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A sufficient condition for the existence of 2-repeated low-density burst error correcting code *

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Abstract

This paper presents upper bound on the number of parity-check digits required for linear codes that correct 2-repeated low-density burst errors of length b(fixed) with weight w or less ($w \leq b$). Further, an illustration for the existence of a linear code that corrects 2-repeated burst errors of length 3(fixed) with weight 2 or less over GF(2) has also been provided.

Keywords and Phrases: *Error correcting code, Burst error, Low-density burst error, Repeated low-density burst error, CT burst.*

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1. Introduction

In the theory of error correcting codes, burst errors have played a dominant role amongst the several kinds of errors (refer Abramson (1959), Fire (1959)). A burst of length b is defined as follows:

Definition 1. A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and the last of which are non-zero.

It was observed by Chien and Tang (1965) that in many channels errors occur in the form of a burst but do not occur towards the end digits of the burst. They modified the definition as follows:

Definition 2. A burst of length b is a vector whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero.

There are cetain channels viz. studied by Alexander, Gryb and Nast (1960) which deals with such bursts commonly known as CT bursts.

A further modification to this definition was made by Dass (1980) which is useful for channels not producing errors near the end of a code word and is as follows:

Definition 3. A burst of length b(fixed) is an n-tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions in an n-tuple is the first $n-b+1$ positions.

In situations like lightning or other similar disturbances which introduce burst errors usually operate in a way that, over a given length, some digits are received correctly while others get corrupted i.e. errors occur in the form of low-density bursts. The study of low-density bursts was initiated by Wyner (1963). Further study on low-density burst error correcting codes has been made by Sharma and Dass (1974), Dass (1975), Dass (1983) and others.

As has been observed that in very busy communication channels errors repeat themselves, Dass and Garg (2009) studied 2-repeated burst errors of length b(fixed) and this concept was generalized for m-repeated bursts of length b(fixed) by Dass, Garg and Zannetti (2008a). Further results have also been obtained by Dass, Garg and Zannetti (2008b).

Different situations demand development of codes which correct those errors that are repeated burst errors of specified length, i.e., repeated low-density burst errors of length b(fixed) with weight w or less. Such 2-repeated low-density burst errors and also the general case of m-repeated bursts have been studied by Dass and Garg (2011). An m-repeated low-density burst of length b(fixed) with weight w or less ($w \leq b$) has been defined as follows:

Definition 4. An m-repeated low-density burst of length b(fixed) with weight w or less is an n-tuple whose only non-zero components are confined to m distinct sets of b consecutive positions, the first component of each set is non-zero where each set can have at the most w non-zero components ($w \leq b$), and the number of its starting positions in an n-tuple is among the first $n-mb+1$ positions.

In particular, 2-repeated low-density burst error of length b(fixed) with weight w or less becomes as follows:

Definition 5. A 2-repeated low-density burst of length b(fixed) with weight w or less is an n-tuple whose only non-zero components are confined to two distinct sets of b consecutive positions, the first component of each set is non-zero where each set can have at the most w non-zero components ($w \leq b$), and the number of its starting positions in an n-tuple is among the first $n-2b+1$ positions.

It may be noted that according to Definition 5, when the first low-density burst of length b(fixed) with weight w or less starts from the first position of the vector then the second low-density burst of length b(fixed) with weight w or less is in the last $n-b$ components. When the first low-density burst of length b(fixed) with weight w or less starts from the second position of the vector then the second low-density burst of length b(fixed) with weight w or less will be in the last $n-b-1$ components. In general, when the first low-density burst of length b(fixed) with weight w or less starts from the i-th position, then the second low-density burst of length b(fixed) with weight w or less will be in the remaining last $(n-b-i+1)$ components where i can take the values from 1 to $n-2b+1$ since the starting positions are among the first $n-2b+1$ components. Further, in the last $2b-1$ components only single low-density burst of length b(fixed) with weight w or less can exist, with the starting positions to be atmost upto $n-b+1$.

Lower bound for the correction of m-repeated low-density bursts of length b(fixed) with weight w or less ($w \leq b$) has been obtained by Dass and Garg (2012). In the same paper, an upper bound for codes which can detect such errors has also been obtained.

This paper has been organized as follows:

In section 2, an upper bound for the existence of linear codes that can correct any 2-repeated low-density burst errors of length b(fixed) with weight w or less is given. The paper concludes with an illustration of such a code.

In what follows a linear code will be considered as a subspace of the space of all n-tuples over $GF(q)$. The distance between two vectors shall be considered in the Hamming sense.

2. Bound for codes correcting 2-repeated low-density burst errors

In this section, we derive an upper bound on the number of parity-check digits that assures the existence of a code capable of correcting 2-repeated low-density burst errors of length b(fixed) with weight w or less. The proof involves a technique given by Dass (1983) which is a suitable modification of the technique used by Sacks (1958) in establishing the well-known Varsharmov-Gilbert bound. Before deriving the main result, we state below a result obtained by Dass and Garg (2012, Theorem 1) to be used.

Result. An (n, k) linear code over GF(q) that corrects m-repeated low-density bursts of length b(fixed) with weight w or less ($w \leq b$) must satisfy:

$$q^{n-k} \geq \sum_{i=0}^{m} \binom{n-ib+i}{i} (q-1)^i [1+(q-1)]^{i(b-1,w-1)},$$

where $[1+x]^{(m,r)} = 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \ldots + \binom{m}{r}x^r$.

We now derive the following theorem:

Theorem 1. *Given positive integers w and b such that $w \leq b$, there exists an (n,k) linear code that corrects all 2-repeated low-density burst errors of length b(fixed) $(n > 4b)$ with weight w or less provided that*

$$
\begin{aligned}
q^{n-k} > & ([1+(q-1)]^{(b-1,w-1)}) \\
& \times\Bigg\{\sum_{i=0}^{3}\binom{n-(i+1)b+i}{i}(q-1)^{i}[1+(q-1)]^{i(b-1,w-1)} \\
& + \sum_{i=1}^{n-4b+1}\Bigg\{\sum_{k_2=1}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6) \\
& \times\{(n-4b+k_2-i+2)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\Bigg\} \\
& + \sum_{\substack{i=n-4b+1+k_3\\1\leq k_3\leq b-1}}\Bigg\{\sum_{k_2=k_3}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6) \\
& \times\{(n-4b+k_2-i+2)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\Bigg\} \\
& + \sum_{i=1}^{n-4b+1}\Bigg\{\Big((q-1)[1+(q-1)]^{(b-1,w-1)}\Big) \\
& \times\Bigg\{(n-4b-i+2)\sum_{k_2=1}^{b-1}L_1(b,k_2,r_4,r_5,r_6) \\
& + \sum_{\substack{k_2=k_3\\1\leq k_3\leq b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\Bigg\}\Bigg\} \\
& + \Bigg\{\big((q-1)[1+(q-1)]^{(b-1,w-1)}\big)\sum_{\substack{k_2=k_4\\1\leq k_4\leq b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\Bigg\} \\
& + (n-3b+1)\sum_{k_2=1}^{b-1}L_1(b,k_2,r_4,r_5,r_6)+\sum_{\substack{k_2=k_4\\1\leq k_4\leq b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6) \\
& + (n-4b+3)\sum_{\substack{k_5,k_6\\1\leq k_6\leq b-1\\1\leq k_5\leq b-k_6}}L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11}) \\
& + \sum_{\substack{i_1,k_5,k_6\\1\leq i_1\leq b-2\\i_1+1\leq k_6\leq b-1\\1\leq k_5\leq b-k_6}}L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{\substack{i_1,k_5,k_6\\ 1\le i_1\le b-2\\ 1\le k_6\le b-1\\ i_1+1\le k_5\le b-k_6}} L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})\Big\}\\
&+\sum_{k_1=1}^{b-1}\Big\{L(b,k_1,r_1,r_2,r_3)\Big\{\sum_{i=0}^{2}\binom{n-(2+i)b+(k_1+i)}{i}\\
&\times(q-1)^i[1+(q-1)]^{i(b-1,w-1)}+(n-4b+k_1+1)\\
&\times\sum_{k_2=1}^{b-1}L_1(b,k_2,r_4,r_5,r_6)+\sum_{\substack{k_2=k_3\\ 1\le k_3\le b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\Big\}\Big\}\\
&+\sum_{\substack{k_5,k_6\\ 1\le k_6\le b-1\\ 1\le k_5\le b-k_6}} L_2(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})\\
&\times\big\{1+(n-4b+k_5+k_6+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\big\}\\
&+\sum_{\substack{k_8,k_9,k_{10}\\ 1\le k_{10}\le b-1\\ 1\le k_9\le b-k_{10}\\ 1\le k_8\le b-k_9}} L_4(b,k_8,k_9,k_{10},r_{12},r_{13},r_{14},r_{15},r_{16},r_{17},r_{18}). \qquad (1)
\end{aligned}
$$

where

$$[1+x]^{(m,r)} = 1+\binom{m}{1}x+\binom{m}{2}x^2+\ldots+\binom{m}{r}x^r,$$

$$
\begin{aligned}
&L(b,k_1,r_1,r_2,r_3)\\
&\quad=\sum_{r_1,r_2,r_3}\binom{b-k_1}{r_1}\binom{k_1-1}{r_2}\binom{b-k_1-1}{r_3}(q-1)^{r_1+r_2+r_3+1},\\
&L_1(b,k_2,r_4,r_5,r_6)\\
&\quad=\sum_{r_4,r_5,r_6}\binom{b-k_2}{r_4}\binom{k_2-1}{r_5}\binom{b-k_2-1}{r_6}(q-1)^{r_4+r_5+r_6+2},\\
&L_2(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})\\
&\quad=\sum_{\substack{r_7,r_8,r_9,\\ r_{10},r_{11}}}\binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}}\\
&\quad\times\binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}+2},
\end{aligned}
$$

$$L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11}) = \sum_{\substack{r_7,r_8,r_9,\\ r_{10},r_{11}}} \binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}} \times \binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}+3},$$

$$L_4(b,k_8,k_9,k_{10},r_{12},r_{13},r_{14},r_{15},r_{16},r_{17},r_{18}) = \sum_{r_{12},\dots,r_{18}} \binom{b-k_8}{r_{12}}\binom{k_8-1}{r_{13}}\binom{b-k_8-k_9}{r_{14}}\binom{k_9-1}{r_{15}}\binom{b-k_9-k_{10}}{r_{16}} \times \binom{k_{10}-1}{r_{17}}\binom{b-k_{10}-1}{r_{18}}(q-1)^{r_{12}+r_{13}+r_{14}+r_{15}+r_{16}+r_{17}+r_{18}+3}$$

with

$$0 \le r_1 \le w-1, 0 \le r_2 \le 2w-2, 0 \le r_3 \le w-1,$$

$$r_2 + r_3 \ge w-1, r_1 + r_2 + r_3 \le 2w-2,$$

$$0 \le r_4 \le w-1, 0 \le r_5 \le 2w-2, 0 \le r_6 \le w-1,$$

$$r_5 + r_6 \ge w-1, r_4 + r_5 + r_6 \le 2w-2,$$

$$0 \le r_7 \le w-1, 0 \le r_8 \le 2w-2, 0 \le r_9 \le w-1, 0 \le r_{10} \le 2w-2,$$

$$0 \le r_{11} \le w-1, r_{10} + r_{11} \ge w-1, r_8 + r_9 + r_{10} + r_{11} \ge 2w-2,$$

$$r_7 + r_8 + r_9 + r_{10} + r_{11} \le 3w-3,$$

$$0 \le r_{12} \le w-1, 0 \le r_{13} \le 2w-2, 0 \le r_{14} \le w-1,$$

$$0 \le r_{15} \le 2w-2, 0 \le r_{16} \le w-1, 0 \le r_{17} \le 2w-2, 0 \le r_{18} \le w-1,$$

$$r_{17} + r_{18} \ge w-1,$$

$$r_{15} + r_{16} + r_{17} + r_{18} \ge 2w-2,$$

$$r_{13} + r_{14} + r_{15} + r_{16} + r_{17} + r_{18} \ge 3w-3,$$

$$r_{12} + r_{13} + r_{14} + r_{15} + r_{16} + r_{17} + r_{18} \le 4w-4\,.$$

Proof. The existence of such a code will be shown by constructing an appropriate $(n-k) \times n$ parity-check matrix H. Firstly, we construct a matrix H'

from which the requisite parity-check matrix H shall be obtained by reversing the order of its columns altogether. Any non-zero $(n-k)$-tuple is chosen as the first column h_1 of H'. Subsequent columns are added to H' such that after having selected the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$, the jth column h_j may be added provided that it is not a linear combination of any $w-1$ or fewer columns from amongst the immediately preceding $b-1$ columns and w or fewer columns from amongst any b consecutive columns from the first $j-b$ columns, together with any two sets of w or fewer columns, each chosen from a distinct set of b consecutive columns from amongst all the $j-1$ columns (note that b consecutive columns here do not include less than b columns).

