \mathcal{L}$. By (iv), there exists $y_0 \in \mathcal{L}$ such that $x \perp y_0$ and $x + y_0 \perp x - y_0$. Since $\perp$ is symmetric $x - y_0 \perp x + y_0$, too. Using inequality (2.4) and the oddness of f we obtain

$$\rho(f(x + y_0) + f(x - y_0) - 2f(x)) \leq \left(\frac{\kappa}{2} + \frac{\kappa^2}{2} \right) \varepsilon$$

$$\rho(f(2x) + f(2y_0) - 2f(x + y_0)) \leq \left(\frac{\kappa}{2} + \frac{\kappa^2}{2} \right) \varepsilon$$

$$\rho(f(2x) - f(2y_0) - 2f(x - y_0)) \leq \left(\frac{\kappa}{2} + \frac{\kappa^2}{2} \right) \varepsilon.$$

Hence

$$\begin{aligned}
\rho(f(2x) - 2f(x)) &\leq \frac{1}{2}\rho(2[f(x + y_0) + f(x - y_0) - 2f(x)]) \\
&\quad + \frac{1}{2}\rho[(f(2x) + f(2y_0) - 2f(x + y_0)) \\
&\quad + f(2x) - f(2y_0) - 2f(x - y_0))] \\
&\leq \frac{\kappa}{2} \left(\frac{\kappa}{2} + \frac{\kappa^2}{2} \right) \varepsilon + \frac{\kappa}{4}\rho(f(2x) + f(2y_0) - 2f(x + y_0)) \\
&\quad + \frac{\kappa}{4}\rho(f(2x) - f(2y_0) - 2f(x - y_0)) \\
&\leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon
\end{aligned}$$

for all $x \in \mathcal{L}$ and so

$$\rho\left(\frac{f(2x)}{2} - f(x) \right) \leq \frac{1}{2}\frac{\kappa^2}{2}(1 + \kappa)\varepsilon. \tag{2.5}$$

Continuing in this way, we may have

$$\rho\left(\frac{f(2^2 x)}{2^2} - f(x) \right) \leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon \left(\frac{\kappa}{2^3} + \frac{\kappa^2}{2^3} \right) \leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon \left(\frac{1}{2^2} + \frac{\kappa}{2^2} \right) \tag{2.6}$$

for all $x \in \mathcal{L}$. By using (2.5), (2.6) and the principle of mathematical induction, we can easily see that

$$\rho\left(\frac{f(2^n x)}{2^n} - f(x) \right) \leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon \sum_{i=1}^{n} \frac{\kappa^{n-i}}{2^n} \leq \frac{\kappa^2}{2}(1 + \kappa)\varepsilon \sum_{i=1}^{n} \frac{1}{2^i}$$

for all $x \in \mathcal{L}$, $n \in \mathbb{N}$ and

$$\rho\left(\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m}\right) \leq \frac{\kappa^2}{2}(1+\kappa)\varepsilon \sum_{i=m+1}^{n} \frac{1}{2^i}$$

for all $x \in \mathcal{L}$, $n > m$. Therefore, the sequence $\{\frac{f(2^n x)}{2^n}\}$ is a ρ–Cauchy sequence in the ρ–complete modular space $\mathcal{X}_\rho$. Hence ρ–$\lim_{n \to \infty} \frac{f(2^n x)}{2^n}$ exists and well define the odd mapping $L(x) = \lim_{n \to \infty} \frac{f(2^n x)}{2^n}$ from $\mathcal{L}$ into $\mathcal{X}_\rho$ satisfying

$$\rho(f(x) - L(x)) \leq \frac{\kappa^2}{2}(1+\kappa)\varepsilon \tag{2.7}$$

for all $x \in \mathcal{L}$, since ρ has Fatou property.