In other words, h_j may be added provided that

$$\begin{aligned} h_j \neq\ & \alpha_{j_1}h_{j_1} + \alpha_{j_2}h_{j_2} + \ldots + \alpha_{j_{w-1}}h_{j_{w-1}} + \beta_{r_1}h_{r_1} + \beta_{r_2}h_{r_2} + \ldots + \beta_{r_w}h_{r_w} \\ & + \gamma_{\ell_1}h_{\ell_1} + \gamma_{\ell_2}h_{\ell_2} + \ldots + \gamma_{\ell_w}h_{\ell_w} + \delta_{p_1}h_{p_1} + \delta_{p_2}h_{p_2} + \ldots + \delta_{p_w}h_{p_w} \quad (2) \end{aligned}$$

where the $h_{j_1}, h_{j_2}, \ldots, h_{j_{w-1}}$ are any $w-1$ columns among $h_{j-(b-1)}, h_{j-(b-2)}, \ldots, h_{j-1}$ and h_r, h_ℓ, h_p are any w columns each from three sets of b consecutive columns such that one of the sets of b consecutive columns is amongst the first $j-b$ columns, and the other two sets of b consecutive columns are distinct and from amongst all the $j-1$ columns.

It may be noted that either all β_r, γ_ℓ, δ_p are zero or if β_t is the last non-zero coefficient of the first set of b consecutive columns, then

$$b \leq t \leq j-b, \alpha_j, \beta_r, \gamma_\ell, \delta_p \text{ are in GF}(q). \quad (3)$$

Also if γ_{t_1}, is the last non-zero coefficient of the second set of b consecutive columns, then

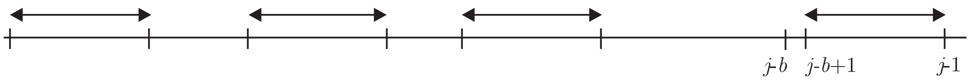
$$b \leq t_1 \leq j-1\,. \quad (4)$$

The restriction $t_1 \leq j-1$ is obviously satisfied since the selection of the columns is out of all the $j-1$ previously chosen columns.

The condition (2) ensures that there would not be a code word which is expressible as sum or difference of two vectors, each of which is a 2-repeated low-density burst of length b(fixed) with weight w or less.

To enumerate all possible linear combinations on the R.H.S of (2), there are as many as 9 different cases to be examined, We analyze these as follows:

Case 1. *When h_j are selected from $h_{j-b+1}, h_{j-b+2}, \ldots, h_{j-1}$ and the h_r, h_ℓ, h_p are selected from three distinct sets of b consecutive columns from amongst the first $j-b$ columns.*



In this case, the number of ways in which coefficients α_j can be selected is

$$[1+(q-1)]^{(b-1,w-1)}. \tag{5}$$

The number of ways in which the β_r, γ_ℓ, δ_p can be selected is equivalent to enumerate the number of 3-repeated low-density bursts of length b(fixed) with weight w or less in a vector of length $j-b$, which is (refer Theorem 1, Dass and Garg (2012))

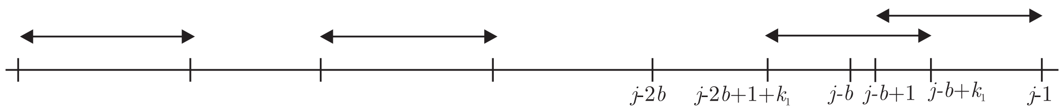
$$\sum_{i=0}^{3} \binom{(j-b)-ib+i}{i} (q-1)^i [1+(q-1)]^{i(b-1,w-1)}. \tag{6}$$

Therefore, the total number of choices of coefficients in this case is

$$[1+(q-1)]^{(b-1,w-1)} \left\{ \sum_{i=0}^{3} \binom{j-(i+1)b+i}{i} (q-1)^i [1+(q-1)]^{i(b-1,w-1)} \right\}. \tag{7}$$

Case 2. *When the h_p are selected from $h_{j-2b+2}, \ldots, h_{j-1}$ such that all the h_p are neither taken from $h_{j-2b+2}, \ldots, h_{j-b}$ nor from $h_{j-b+1}, \ldots, h_{j-1}$ i.e. the last*

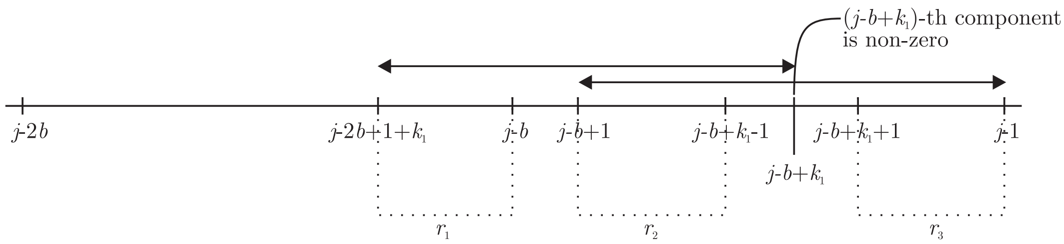
h_p is among $h_{j-b+1}, \ldots, h_{j-1}$. The h_r and h_ℓ are selected from two distinct sets of b consecutive columns among the first $j-2b+k_1$ columns, $1 \leq k_1 \leq b-1$.



In this case, the coefficients δ_p are selected from b consecutive components as w or less non-zero components, which starts from the $(j-2b+1+k_1)$-th component which may obviously continue upto $(j-b+k_1)$-th component. We shall first select $w-1$ or less non-zero components among $(j-2b+1+k_1, \ldots, j-b+k_1-1)$-th positions, the $(j-b+k_1)$-th component is non-zero, together with $w-1$ or less non-zero components among $(j-b+1, \ldots, j-1)$-th positions. Now the aim is to select the coefficients β_r and γ_ℓ which are w or less non-zero components each selected from a distinct set of b consecutive components among the first $j-2b+k_1$ positions, $1 \leq k_1 \leq b-1$.

In order to do so, let us choose r_1 components from the $(j-2b+k_1+1, \ldots, j-b)$-th positions, r_2 components from the $(j-b+1, \ldots, j-b+k_1-1)$-th positions and r_3 components from the $(j-b+k_1+1, \ldots, j-1)$-th positions, where

$$0 \leq r_1 \leq w-1, \quad 0 \leq r_2 \leq 2w-2, \quad 0 \leq r_3 \leq w-1. \tag{8}$$



Keeping in view the situations considered in case 1, r_1, r_2, r_3 should be such that

$$r_2 + r_3 \geq w - 1, r_1 + r_2 + r_3 \leq 2w - 2. \tag{9}$$

Such a selection of coefficients gives us

$$\sum_{r_1,r_2,r_3} \binom{b-k_1}{r_1}\binom{k_1-1}{r_2}\binom{b-k_1-1}{r_3}(q-1)^{r_1+r_2+r_3}$$

possible linear combinations where r_1, r_2, r_3 each satisfy the constraints stated in (8) and (9). Also, the $(j - b + k_1)$-th component can be selected in $(q - 1)$ ways, therefore selection of coefficients give us

$$\sum_{r_1,r_2,r_3} \binom{b-k_1}{r_1}\binom{k_1-1}{r_2}\binom{b-k_1-1}{r_3}(q-1)^{r_1+r_2+r_3+1} \tag{10}$$

choices.

Suppose $L(b, k_1, r_1, r_2, r_3)$ represents the expression in (10) with conditions in (8) and (9). Now to select β_r and γ_ℓ, it is equivalent to enumerate the number of 2-repeated low-density bursts of length b(fixed) with weight w or less in a vector of length $j-2b+k_1$, which gives us (refer Theorem 1 for m =2, Dass and Garg (2012))

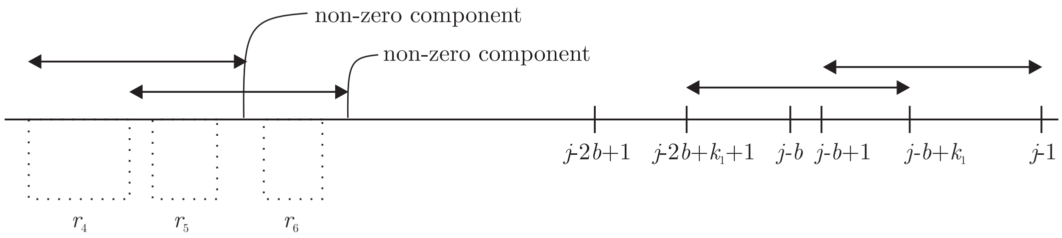
$$\sum_{i=0}^{2} \binom{(j-2b+k_1)-ib+i}{i}(q-1)^i[1+(q-1)]^{i(b-1,w-1)}. \tag{11}$$

Therefore in this case, the total number of choices of coefficients turns out to be

$$\sum_{k_1=1}^{b-1}\{(\text{expr. } (10)) \cdot (\text{expr. } (11))\}. \tag{12}$$

Case 3. *When the h_p are selected from $h_{j-2b+2}, \ldots, h_{j-1}$ such that all the h_p are neither taken from $h_{j-2b+2}, \ldots, h_{j-b}$ nor from $h_{j-b+1}, \ldots, h_{j-1}$, i.e the last*

h_p is among $h_{j-b+1}, \ldots, h_{j-1}$. The h_r and h_ℓ are selected together from $2b-1$ or fewer consecutive columns (but from $b+1$ or more) which are taken from first $j-2b+k_1$ columns, $1 \le k_1 \le b-1$.



In this case, the coefficients δ_p are selected from b consecutive components as w or less non-zero components, which starts from the $(j-2b+1+k_1)$-th component which may obviously continue upto $(j-b+k_1)$-th component. We shall first select $w-1$ or less non-zero components from amongst $(j-2b+1+k_1, \ldots, j-b+k_1-1)$-th positions, the $(j-b+k_1)$-th component is non-zero, together with $w-1$ or less non-zero components amongst $(j-b+1, \ldots, j-1)$-th positions. Such a selection of coefficients δ_p and α_j, following the procedure as in case 2 gives rise to the number of choices of coefficients δ_p and α_j as $L(b, k_1, r_1, r_2, r_3)$ with the constraints stated in (8) and (9). The coefficients β_r and γ_ℓ are selected following the same procedure as in case 2. The last non-zero coefficient of γ_ℓ can be selected in $(q-1)$ ways, thus such a selection of coefficients β_r and γ_ℓ gives us

$$\sum_{k_2=1}^{b-1} \sum_{r_4, r_5, r_6} \binom{b-k_2}{r_4} \binom{k_2-1}{r_5} \binom{b-k_2-1}{r_6} (q-1)^{r_4+r_5+r_6+2} \qquad (13)$$

choices,

$$\text{where } 0 \le r_4 \le w-1, \quad 0 \le r_5 \le 2w-2, \quad 0 \le r_6 \le w-1. \qquad (14)$$