For all $x, y \in \mathcal{L}$ wit $x \perp y$, by applying (2.4) and (iii) we get

$$\rho\left(\frac{f(2^n(x+y))}{2^n} + \frac{f(2^n(x-y))}{2^n} - 2\frac{f(2^n x)}{2^n}\right) \leq \frac{1}{2^n}\frac{\kappa}{2}(1+\kappa)\varepsilon. \tag{2.8}$$

If $n \to \infty$ then, we conclude that

$$L(x+y) + L(x-y) - 2L(x) = 0 \tag{2.9}$$

for all $x, y \in \mathcal{L}$ with $x \perp y$. Moreover, $L(0) = 0$. Using [16, Lemma 2.1] we conclude that L is an orthogonal additive mapping. By [23, Corollary 7], L therefore is of form $T+Q$ with T additive and Q quadratic. If there is another quadratic mapping $\widehat{Q}$ and another additive mapping $\widehat{T}$ satisfying the required inequalities and $\widehat{L} = \widehat{T} + \widehat{Q}$, then

$$\rho(L(x) - \widehat{L}(x)) \leq \frac{\kappa}{2}\rho(L(x) - f(x)) + \frac{\kappa}{2}\rho(\widehat{L}(x) - f(x)) \leq \frac{\kappa^3}{2}(1+\kappa)\varepsilon \tag{2.10}$$

for all $x \in \mathcal{L}$. Using the fact that additive mappings are odd and quadratic mappings are even we obtain

$$\rho(T(x) - \widehat{T}(x)) = \rho\left(\frac{1}{2}\left[(T(x) + Q(x) - \widehat{T}(x) - \widehat{Q}(x)) + (T(x) - Q(x) - \widehat{T}(x) + \widehat{Q}(x))\right]\right)$$

$$\leq \frac{1}{2}\rho(T(x) + Q(x) - \widehat{T}(x) - \widehat{Q}(x)) + \frac{1}{2}\rho(T(x) - Q(x) - \widehat{T}(x) + \widehat{Q}(x))$$

$$= \frac{1}{2}\rho(L(x) - \widehat{L}(x)) + \frac{1}{2}\rho(L(-x) - \widehat{L}(-x))$$

$$\leq \frac{\kappa^3}{2}(1+\kappa)\varepsilon$$

for all $x \in \mathcal{L}$. Thus

$$\rho(T(x) - \widehat{T}(x)) = \rho\left(\frac{T(nx) - \widehat{T}(nx)}{n}\right) \leq \frac{1}{n}\rho\left(T(nx) - \widehat{T}(nx)\right) \leq \frac{\kappa^3}{2n}(1+\kappa)\varepsilon$$

for all $x \in \mathcal{L}$. If $n \to \infty$, we get $T = \widehat{T}$. Similarly,

$$\begin{aligned}
\rho(Q(x) - \widehat{Q}(x)) &= \rho\left(\frac{1}{2}\left[(T(x) + Q(x) - \widehat{T}(x) - \widehat{Q}(x)) + (T(x) - Q(x) - \widehat{T}(x) + \widehat{Q}(x))\right]\right) \\
&\leq \frac{1}{2}\rho(T(x) + Q(x) - \widehat{T}(x) - \widehat{Q}(x)) + \frac{1}{2}\rho(T(x) - Q(x) - \widehat{T}(x) + \widehat{Q}(x)) \\
&= \frac{1}{2}\rho(L(x) - \widehat{L}(x)) + \frac{1}{2}\rho(L(-x) - \widehat{L}(-x)) \\
&\leq \frac{\kappa^3}{2}(1+\kappa)\varepsilon
\end{aligned}$$

for all $x \in \mathcal{L}$. Hence

$$\rho(Q(x) - \widehat{Q}(x)) = \rho\left(\frac{Q(nx) - \widehat{Q}(nx)}{n^2}\right) \leq \frac{1}{n^2}\rho\left(Q(nx) - \widehat{Q}(nx)\right) \leq \frac{\kappa^3}{2n}(1+\kappa)\varepsilon$$