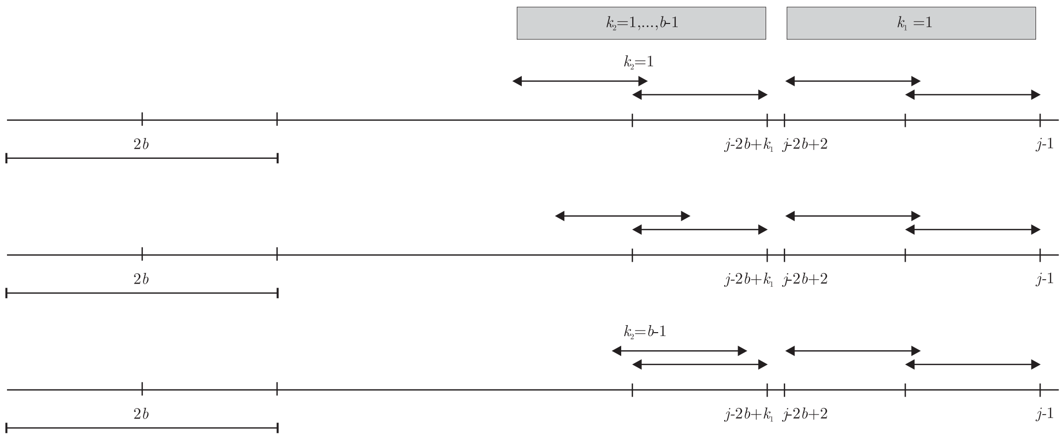
Keeping in view the situations considered in cases 1 and 2, r_4, r_5, r_6 should be such that

$$r_5 + r_6 \geq w - 1, r_4 + r_5 + r_6 \leq 2w - 2. \tag{15}$$

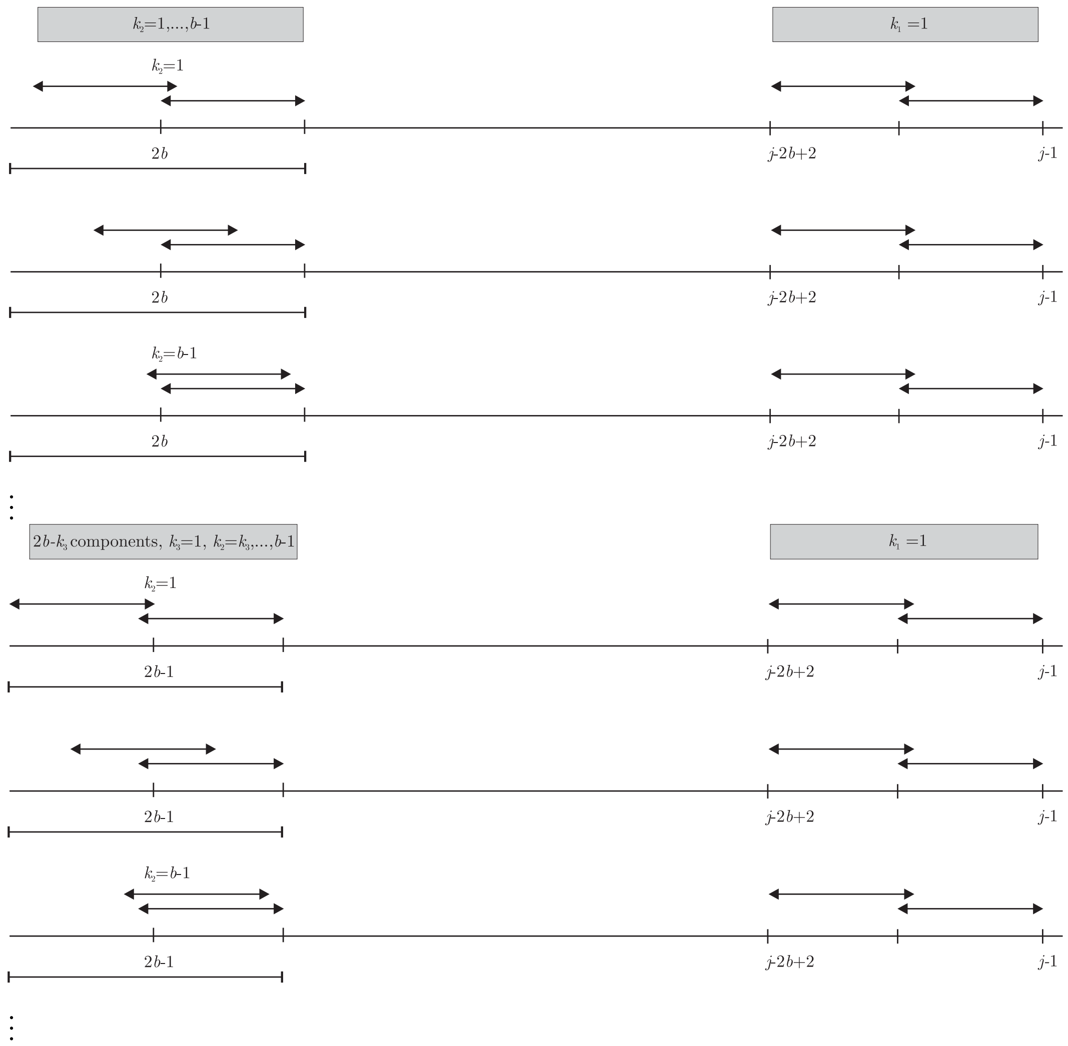
Since the selection of β_r and γ_ℓ is made among the first $(j-2b+k_1)$ components, therefore such a selection of coefficients β_r and γ_ℓ gives us

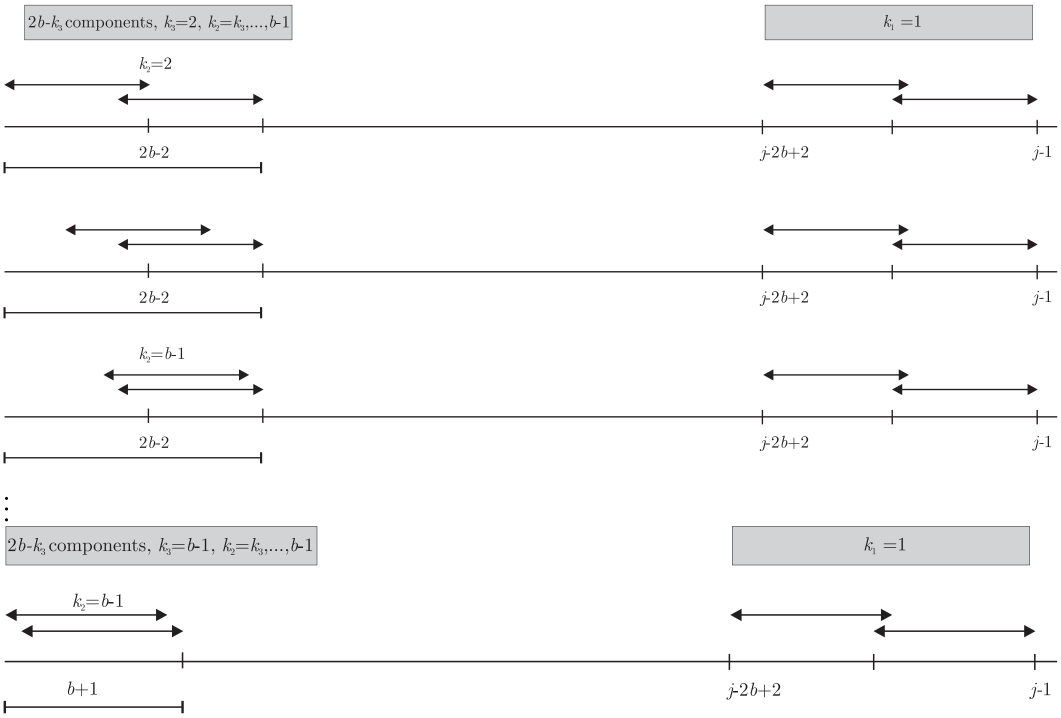
$$\begin{aligned}&\big\{((j-2b+k_1)-2b+1)\cdot(\text{expr. }(13))\big\}\\&\quad+\Bigg\{\sum_{\substack{k_2=k_3\\1\leq k_3\leq b-1}}^{b-1}\sum_{r_4,r_5,r_6}\binom{b-k_2}{r_4}\binom{k_2-1}{r_5}\binom{b-k_2-1}{r_6}(q-1)^{r_4+r_5+r_6+2}\Bigg\},\end{aligned} \tag{16}$$

(the last non-zero coefficient during the selection of coefficients β_r, γ_ℓ has the positions $(j-2b+k_1,\ldots,2b)$-th with $1 \leq k_2 \leq b-1$ and further it has $(2b-k_3)$-th position where $k_3 = 1, 2, \ldots, b-1$ with $k_2 = k_3, \ldots, b-1$).



⋮


k_2=1,...,b-1
k_1 =1
k_2=1
2b
j-2b+2
j-1
k_2=b-1
2b-k_3 components, k_3=1, k_2=k_3,...,b-1
2b-1



Similarly, when $k_1 = 2$, the last non-zero coefficient has the positions $(j - 2b + 2, \ldots, 2b)$ with $k_2 = 1, \ldots, b-1$ and further it has $(2b - k_3)$-th positions where $k_3 = 1, 2, \ldots, b-1$ with $k_2 = k_3, \ldots, b-1$.

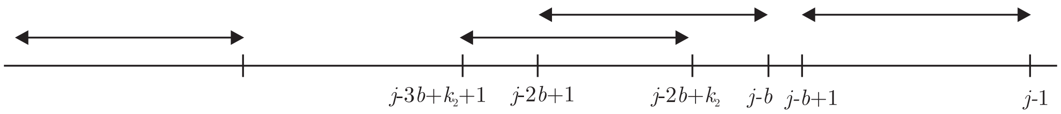
Thus in this case, the total number of choices of coefficients turns out to be

$$\sum_{k_1=1}^{b-1} \left\{ (L(b, k_1, r_1, r_2, r_3)) \cdot (\text{expr. } (16)) \right\}. \tag{17}$$

The expression $\sum_{r_4, r_5, r_6} \binom{b-k_2}{r_4} \binom{k_2-1}{r_5} \binom{b-k_2-1}{r_6} (q-1)^{r_4+r_5+r_6+2}$ with constraints stated in (14) and (15), is denoted with $L_1(b, k_2, r_4, r_5, r_6)$.

Case 4. When the h_r and h_p are selected together from $2b - 1$ or fewer columns (but from $b+1$ or more) from amongst the columns $h_{b+1}, \ldots, h_{j-b}$ i.e. suppose the columns are selected from $h_{j-3b+k_2+1}, \ldots, h_{j-b}$ such that all the h_r

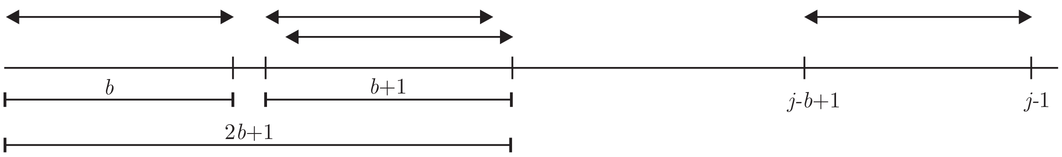
are neither taken from $h_{j-3b+k_2+1}, \ldots, h_{j-2b}$ nor from $h_{j-2b+1}, \ldots, h_{j-b}$, the h_p are selected from $h_{j-2b+1}, \ldots, h_{j-b}$, the h_ℓ are taken from some b consecutive columns amongst the first $j-3b+k_2$ columns, $1 \le k_2 \le b-1$.



In this case, the number of ways in which the coefficients α_j can be selected is

$$[1+(q-1)]^{(b-1,w-1)}. \tag{18}$$

The coefficients β_r and δ_p are selected as in case 3 which gives us $L_1(b, k_2, r_4, r_5, r_6)$. The coefficients γ_ℓ as a single low-density burst of length b(fixed) with weight w or less is selected from amongst the first $j-3b+k_2-i+1$ components, $1 \le k_2 \le b-1$, i represents the positions $(j-b), \ldots, (2b+1)$.

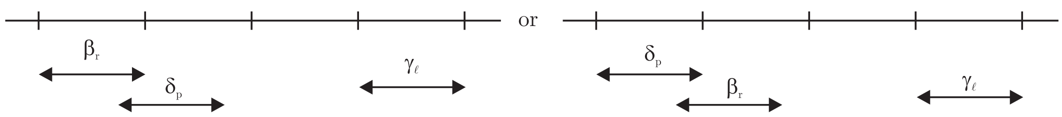


To enumerate the total number of choices of γ_ℓ, β_r and δ_p we prove the following Lemma:

Lemma 1. $L_1(b, k_2, r_4, r_5, r_6)$ *denotes the number of ways for the selection of* β_r *and* δ_p*. The total number of choices of* γ_ℓ, β_r *and* δ_p *with varying starting position of the first non-zero component when* β_r *and* δ_p *are selected together from* $2b-1$ *or fewer components (but from* $b+1$ *or more) along with the selection of* γ_ℓ *which forms a single low-density burst of length* b*(fixed) with weight*

w or less in the remaining components of the vector of length n_1 (considering $(j-b)$-th position as the first position) is

$$\sum_{i=1}^{n_1-3b+1}\left\{\sum_{k_2=1}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6)\right.$$
$$\left.\times\big\{((n_1-2b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\big\}\big\}\right\}$$
$$+\sum_{\substack{i=n_1-3b+1+k_3\\1\le k_3\le b-1}}\left\{\sum_{k_2=k_3}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6)\right.$$
$$\left.\times\big\{((n_1-2b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\big\}\big\}\right\}. \quad (19)$$