for all $x \in \mathcal{L}$, $n \in \mathbb{N}$. If $n \to \infty$ the latter inequality implies that $Q = \widehat{Q}$. Using inequalities (2.3) and (2.7) we obtain

$$\rho(g(x) - L(x)) \leq \frac{\kappa}{2}\rho(g(x) - f(x)) + \frac{\kappa}{2}\rho(f(x) - L(x)) \leq \frac{\kappa}{4}\left[1 + \kappa^2(1+\kappa)\right]\varepsilon$$

for all $x \in \mathcal{L}$. $\square$

Corollary 2.2. *[16] Suppose $\perp$ is symmetric on $\mathcal{L}$ and $\mathcal{X}$ is Banach space. Let $f, g, h : \mathcal{E} \to \mathcal{X}$ be mappings fulfilling*

$$\|f(x+y) + f(x-y) - 2g(x) + 2h(y)\| \leq \varepsilon \tag{2.11}$$

for all $x, y \in \mathcal{L}$ with $x \perp y$, $\varepsilon > 0$ and $f(0) = g(0) = h(0) = 0$. Assume that f is odd. Then there exist unique additive mapping $L : \mathcal{L} \to \mathcal{X}$ and unique quadratic mapping $Q : \mathcal{L} \to \mathcal{X}$ such that

$$\|(f(x) - L(x) - Q(x)\| \leq 6\varepsilon$$

$$\|(g(x) - L(x) - Q(x)\| \leq \frac{13}{2}\varepsilon$$

for all $x \in \mathcal{L}$.

Proof. It is well known that every normed space is a modular space with the modular $\rho(x) = \|x\|$ and $\kappa = 2$. $\qquad\square$

A convex function φ defined on the interval $[0, \infty)$, nondecreasing and continuous for $\alpha \geq 0$ and such that $\varphi(0) = 0, \varphi(\alpha) > 0$ for $\alpha > 0$, $\varphi(\alpha) \to \infty$ as $\alpha \to \infty$, is called an Orlicz function. The Orlicz function φ satisfies the Δ_2–condition if there exists $\kappa > 0$ such that $\varphi(2\alpha) \leq \kappa\varphi(\alpha)$ for all $\alpha > 0$. Let (Ω, Σ, μ) be a measure space. Let us consider the space $L^0(\mu)$ consisting of all measurable real–valued (or complex–valued) functions on Ω. Define for every $f \in L^0(\mu)$ the Orlicz modular $\rho_\varphi(f)$ by the formula

$$\rho_\varphi(f) = \int_\Omega \varphi(|f|)d\mu.$$

The associated modular function space with respect to this modular is called an Orlicz space, and will be denoted by $L^\varphi(\Omega, \mu)$ or briefly L^φ . In other words,

$$L^\varphi = \{f \in L^0(\mu) \quad | \quad \rho_\varphi(\lambda f) \to 0 \text{ as } \lambda \to 0\}$$

or equivalently as

$$L^\varphi = \{f \in L^0(\mu) \quad | \quad \rho_\varphi(\lambda f) < \infty \text{ for some } \lambda > 0\}.$$

It is known that the Orlicz space L^φ is ρ_φ–complete. Moreover, $(L^\varphi, \|.\|_{\rho_\varphi})$ is a Banach space, where the Luxemburg norm $\|.\|_{\rho_\varphi}$ is defined as follows

$$\|f\|_{\rho_\varphi} = \inf\left\{\lambda > 0 \quad : \quad \int_\Omega \varphi\left(\frac{|f|}{\lambda}\right)d\mu \leq 1\right\}.$$

Moreover, if ℓ is the space of sequences $x = \{x_i\}_{i=1}^\infty$ with real or complex terms x_i, $\varphi = \{\varphi_i\}_{i=1}^\infty$, φ_i are Orlicz functions and $\varrho_\varphi(x) = \Sigma_{i=1}^\infty \varphi_i(|x_i|)$, we shall write ℓ^φ in place of L^φ. The space ℓ^φ is called the generalized Orlicz sequence space. The motivation for the study of modular spaces (and Orlicz spaces) and many examples are detailed in [19, 17, 20, 13]. The following examples show that our results in this paper is different from some results of [16].