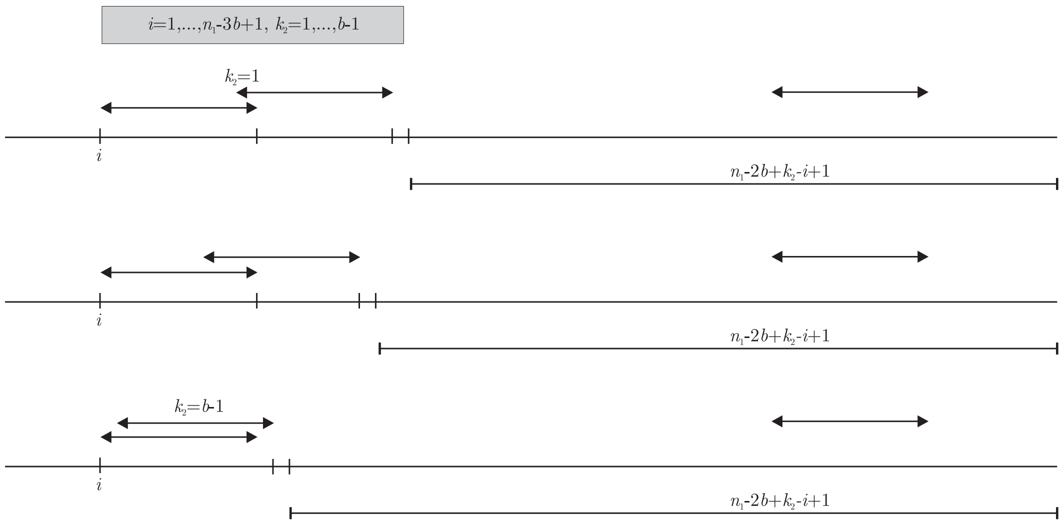
Proof of Lemma 1. Consider a vector of length n_1. When first non-zero component during the selection of β_r and δ_p selected together from $2b-1$ or less components (but from $b+1$ or more) is at the first position then the single low-density burst of length b(fixed) with weight w or less is in the remaining n_1-2b+k_2 components, $1\le k_2\le b-1$. When first non-zero component is at the i-th position then the single low-density burst is in the remaining n_1-2b+k_2-i+1 components, where $1\le i\le n_1-3b+1$. The number for the selection of δ_p and β_r is $L_1(b,k_2,r_4,r_5,r_6)$ as in case 3. The number of single low-density bursts of length b(fixed) with weight w or less in a vector of length (n_1-2b+k_2-i+1) (not including vector of all zeros) is (Dass (1983))

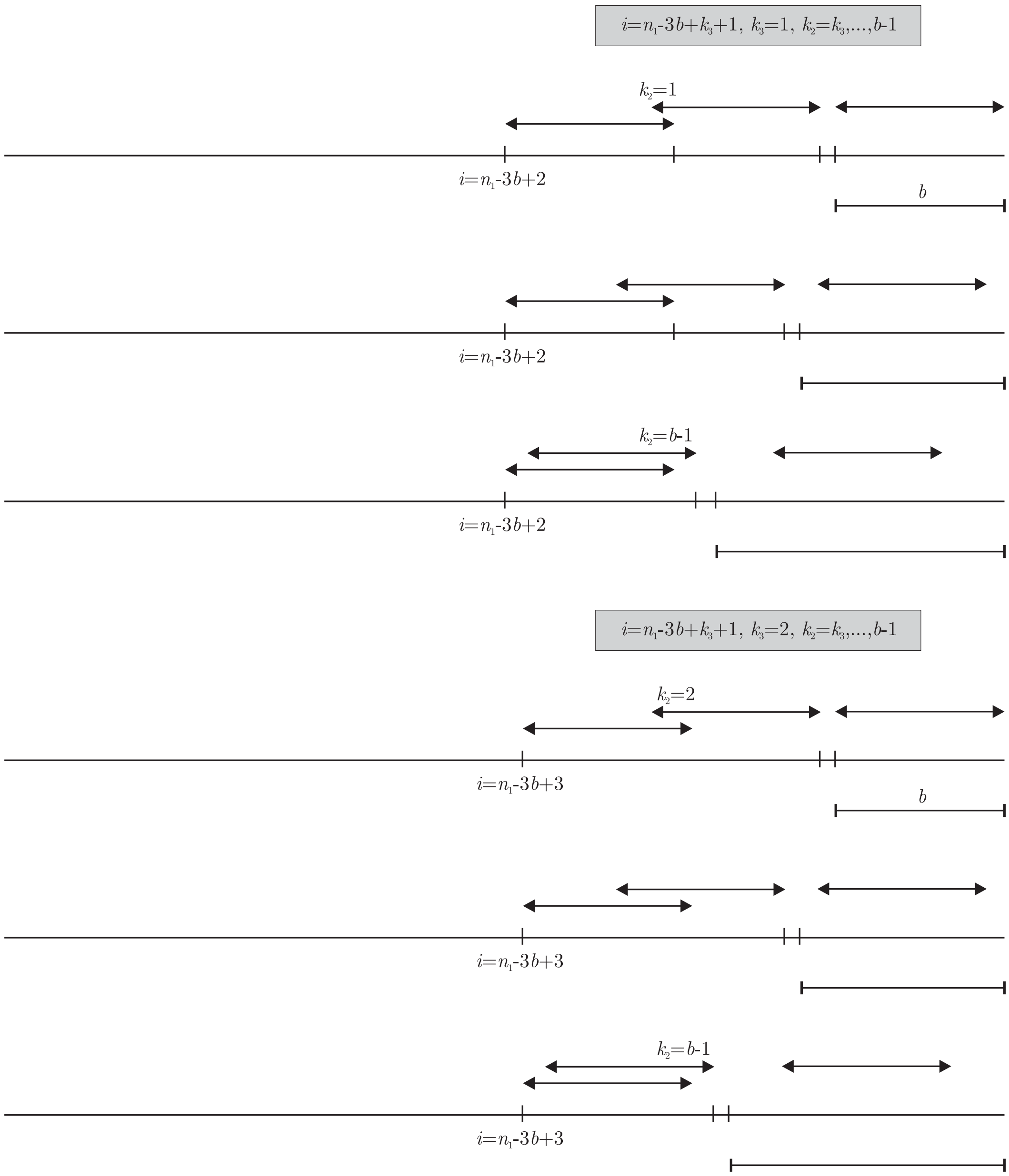
$$((n_1-2b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}.$$

Summing on i, the number of such vectors is

$$\sum_{i=1}^{n_1-3b+1}\left\{\sum_{k_2=1}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6)\right.$$
$$\left.\times\{((n_1-2b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\right\}. \quad (20)$$

Also, when i takes the value n_1-3b+2, k_2 can take values $1,\ldots,b-1$, then the single low-density burst of length b(fixed) with weight w or less is in the remaining n_1-2b+k_2-i+1, i.e., when i takes the value n_1-3b+k_3+1, $1\le k_3\le b-1$, with $k_3\le k_2\le b-1$, then single low-density burst of length b(fixed) with weight w or less is in the remaining n_1-2b+k_2-i+1 components.




i=n_1-3b+k_3+1, k_3=1, k_2=k_3,...,b-1
k_2=1
i=n_1-3b+2
b
i=n_1-3b+2
k_2=b-1
i=n_1-3b+2
i=n_1-3b+k_3+1, k_3=2, k_2=k_3,...,b-1
k_2=2
i=n_1-3b+3
b
i=n_1-3b+3
k_2=b-1
i=n_1-3b+3

⋮

Therefore, further summing on i we get

$$\sum_{\substack{i=n_1-3b+k_3+1\\ 1\leq k_3\leq b-1}} \left\{ \sum_{k_2=k_3}^{b-1} \{L_1(b,k_2,r_4,r_5,r_6)\right.$$

$$\left.\times\{((n_1-2b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\right\}. \quad (21)$$

Thus, the total number of such vectors is

$$\text{(expr. (20))} + \text{(expr. (21))} \quad (22)$$

which completes the proof of Lemma 1.

Now, total number of choices of coefficients β_r, γ_ℓ and δ_p in a vector of length $j-b$ turns out to be (using Lemma 1)

$$\sum_{i=1}^{(j-b)-3b+1} \left\{ \sum_{k_2=1}^{b-1}\{L_1(b,k_2,r_4,r_5,r_6)\right.$$

$$\left.\times\{((j-3b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\right\}$$

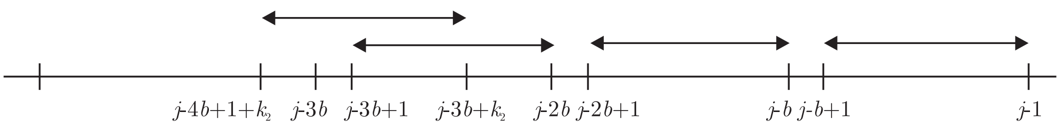
$$+\sum_{\substack{i=(j-b)-3b+1+k_3\\ 1\leq k_3\leq b-1}} \left\{ \sum_{k_2=k_3}^{b-1} \{L_1(b,k_2,r_4,r_5,r_6)\right.$$

$$\left.\times\quad \{((j-3b+k_2-i+1)-b+1)(q-1)[1+(q-1)]^{(b-1,w-1)}\}\}\right\}. \quad (23)$$

Therefore, the number of choices of all the coefficients is

$$\text{(expr. (18))} \cdot \text{(expr. (23))}. \quad (24)$$

Case 5. *When the h_r and h_ℓ are selected together from $2b-1$ or fewer columns (but from $b+1$ or more) from amongst the columns $h_1, \ldots, h_{j-2b}$, i.e, suppose the columns are selected from $h_{j-4b+k_2+1}, \ldots, h_{j-2b}$ such that all h_r are neither taken from $h_{j-4b+k_2+1}, \ldots, h_{j-3b}$ nor from $h_{j-3b+1}, \ldots, h_{j-2b}$, the columns h_ℓ are selected from $h_{j-3b+1}, \ldots, h_{j-2b}$ and the columns h_p are selected from $h_{j-2b+1}, \ldots, h_{j-b}$, $(1 \le k_2 \le b-1)$.*



In this case, the number of ways in which the coefficients α_j can be selected is

$$[1+(q-1)]^{(b-1,w-1)}. \tag{25}$$

The coefficients β_r and γ_ℓ are selected as in case 3 which gives us $L_1(b, k_2, r_4, r_5, r_6)$. To enumerate the total number of choices of the coefficients δ_p along with β_r and γ_ℓ we prove the following Lemma 2.

Lemma 2. *$L_1(b, k_2, r_4, r_5, r_6)$ denotes the number as in case 3 for the selection of β_r and γ_ℓ. The number of vectors with varying starting position of δ_p which forms single low-density burst of length b(fixed) with weight w or less (consider the position $(j-b)$-th as the first position) along with the selection of β_r and γ_ℓ together from $2b-1$ or fewer components (but from $b+1$ or more) in the* *remaining components of the vector of length n_1, is*

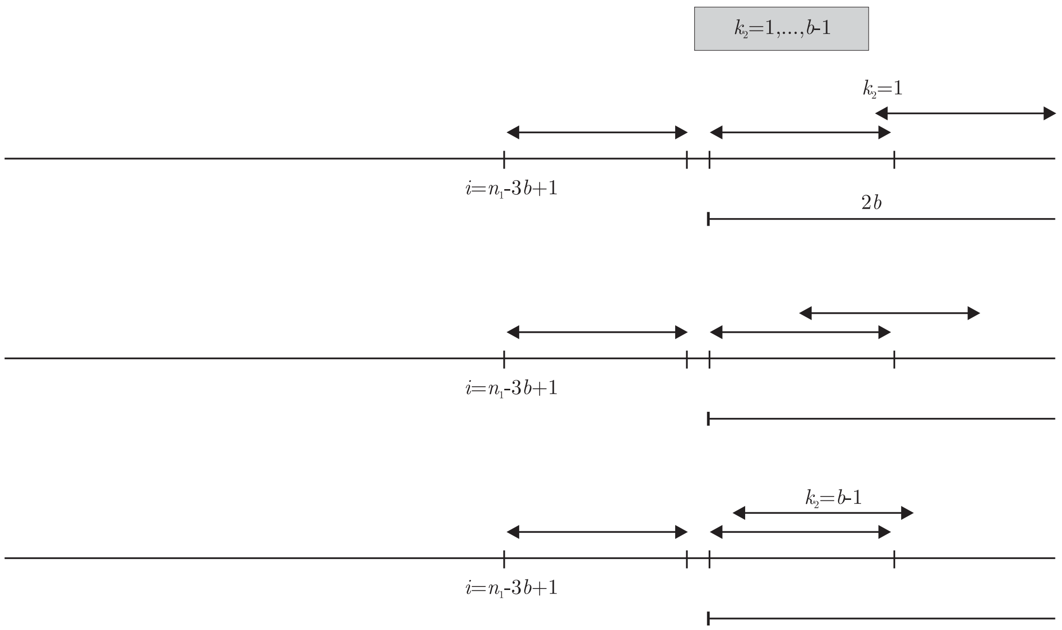
$$\sum_{i=1}^{n_1-3b+1}\Big\{\big((q-1)[1+(q-1)]^{(b-1,w-1)}\big)$$
$$\times\Big\{((n_1-b-i+1)-2b+1)\sum_{k_2=1}^{b-1}L_1(b,k_2,r_4,r_5,r_6)$$
$$+\sum_{\substack{k_2=k_3\\1\le k_3\le b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\Big\}\Big\}$$
$$+\Big\{((q-1)[1+(q-1)]^{(b-1,w-1)})\sum_{\substack{k_2=k_4\\1\le k_4\le b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\Big\}.\quad(26)$$

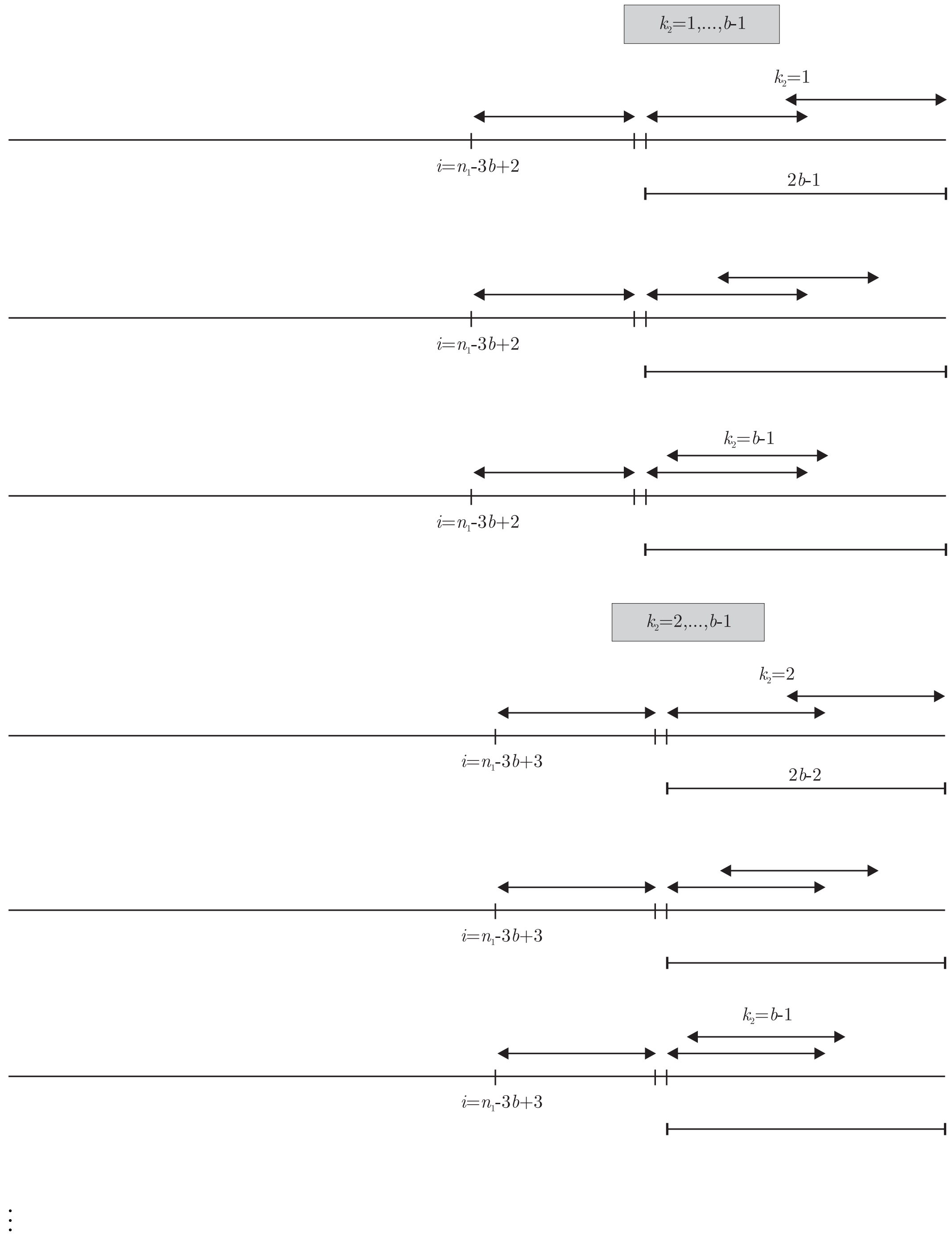
Proof of Lemma 2. Consider a vector of length n_1. When the first non-zero component of δ_p is at the first position then the selection of β_r and γ_ℓ is made (together from $2b-1$ or fewer but from $b+1$ or more components) from the remaining n_1-b components. When the first non-zero component of δ_p is at the i-th position then the selection of β_r and γ_ℓ together is made from the remaining $n_1-b-i+1$ components, $1\le i\le n_1-3b+1$.

Summing on i, the number of such vectors is

$$\sum_{i=1}^{n_1-3b+1}\Big\{((q-1)[1+(q-1)]^{(b-1,w-1)})$$
$$\times\{((n_1-b-i+1)-2b+1)\sum_{k_2=1}^{b-1}L_1(b,k_2,r_4,r_5,r_6)$$
$$+\sum_{\substack{k_2=k_3\\1\le k_3\le b-1}}^{b-1}L_1(b,k_2,r_4,r_5,r_6)\}\Big\}.\quad(27)$$

Further, when i takes the value $n_1 - 3b + 2$, the selection of β_r and γ_ℓ is made from the remaining $n_1 - b - (n_1 - 3b + 2) + 1$ i.e., $2b - 1$ components, k_2 can take the values as $1, \ldots, b - 1$, i.e., when i takes the value $n_1 - 3b + k_4 + 1$, $1 \leq k_4 \leq b - 1$, selection of β_r and γ_ℓ is made from the remaining $n_1 - b - i + 1$, k_2 can take the values as $k_4, \ldots, b - 1$.





k_2=1,...,b-1
k_2=1
i=n_1-3b+2
2b-1
i=n_1-3b+2
k_2=b-1
i=n_1-3b+2
k_2=2,...,b-1
k_2=2
i=n_1-3b+3
2b-2
i=n_1-3b+3
k_2=b-1
i=n_1-3b+3

Therefore, further summing on i we get

$$\sum_{\substack{i=n_1-3b+1+k_4\\ 1\le k_4\le b-1}} \left\{\left((q-1)[1+(q-1)]^{(b-1,w-1)}\right)\sum_{k_2=k_4}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\right\}$$

i.e.

$$\left\{\left((q-1)[1+(q-1)]^{(b-1,w-1)}\right)\sum_{\substack{k_2=k_4\\ 1\le k_4\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\right\}. \tag{28}$$

Thus, total number of such vectors is

$$(\text{expr. } (27)) + (\text{expr. } (28)) \tag{29}$$

which completes the proof of the Lemma 2.

Now, the number of choices of coefficients β_r, γ_ℓ and δ_p in a vector of length

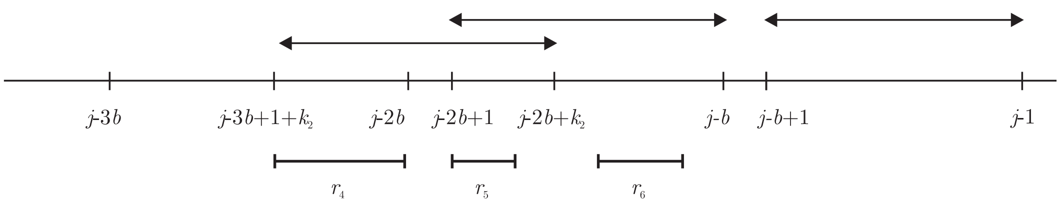
$j-b$ is obtained by replacing n_1 by $j-b$ in expression (29) giving rise to

$$\sum_{i=1}^{(j-b)-3b+1} \Big\{((q-1)[1+(q-1)]^{(b-1,w-1)}) \times \Big\{((j-2b-i+1)-2b+1)\sum_{k_2=1}^{b-1} L_1(b,k_2,r_4,r_5,r_6) + \sum_{\substack{k_2=k_3\\ 1\le k_3\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\Big\}\Big\} + \Big\{((q-1)[1+(q-1)]^{(b-1,w-1)}) \times \sum_{\substack{k_2=k_4\\ 1\le k_4\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\Big\}. \tag{30}$$

Therefore, the total number of choices of all the coefficients is

$$(\text{expr. } (25)) \cdot (\text{expr. } (30)). \tag{31}$$

Case 6. *When the h_r and h_ℓ are selected together from $2b-1$ or fewer columns (but from $b+1$ or more) from amongst the columns $h_1, \ldots, h_{j-b}$ i.e. suppose the h_r and h_ℓ are selected from $h_{j-3b+1+k_2}, \ldots, h_{j-b}$, the h_ℓ are selected from $h_{j-2b+1}, \ldots, h_{j-b}$ and the h_r are selected from $h_{j-3b+1+k_2}, \ldots, h_{j-2b+k_2}$ such that all the h_r are neither from $h_{j-3b+1+k_2}, \ldots, h_{j-2b}$ nor from $h_{j-2b+1}, \ldots, h_{j-b}, 1 \le k_2 \le b-1$.*



In this case, the number of ways in which the coefficients α_j can be selected is

$$[1+(q-1)]^{(b-1,w-1)}. \tag{32}$$

The number of ways in which the coefficients β_r and γ_ℓ are selected is $L_1(b,k_2,r_4,r_5,r_6)$ (refer case 3). Further, β_r and γ_ℓ are to be selected such that the last non-zero coefficient of γ_ℓ can take position $j-b, j-b-1,\ldots,b+1$ in a vector of length $j-b$.

To enumerate total number of choices of the coefficients β_r and γ_ℓ we prove the following Lemma 3 (consider the position $(j-b)$-th as the first position).

Lemma 3. *$L_1(b,k_2,r_4,r_5,r_6)$ represents the number as in case 3 for the selection of γ_ℓ and β_r. The number of such vectors with the varying starting position of γ_ℓ in a vector of length n_1 is*

$$(n_1-2b+1)\left(\sum_{k_2=1}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\right)+\sum_{\substack{k_2=k_4\\1\le k_4\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\,. \tag{33}$$

Proof of Lemma 3. Consider a vector of length n_1. The first non-zero component can take the position $i=1,\ldots,n_1-b$. Obviously, for $i=1,\ldots,n_1-2b+1$, k_2 can take values as $1\le k_2\le b-1$ and for $i=n_1-2b+2,\ldots,n_1-b$ i.e. for $i=n_1-2b+1+k_4$, $1\le k_4\le b-1$, k_2 can take values as $k_4\le k_2\le b-1$. Thus total number of such vectors is

$$(n_1-2b+1)\left(\sum_{k_2=1}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\right)+\sum_{\substack{k_2=k_4\\1\le k_4\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)$$

which completes the proof of Lemma 3.

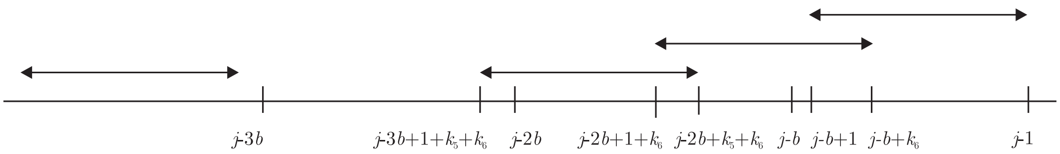
Now the number of choices of the coefficients β_r and γ_ℓ in a vector of length $j-b$ can be obtained by replacing n_1 by $j-b$ in expression (33) which gives

$$(j-3b+1)\sum_{k_2=1}^{b-1} L_1(b,k_2,r_4,r_5,r_6)+\sum_{\substack{k_2=k_4\\ 1\le k_4\le b-1}}^{b-1} L_1(b,k_2,r_4,r_5,r_6)\,. \tag{34}$$

Therefore, total number of choices of all the coefficients is

$$(\text{expr. }(32))\cdot(\text{expr. }(34)). \tag{35}$$

Case 7. *When the h_r, h_p and h_j are selected together from $3b-3$ or fewer columns (but from $2b-1$ or more) i.e. the h_p are selected from b consecutive columns amongst $h_{j-2b+2},\ldots,h_{j-1}$ with the last h_p among $h_{j-b+1},\ldots h_{j-1}$ and the h_r are selected from b consecutive columns amongst $h_{j-3b+3},\ldots,h_{j-b}$ with the last h_r among $h_{j-2b+1+k_6,\ldots,h_{j-b}}$ (depending on the selection of h_p's), $1\le k_6\le b-1$, together with the h_ℓ being selected from some b consecutive columns from the first $j-3b+1+k_7$ columns, $1\le k_7\le b-1$ (depending upon the b consecutive columns from which h_r's are selected).*



In this case, the coefficients δ_p are selected from b consecutive components as w or less non-zero components, which starts from $(j-2b+1+k_6)$-th component which may obviously continue upto $(j-b+k_6)$-th component. The coefficients β_r are selected from b consecutive components as w or less non-zero components, which starts from $(j-3b+1+k_5+k_6)$-th component which may continue obviously upto $(j-2b+k_5+k_6)$-th component, $1 \leq k_6 \leq b-1$, $1 \leq k_5 \leq b-k_6$. Suppose $k_5+k_6-1=k_7$.