Example 2.3. *Suppose $\perp$ is symmetric on $\mathcal{L}$ and φ is an Orlicz function and satisfy the Δ_2–condition with $0 < \kappa \leq 2$. Let $f, g, h : \mathcal{E} \to L^\varphi$ be mappings with $f(0) = g(0) = h(0) = 0$ satisfying*

$$\int_\Omega \varphi(|f(x + y) + f(x - y) - 2g(x) - 2h(y)|)d\mu \leq \varepsilon \qquad (2.12)$$

for all $x, y \in \mathcal{L}$, with $x \perp y$ and $\varepsilon > 0$. Assume that f is odd. Then there exists a unique additive mapping $T : \mathcal{L} \to L^\varphi$ and unique quadratic mapping $Q : \mathcal{L} \to L^\varphi$ such that

$$\int_\Omega \varphi(|f(x) - T(x) - Q(x)|)d\mu \leq \frac{\kappa^2}{2}(1 + \kappa)$$

$$\int_\Omega \varphi(|g(x) - T(x) - Q(x)|)d\mu \leq \frac{\kappa}{4}[1 + \kappa^2(1 + \kappa)]$$

for all $x \in \mathcal{L}$.

Example 2.4. *Suppose $\perp$ is symmetric on $\mathcal{L}$. Let $\hat{\varphi} = \{\varphi_i\}$ be a sequence of Orlicz functions and satisfy the Δ_2–condition with $0 < \kappa \leq 2$ and let $(\ell^{\hat{\varphi}}, \varrho_{\hat{\varphi}})$ be a generalized Orlicz sequence space associated to $\hat{\varphi} = \{\varphi_i\}$. Let $f, g, h : \mathcal{L} \to \ell^{\hat{\varphi}}$ be mappings with $f(0) = g(0) = h(0) = 0$ satisfying*

$$\varrho_{\hat{\varphi}}(f(x + y) + f(x - y) - 2g(x) - 2h(y)) \leq \varepsilon$$

for all $x, y \in \mathcal{L}$, with $x \perp y$ and $\varepsilon > 0$. Assume that f is odd. Then there exists a unique additive mapping $T : \mathcal{L} \to L^\varphi$ and unique quadratic mapping $Q : \mathcal{L} \to L^\varphi$ such that

$$\varrho_{\hat{\varphi}}(f(x) - T(x) - Q(x))d\mu \leq \frac{\kappa^2}{2}(1 + \kappa)$$

$$\varrho_{\hat{\varphi}}(g(x) - T(x) - Q(x))d\mu \leq \frac{\kappa}{4}[1 + \kappa^2(1 + \kappa)]$$

for all $x \in \mathcal{L}$.

Problem 2.5. *Suppose $f, g, h : \mathcal{L} \to \mathcal{X}_\rho$ are mappings fulfilling*

$$\rho\left(f(x + y) + f(x - y) - 2g(x) - 2h(y)\right) \leq \varepsilon$$

for some ε and all $x, y \in \mathcal{L}$ with $x \perp y$. Assume that f is even. Does there exists an orthogonally quadratic mapping $Q : \mathcal{L} \to \mathcal{X}_\rho$, under certain conditions, such that

$$\rho\left(f(x) - Q(x)\right) \leq \alpha\varepsilon$$
$$\rho\left(g(x) - Q(x)\right) \leq \beta\varepsilon$$
$$\rho\left(h(x) - Q(x)\right) \leq \gamma\varepsilon$$

for some scalars α, β, γ and for all $x \in \mathcal{L}$.

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