Our main objective is to select $w-1$ or less non-zero components amongst $(j-3b+1+k_5+k_6,\ldots,j-2b+k_5+k_6-1)$-th positions, $(j-2b+k_5+k_6)$-th component is non-zero, $w-1$ or less non-zero components amongst $(j-2b+k_6+1,\ldots,j-b+k_6-1)$-th positions, $(j-b+k_6)$-th component is non-zero and $w-1$ or less non-zero components amongst $(j-b+1,\ldots,j-1)$-th positions. Also we have to select the coefficients γ_ℓ which appear as w or less non-zero components from a set of b consecutive components among the first $j-3b+k_5+k_6$ components where $1 \leq k_6 \leq b-1$, $1 \leq k_5 \leq b-k_6$.

In order to do so, let us choose

r_7 components from the $(j-3b+1+k_5+k_6,\ldots,j-2b+k_6)$-th positions,

r_8 components from the $(j-2b+k_6+1,\ldots,j-2b+k_5+k_6-1)$-th positions,

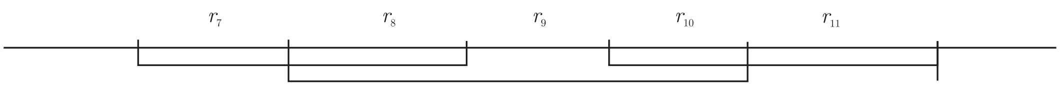
r_9 components from the $(j-2b+k_5+k_6+1,\ldots,j-b)$-th positions,

r_{10} components from the $(j-b+1,\ldots,j-b+k_6-1)$-th positions,

r_{11} components from the $(j-b+k_6+1,\ldots,j-1)$-th positions,

where

$$0 \leq r_7 \leq w-1,\ 0 \leq r_8 \leq 2w-2,\ 0 \leq r_9 \leq w-1,\ 0 \leq r_{10} \leq 2w-2,$$
$$0 \leq r_{11} \leq w-1. \tag{36}$$



Keeping in view the situations considered in cases 1, 2 and 4, $r_7, r_8, r_9, r_{10}, r_{11}$

should be such that

$$r_{10} + r_{11} \geq w - 1, r_8 + r_9 + r_{10} + r_{11} \geq 2w - 2,$$
$$r_7 + r_8 + r_9 + r_{10} + r_{11} \leq 3w - 3. \tag{37}$$

Such a selection of coefficients give us

$$\sum_{\substack{r_7, r_8, r_9, \\ r_{10}, r_{11}}} \binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}} \times \binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}}$$

possible linear combinations where $r_7, r_8, r_9, r_{10}, r_{11}$ each satisfy the constraints stated in (36) and (37). The $(j-2b+k_5+k_6)$-th and $(j-b+k_6)$-th components can be selected in $(q-1)$ ways each, therefore selection of coefficients give us

$$\sum_{\substack{r_7, r_8, r_9, \\ r_{10}, r_{11}}} \binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}} \times \binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}+2}. \tag{38}$$

Now to select the coefficients γ_ℓ it is equivalent to enumerate the single low-density burst of length b(fixed) with weight w or less in a vector of length $j-3b+k_5+k_6$, which gives us (Dass (1983))

$$1 + ((j-3b+k_5+k_6) - b + 1)(q-1)[1+(q-1)]^{(b-1,w-1)}. \tag{39}$$

Therefore, in this case total number of choices of coefficients turns out to be

$$\sum_{\substack{k_5, k_6 \\ 1 \leq k_6 \leq b-1 \\ 1 \leq k_5 \leq b-k_6}} (\text{expr. (38)}) \cdot (\text{expr. (39)}). \tag{40}$$

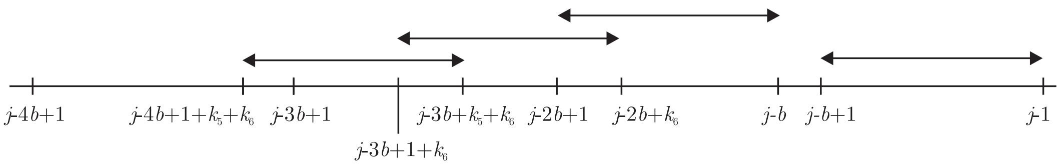
The expression

$$\sum_{\substack{r_7,r_8,r_9,\\ r_{10},r_{11}}} \binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}} \times \binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}+2}$$

with constraints stated in (36) and (37) is denoted as

$$L_2(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11}).$$

Case 8. *When the h_ℓ, h_r, h_p are selected together from $3b-2$ or fewer columns (but from $2b$ or more) from amongst the first $j-b$ columns, i.e., suppose the h_p are selected from b consecutive columns amongst $h_{j-2b+1},\ldots,h_{j-b}$, the h_r are selected from b consecutive columns amongst $h_{j-3b+2},\ldots,h_{j-b-1}$ with the last h_r among $h_{j-2b+1},\ldots,h_{j-b-1}$ and the h_ℓ are selected from b consecutive columns amongst $h_{j-4b+3},\ldots,h_{j-2b}$ with last column h_ℓ among $h_{j-3b+1+k_6},\ldots,h_{j-2b}$ (depending on the selection of h_r's), $1\le k_6\le b-1$.*



In this case, the number of ways in which the coefficients α_j can be selected is

$$[1+(q-1)]^{(b-1,w-1)}. \tag{41}$$

The coefficients δ_p are selected as w or less non-zero components from b consecutive components starting from $(j-2b+1)$-th component which may obviously continue upto $(j-b)$-th component. The coefficients β_r are selected from the components starting from $(j-3b+1+k_6)$-th component which may obviously continue upto $(j-2b+k_6)$-th component $(1 \leq k_6 \leq b-1)$ as w or less non-zero components from b consecutive components. The coefficients γ_ℓ are selected as w or less non-zero components from $(j-4b+1+k_5+k_6)$-th component which may obviously continue upto $(j-3b+k_5+k_6)$-th component, $1 \leq k_6 \leq b-1, 1 \leq k_5 \leq b-k_6$.

Our main objective is to select $w-1$ or less non-zero components amongst $(j-4b+1+k_5+k_6, \ldots, j-3b+k_5+k_6-1)$-th positions, the $(j-3b+k_5+k_6)$-th component is non-zero, $w-1$ or less non-zero components amongst $(j-3b+1+k_6, \ldots, j-2b+k_6-1)$-th positions, the $(j-2b+k_6)$-th component is non-zero, and $w-1$ or less non-zero components amongst $(j-2b+1, \ldots, j-b-1)$-th positions, the $(j-b)$-th component is non-zero. Following the procedure for the selection of coefficients γ_ℓ, β_r and δ_p as in case 7, with the selection of $(j-b)$-th component in $(q-1)$ ways, we get the expression as

$$\sum_{\substack{r_7,r_8,r_9,\\ r_{10},r_{11}}} \binom{b-k_5}{r_7}\binom{k_5-1}{r_8}\binom{b-k_5-k_6}{r_9}\binom{k_6-1}{r_{10}} \times \binom{b-k_6-1}{r_{11}}(q-1)^{r_7+r_8+r_9+r_{10}+r_{11}+3} \tag{42}$$

where

r_7 components are chosen from the $(j-4b+1+k_5+k_6, \ldots, j-3b+k_6)$-th positions,

r_8 components are chosen from the $(j-3b+k_6+1, \ldots, j-3b+k_5+k_6-1)$-th positions,

r_9 components are chosen from the $(j-3b+k_5+k_6+1, \ldots, j-2b)$-th positions,

r_{10} components are chosen from the $(j-2b+1, \ldots, j-2b+k_6-1)$-th positions,

r_{11} components are chosen from the $(j-2b+k_6+1, \ldots, j-b-1)$-th positions, where

$$0 \le r_7 \le w-1, 0 \le r_8 \le 2w-2, 0 \le r_9$$
$$\le w-1, 0 \le r_{10} \le 2w-2, 0 \le r_{11} \le w-1. \quad (43)$$

Keeping in view the situations considered in cases 1, 4 and 5, r_7, r_8, r_9, r_{10}, r_{11} should be such that

$$r_{10}+r_{11} \ge w-1, \quad r_8+r_9+r_{10}+r_{11} \ge 2w-2,$$
$$r_7+r_8+r_9+r_{10}+r_{11} \le 3w-3. \quad (44)$$

The (expr. (42)) with constraints stated in (43) and (44) is denoted by $L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})$.

The non-zero component at $(j-b)$-th position during the selection of coefficients, can take positions $(j-b),(j-b-1),\ldots,2b$.

Total number of ways in which β_r, γ_ℓ and δ_p are selected is

$$((j-b)-3b+3)\sum_{\substack{k_5,k_6\\1\le k_6\le b-1\\1\le k_5\le b-k_6}} L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})$$
$$+\sum_{\substack{i_1,k_5,k_6\\1\le i_1\le b-2\\i_1+1\le k_6\le b-1\\1\le k_5\le b-k_6}} L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11})$$
$$+\sum_{\substack{i_1,k_5,k_6\\1\le i_1\le b-2\\1\le k_6\le b-1\\i_1+1\le k_5\le b-k_6}} L_3(b,k_5,k_6,r_7,r_8,r_9,r_{10},r_{11}) \quad (45)$$

with last non-zero component during the selection of coefficients taking positions

(i) $j-b,\ldots,3b-2$, values of k_5,k_6 varies as $1\le k_6\le b-1$, $1\le k_5\le b-k_6$, further

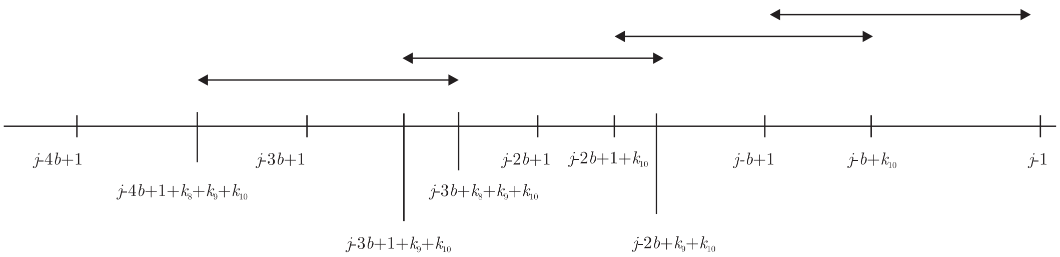
(ii) (a) $(3b-2-i_1), 1 \le i_1 \le b-2$ positions, values of k_5, k_6 varies as $i_1+1 \le k_6 \le b-1$, $1 \le k_5 \le b-k_6$.

(b) $(3b-2-i_1), 1 \le i_1 \le b-2$ positions, values of k_5, k_6 varies as $1 \le k_6 \le b-1$, $i_1+1 \le k_5 \le b-k_6$.

Therefore, in this case total number of choices of coefficients turns out to be

$$(\text{expr. } (41)) \cdot (\text{expr. } (45)). \tag{46}$$

Case 9. *When the h_ℓ, h_r, h_p and h_j are selected together from $4b-4$ or fewer columns (but from $2b$ or more), the h_p are selected from b consecutive columns amongst $h_{j-2b+2}, \ldots, h_{j-1}$ with the last h_p among $h_{j-b+1}, \ldots, h_{j-1}$, the h_r are selected from b consecutive columns amongst $h_{j-3b+3}, \ldots, h_{j-b}$ with the last h_r among $h_{j-2b+1+k_{10}}, \ldots, h_{j-b}$ (depending on the selection of h_p's), $1 \le k_{10} \le b-1$, the h_ℓ are selected from b consecutive columns from amongst $h_{j-4b+4}, \ldots, h_{j-2b+k_{10}}$, with last column h_ℓ among $h_{j-3b+1+k_9+k_{10}}, \ldots, h_{j-2b+k_{10}}, 1 \le k_{10} \le b-1, 1 \le k_9 \le b-k_{10}$, i.e., among the starting of selection of b consecutive columns for h_r upto one column before the starting of selection of h_p.*



In this case, the coefficients δ_p are selected from b consecutive components as w or less non-zero components which start from $(j-2b+1+k_{10})$-th component and may obviously continue upto $(j-b+k_{10})$-th component,

$1 \leq k_{10} \leq b-1$. The coefficients β_r are selected from b consecutive components as w or less non-zero components which start from $(j-3b+1+k_9+k_{10})$-th component and may obviously continue upto $(j-2b+k_9+k_{10})$-th component, $1 \leq k_{10} \leq b-1, 1 \leq k_9 \leq b-k_{10}$. The coefficients γ_ℓ are selected from b consecutive components as w or less non-zero components which start from $(j-4b+1+k_8+k_9+k_{10})$-th component which may obviously continue upto $(j-3b+k_8+k_9+k_{10})$-th component, $1 \leq k_{10} \leq b-1$, $1 \leq k_9 \leq b-k_{10}$, $1 \leq k_8 \leq b-k_9$.

Our main objective is to select $w-1$ or less non-zero components amongst $(j-4b+1+k_8+k_9+k_{10}, \ldots, j-3b+k_8+k_9+k_{10}-1)$-th positions, the $(j-3b+k_8+k_9+k_{10})$-th component is non-zero, $w-1$ or less non-zero components among $(j-3b+1+k_9+k_{10}, \ldots, j-2b+k_9+k_{10}-1)$-th positions, the $(j-2b+k_9+k_{10})$-th component is non-zero, $w-1$ or less non-zero components among $(j-2b+1+k_{10}, \ldots, j-b+k_{10}-1)$-th positions, the $(j-b+k_{10})$-th component is non-zero and $w-1$ or less non-zero components amongst $(j-b+1, \ldots, j-1)$-th positions.

In order to do so, let us choose

r_{12} components from the $(j-4b+1+k_8+k_9+k_{10}, \ldots, j-3b+k_9+k_{10})$-th positions,

r_{13} components from the $(j-3b+k_9+k_{10}+1, \ldots, j-3b+k_8+k_9+k_{10}-1)$-th positions,

r_{14} components from the $(j-3b+k_8+k_9+k_{10}+1, \ldots, j-2b+k_{10})$-th positions,

r_{15} components from the $(j-2b+k_{10}+1, \ldots, j-2b+k_9+k_{10}-1)$-th positions,

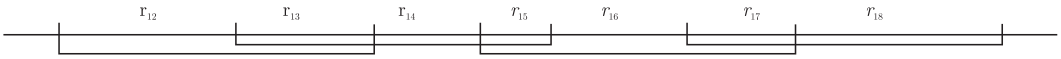
r_{16} components from the $(j-2b+k_9+k_{10}+1, \ldots, j-b)$-th positions,

r_{17} components from the $(j-b+1, \ldots, j-b+k_{10}-1)$-th positions,

r_{18} components from the $(j-b+k_{10}+1,\ldots,j-1)$-th positions,

where

$$0 \le r_{12} \le w-1, 0 \le r_{13} \le 2w-2, 0 \le r_{14} \le w-1,$$
$$0 \le r_{15} \le 2w-2, 0 \le r_{16} \le w-1, 0 \le r_{17} \le 2w-2, 0 \le r_{18} \le w-1. \qquad (47)$$



Keeping in view the situations considered in cases 3, 7 and 8, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, r_{17}, r_{18} should be such that

$$\begin{aligned} & r_{17}+r_{18} \ge w-1, r_{15}+r_{16}+r_{17}+r_{18} \ge 2w-2, \\ & r_{13}+r_{14}+r_{15}+r_{16}+r_{17}+r_{18} \ge 3w-3, \\ & r_{12}+r_{13}+r_{14}+r_{15}+r_{16}+r_{17}+r_{18} \le 4w-4. \end{aligned} \qquad (48)$$

Such a selection of coefficients give us

$$\sum_{r_{12},r_{13},\ldots,r_{18}} \binom{b-k_8}{r_{12}}\binom{k_8-1}{r_{13}}\binom{b-k_8-k_9}{r_{14}}\binom{k_9-1}{r_{15}}\binom{b-k_9-k_{10}}{r_{16}} \times \binom{k_{10}-1}{r_{17}}\binom{b-k_{10}-1}{r_{18}}(q-1)^{r_{12}+r_{13}+r_{14}+r_{!5}+r_{16}+r_{17}+r_{18}} \qquad (49)$$

possible linear combinations where $r_{12},\ldots,r_{18}$ each satisfy the constraints stated in (47) and (48). The $(j-b+k_{10})$-th, $(j-2b+k_9+k_{10})$-th and $(j-3b+k_8+k_9+k_{10})$-th components can be selected in $(q-1)$ ways each, therefore selection of coefficients give us

$$\sum_{\substack{k_8,k_9,k_{10} \\ 1\le k_{10}\le b-1 \\ 1\le k_9\le b-k_{10} \\ 1\le k_8\le b-k_9}} \sum_{r_{12},\ldots,r_{18}} \binom{b-k_8}{r_{12}}\binom{k_8-1}{r_{13}}\binom{b-k_8-k_9}{r_{14}}\binom{k_9-1}{r_{15}}\binom{b-k_9-k_{10}}{r_{16}} \times \binom{k_{10}-1}{r_{17}}\binom{b-k_{10}-1}{r_{18}}(q-1)^{r_{12}+r_{13}+r_{14}+r_{!5}+r_{16}+r_{17}+r_{18}+3}. \qquad (50)$$

The expression

$$\sum_{r_{12},\dots,r_{18}} \binom{b-k_8}{r_{12}}\binom{k_8-1}{r_{13}}\binom{b-k_8-k_9}{r_{14}}\binom{k_9-1}{r_{15}}\binom{b-k_9-k_{10}}{r_{16}} \\ \times\binom{k_{10}-1}{r_{17}}\binom{b-k_{10}-1}{r_{18}}(q-1)^{r_{12}+r_{13}+r_{14}+r_{!5}+r_{16}+r_{17}+r_{18}+3}$$

with constraints stated in (47) and (48) is denoted by
$L_4(b, k_8, k_9, k_{10}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, r_{17}, r_{18})$.

Thus, the total number of possible combinations that h_j cannot be equal to, is

$$\begin{aligned} &(\text{expr. } (7)) + (\text{expr. } (12)) + (\text{expr. } (17)) \\ &\quad +(\text{expr. } (24)) + (\text{expr. } (31)) + (\text{expr. } (35)) \\ &\quad +(\text{expr. } (40)) + (\text{expr. } (46)) + (\text{expr. } (50))\,. \end{aligned} \tag{51}$$

At worst, all these linear combinations may yield a distinct sum. Therefore a column h_j can be added to H' provided that

$$q^{n-k} > (\text{expr. } (51)). \tag{52}$$

Now reverse the columns of H' to obtain the requisite parity-check matrix $H = [H_1, H_2 \dots H_n], (h_i \to H_{n-i+1})$. Thus, to achieve code of length n, replace j by n which gives the result.

Remark 1. The result just obtained holds for $w \leq b$. If we take $w = b$, the weight consideration over the burst becomes redundant. The situations giving rise to the expressions except (expr. (7)) does not arise. The bound then reduces to

$$\begin{aligned} q^{n-k} &> [1+(q-1)]^{(b-1,b-1)} \\ &\quad \times \left\{ \sum_{i=0}^{3} \binom{n-(i+1)b+i}{i} (q-1)^i [1+(q-1)]^{i(b-1,b-1)} \right\} \end{aligned}$$

i.e.

$$q^{n-k} > q^{b-1} \left\{ \sum_{i=0}^{3} \binom{n-(i+1)b+i}{i} (q-1)^i q^{i(b-1)} \right\}$$

which coincides with a result due to Dass, Garg and Zannetti (2008b), when bursts considered are 2-repeated bursts of length b(fixed).

We conclude this section with an example.

Example 1. Consider the following 12×16 matrix over GF(2)

$$H = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 1&0&1&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0 \\ 1&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&1&0&0&0&0&0&1&0&0 \\ 1&1&0&0&0&0&0&0&1&0&0&0&0&0&1&0 \\ 1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0 \\ 0&1&1&0&0&0&0&0&0&0&0&1&0&0&1&0 \\ 1&1&0&0&0&0&0&0&0&0&0&0&1&0&0&1 \end{bmatrix}.$$

This matrix has been constructed by the synthesis procedure outlined in the proof of Theorem 1 by taking $b = 3$ and $w = 2$. Considered as a parity-check matrix, this matrix gives rise to a $(16, 4)$ binary code. It can be seen from Table 1 that the syndromes of bursts of length 3(fixed) with weight 2 or less are distinct, showing thereby that the code that is the null space of the matrix given in the example corrects all bursts of length 3(fixed) with weight 2 or less. It should be noted that this code does not correct all bursts of length 3(fixed) with weight 3, e.g.,

(0000000011100111) as its syndrome is the same as that of (1100000000001000),
(0000000000101111) as its syndrome is the same as that of (1100000011000000),
(0000000011000111) as its syndrome is the same as that of (1100000000101000),
(0000000011100101) as its syndrome is the same as that of (1100000000001010),
(0000000011101010) as its syndrome is the same as that of (1100000000000101),
(0000000011101000) as its syndrome is the same as that of (1100000000000111).

Table 1

Syndromes of Correctable error-vectors

Error vectors	Syndromes
0000000000000000	000000000000
1000000000000000	011111111001
1001000000000000	010111111001
1000100000000000	011011111001
1000010000000000	011101111001
1000001000000000	011110111001
1000000100000000	011111011001
1000000010000000	011111101001
1000000001000000	011111110001
1000000000100000	011111111101
1000000000010000	011111111011
1000000000001000	011111111000
1000000000000100	111011011101
1001100000000000	010011111001
1000110000000000	011001111001
1000011000000000	011100111001
1000001100000000	011110011001
1000000110000000	011111001001
1000000011000000	011111100001
1000000001100000	011111110101
1000000000110000	011111111111
1000000000011000	011111111010
1000000000001100	111011011100
1000000000000110	101001001111
1001010000000000	010101111001
1000101000000000	011010111001
1000010100000000	011101011001
1000001010000000	011110101001
1000000101000000	011111010001
1000000010100000	011111101101
1000000001010000	011111110011
1000000000101000	011111111100
1000000000010100	111011011111
1000000000001010	001101101010

Error vectors	Syndromes
1000000000000101	110010010100
1100000000000000	111111100010
1101000000000000	110111100010
1100100000000000	111011100010
1100010000000000	111101100010
1100001000000000	111110100010
1100000100000000	111111000010
1100000010000000	111111110010
1100000001000000	111111101010
1100000000100000	111111100110
1100000000010000	111111100000
1100000000001000	111111100011
1100000000000100	011011000110
1101100000000000	110011100010
1100110000000000	111001100010
1100011000000000	111100100010
1100001100000000	111110000010
1100000110000000	111111010010
1100000011000000	111111111010
1100000001100000	111111101110
1100000000110000	111111100100
1100000000011000	111111100001
1100000000001100	011011000111
1100000000000110	001001010100
1101010000000000	110101100010
1100101000000000	111010100010
1100010100000000	111101000010
1100001010000000	111110110010
1100000101000000	111111001010
1100000010100000	111111110110
1100000001010000	111111101000
1100000000101000	111111100111
1100000000010100	011011000100
1100000000001010	101101110001

(Contd.)

Error vectors	Syndromes
1100000000000101	010010001111
1010000000000000	001111111011
1011000000000000	000111111011
1010100000000000	001011111011
1010010000000000	001101111011
1010001000000000	001110111011
1010000100000000	001111011011
1010000010000000	001111101011
1010000001000000	001111110011
1010000000100000	001111111111
1010000000010000	001111111001
1010000000001000	001111111010
1010000000000100	101011011111
1011100000000000	000011111011
1010110000000000	001001111011
1010011000000000	001100111011
1010001100000000	001110011011
1010000110000000	001111001011
1010000011000000	001111100011
1010000001100000	001111110111
1010000000110000	001111111101
1010000000011000	001111111000
1010000000001100	101011011110
1010000000000110	111001001101
1011010000000000	000101111011
1010101000000000	001010111011
1010010100000000	001101011011
1010001010000000	001110101011
1010000101000000	001111010011
1010000010100000	001111101111
1010000001010000	001111110001
1010000000101000	001111111110
1010000000010100	101011011101
1010000000001010	011101101000
1010000000000101	100010010110
0100000000000000	100000011011
0100100000000000	100100011011

Error vectors	Syndromes
0100010000000000	100010011011
0100001000000000	100001011011
0100000100000000	100000111011
0100000010000000	100000001011
0100000001000000	100000010011
0100000000100000	100000011111
0100000000010000	100000011001
0100000000001000	100000011010
0100000000000100	000100111111
0100110000000000	100110011011
0100011000000000	100011011011
0100001100000000	100001111011
0100000110000000	100000101011
0100000011000000	100000000011
0100000001100000	100000010111
0100000000110000	100000011101
0100000000011000	100000011000
0100000000001100	000100111110
0100000000000110	010110101101
0100101000000000	100101011011
0100010100000000	100010111011
0100001010000000	100001001011
0100000101000000	100000110011
0100000010100000	100000001111
0100000001010000	100000010001
0100000000101000	100000011110
0100000000010100	000100111101
0100000000001010	110010001000
0100000000000101	001101110110
0110000000000000	110000011001
0110100000000000	110100011001
0110010000000000	110010011001
0110001000000000	110001011001
0110000100000000	110000111001
0110000010000000	110000001001
0110000001000000	110000010001
0110000000100000	110000011101

(Contd.)

Error vectors	Syndromes
0110000000010000	110000011011
0110000000001000	110000011000
0110000000000100	010100111101
0110110000000000	110110011001
0110011000000000	110011011001
0110001100000000	110001111001
0110000110000000	110000101001
0110000011000000	110000000001
0110000001100000	110000010101
0110000000110000	110000011111
0110000000011000	110000011010
0110000000001100	010100111100
0110000000000110	000110101111
0110101000000000	110101011001
0110010100000000	110010111001
0110001010000000	110001001001
0110000101000000	110000110001
0110000010100000	110000001101
0110000001010000	110000010011
0110000000101000	110000011100
0110000000010100	010100111111
0110000000001010	100010001010
0110000000000101	011101110100
0101000000000000	101000011011
0101100000000000	101100011011
0101010000000000	101010011011
0101001000000000	101001011011
0101000100000000	101000111011
0101000010000000	101000001011
0101000001000000	101000010011
0101000000100000	101000011111
0101000000010000	101000011001
0101000000001000	101000011010
0101000000000100	001100111111
0101110000000000	101110011011
0101011000000000	101011011011
0101001100000000	101001111011

Error vectors	Syndromes
0101000110000000	101000101011
0101000011000000	101000000011
0101000001100000	101000010111
0101000000110000	101000011101
0101000000011000	101000011000
0101000000001100	001100111110
0101000000000110	011110101101
0101101000000000	101101011011
0101010100000000	101010111011
0101001010000000	101001001011
0101000101000000	101000110011
0101000010100000	101000001111
0101000001010000	101000010001
0101000000101000	101000011110
0101000000010100	001100111101
0101000000001010	111010001000
0101000000000101	000101110110
0010000000000000	010000000010
0010010000000000	010010000010
0010001000000000	010001000010
0010000100000000	010000100010
0010000010000000	010000010010
0010000001000000	010000001010
0010000000100000	010000000110
0010000000010000	010000000000
0010000000001000	010000000011
0010000000000100	110100100110
0010011000000000	010011000010
0010001100000000	010001100010
0010000110000000	010000110010
0010000011000000	010000011010
0010000001100000	010000001110
0010000000110000	010000000100
0010000000011000	010000000001
0010000000001100	110100100111
0010000000000110	100110110100
0010010100000000	010010100010

(Contd.)

Error vectors	Syndromes
0010001010000000	010001010010
0010000101000000	010000101010
0010000010100000	010000010110
0010000001010000	010000001000
0010000000101000	010000000111
0010000000010100	110100100100
0010000000001010	000010010001
0010000000000101	111101101111
0011000000000000	011000000010
0011010000000000	011010000010
0011001000000000	011001000010
0011000100000000	011000100010
0011000010000000	011000010010
0011000001000000	011000001010
0011000000100000	011000000110
0011000000010000	011000000000
0011000000001000	011000000011
0011000000000100	111100100110
0011011000000000	011011000010
0011001100000000	011001100010
0011000110000000	011000110010
0011000011000000	011000011010
0011000001100000	011000001110
0011000000110000	011000000100
0011000000011000	011000000001
0011000000001100	111100100111
0011000000000110	101110110100
0011010100000000	011010100010
0011001010000000	011001010010
0011000101000000	011000101010
0011000010100000	011000010110
0011000001010000	011000001000
0011000000101000	011000000111
0011000000010100	111100100100
0011000000001010	001010010001
0011000000000101	110101101111
0010100000000000	010100000010

Error vectors	Syndromes
0010110000000000	010110000010
0010101000000000	010101000010
0010100100000000	010100100010
0010100010000000	010100010010
0010100001000000	010100001010
0010100000100000	010100000110
0010100000010000	010100000000
0010100000001000	010100000011
0010100000000100	110000100110
0010111000000000	010111000010
0010101100000000	010101100010
0010100110000000	010100110010
0010100011000000	010100011010
0010100001100000	010100001110
0010100000110000	010100000100
0010100000011000	010100000001
0010100000001100	110000100111
0010100000000110	100010110100
0010110100000000	010110100010
0010101010000000	010101010010
0010100101000000	010100101010
0010100010100000	010100010110
0010100001010000	010100001000
0010100000101000	010100000111
0010100000010100	110000100100
0010100000001010	000110010001
0010100000000101	111001101111
0001000000000000	001000000000
0001001000000000	001001000000
0001000100000000	001000100000
0001000010000000	001000010000
0001000001000000	001000001000
0001000000100000	001000000100
0001000000010000	001000000010
0001000000001000	001000000001
0001000000000100	101100100100
0001001100000000	001001100000

(Contd.)

Error vectors	Syndromes
0001000110000000	001000110000
0001000011000000	001000011000
0001000001100000	001000001100
0001000000110000	001000000110
0001000000011000	001000000011
0001000000001100	101100100101
0001000000000110	111110110110
0001001010000000	001001010000
0001000101000000	001000101000
0001000010100000	001000010100
0001000001010000	001000001010
0001000000101000	001000000101
0001000000010100	101100100110
0001000000001010	011010010011
0001000000000101	100101101101
0001100000000000	001100000000
0001101000000000	001101000000
0001100100000000	001100100000
0001100010000000	001100010000
0001100001000000	001100001000
0001100000100000	001100000100
0001100000010000	001100000010
0001100000001000	001100000001
0001100000000100	101000100100
0001101100000000	001101100000
0001100110000000	001100110000
0001100011000000	001100011000
0001100001100000	001100001100
0001100000110000	001100000110
0001100000011000	001100000011
0001100000001100	101000100101
0001100000000110	111010110110
0001101010000000	001101010000
0001100101000000	001100101000
0001100010100000	001100010100
0001100001010000	001100001010
0001100000101000	001100000101

Error vectors	Syndromes
0001100000010100	101000100110
0001100000001010	011110010011
0001100000000101	100001101101
0001010000000000	001010000000
0001011000000000	001011000000
0001010100000000	001010100000
0001010010000000	001010010000
0001010001000000	001010001000
0001010000100000	001010000100
0001010000010000	001010000010
0001010000001000	001010000001
0001010000000100	101110100100
0001011100000000	001011100000
0001010110000000	001010110000
0001010011000000	001010011000
0001010001100000	001010001100
0001010000110000	001010000110
0001010000011000	001010000011
0001010000001100	101110100101
0001010000000110	111100110110
0001011010000000	001011010000
0001010101000000	001010101000
0001010010100000	001010010100
0001010001010000	001010001010
0001010000101000	001010000101
0001010000010100	101110100110
0001010000001010	011000010011
0001010000000101	100111101101
0000100000000000	000100000000
0000100100000000	000100100000
0000100010000000	000100010000
0000100001000000	000100001000
0000100000100000	000100000100
0000100000010000	000100000010
0000100000001000	000100000001
0000100000000100	100000100100
0000100110000000	000100110000

(Contd.)

Error vectors	Syndromes
0000100011000000	000100011000
0000100001100000	000100001100
0000100000110000	000100000110
0000100000011000	000100000011
0000100000001100	100000100101
0000100000000110	110010110110
0000100101000000	000100101000
0000100010100000	000100010100
0000100001010000	000100001010
0000100000101000	000100000101
0000100000010100	100000100110
0000100000001010	010110010011
0000100000000101	101001101101
0000110000000000	000110000000
0000110100000000	000110100000
0000110010000000	000110010000
0000110001000000	000110001000
0000110000100000	000110000100
0000110000010000	000110000010
0000110000001000	000110000001
0000110000000100	100010100100
0000110110000000	000110110000
0000110011000000	000110011000
0000110001100000	000110001100
0000110000110000	000110000110
0000110000011000	000110000011
0000110000001100	100010100101
0000110000000110	110000110110
0000110101000000	000110101000
0000110010100000	000110010100
0000110001010000	000110001010
0000110000101000	000110000101
0000110000010100	100010100110
0000110000001010	010100010011
0000110000000101	101011101101
0000101000000000	000101000000
0000101100000000	000101100000

Error vectors	Syndromes
0000101010000000	000101010000
0000101001000000	000101001000
0000101000100000	000101000100
0000101000010000	000101000010
0000101000001000	000101000001
0000101000000100	100001100100
0000101110000000	000101110000
0000101011000000	000101011000
0000101001100000	000101001100
0000101000110000	000101000110
0000101000011000	000101000011
0000101000001100	100001100101
0000101000000110	110011110110
0000101101000000	000101101000
0000101010100000	000101010100
0000101001010000	000101001010
0000101000101000	000101000101
0000101000010100	100001100110
0000101000001010	010111010011
0000101000000101	101000101101
0000010000000000	000010000000
0000010010000000	000010010000
0000010001000000	000010001000
0000010000100000	000010000100
0000010000010000	000010000010
0000010000001000	000010000001
0000010000000100	100110100100
0000010011000000	000010011000
0000010001100000	000010001100
0000010000110000	000010000110
0000010000011000	000010000011
0000010000001100	100110100101
0000010000000110	110100110110
0000010010100000	000010010100
0000010001010000	000010001010
0000010000101000	000010000101
0000010000010100	100110100110

(Contd.)

Error vectors	Syndromes
0000010000001010	010000010011
0000010000000101	101111101101
0000011000000000	000011000000
0000011010000000	000011010000
0000011001000000	000011001000
0000011000100000	000011000100
0000011000010000	000011000010
0000011000001000	000011000001
0000011000000100	100111100100
0000011011000000	000011011000
0000011001100000	000011001100
0000011000110000	000011000110
0000011000011000	000011000011
0000011000001100	100111100101
0000011000000110	110101110110
0000011010100000	000011010100
0000011001010000	000011001010
0000011000101000	000011000101
0000011000010100	100111100110
0000011000001010	010001010011
0000011000000101	101110101101
0000010100000000	000010100000
0000010110000000	000010110000
0000010101000000	000010101000
0000010100100000	000010100100
0000010100010000	000010100010
0000010100001000	000010100001
0000010100000100	100110000100
0000010111000000	000010111000
0000010101100000	000010101100
0000010100110000	000010100110
0000010100011000	000010100011
0000010100001100	100110000101
0000010100000110	110100010110
0000010110100000	000010110100
0000010101010000	000010101010
0000010100101000	000010100101

Error vectors	Syndromes
0000010100010100	100110000110
0000010100001010	010000110011
0000010100000101	101111001101
0000001000000000	000001000000
0000001001000000	000001001000
0000001000100000	000001000100
0000001000010000	000001000010
0000001000001000	000001000001
0000001000000100	100101100100
0000001001100000	000001001100
0000001000110000	000001000110
0000001000011000	000001000011
0000001000001100	100101100101
0000001000000110	110111110110
0000001001010000	000001001010
0000001000101000	000001000101
0000001000010100	100101100110
0000001000001010	010011010011
0000001000000101	101100101101
0000001100000000	000001100000
0000001101000000	000001101000
0000001100100000	000001100100
0000001100010000	000001100010
0000001100001000	000001100001
0000001100000100	100101000100
0000001101100000	000001101100
0000001100110000	000001100110
0000001100011000	000001100011
0000001100001100	100101000101
0000001100000110	110111010110
0000001101010000	000001101010
0000001100101000	000001100101
0000001100010100	100101000110
0000001100001010	010011110011
0000001100000101	101100001101
0000001010000000	000001010000
0000001011000000	000001011000

(Contd.)

Error vectors	Syndromes
0000001010100000	000001010100
0000001010010000	000001010010
0000001010001000	000001010001
0000001010000100	100101110100
0000001011100000	000001011100
0000001010110000	000001010110
0000001010011000	000001010011
0000001010001100	100101110101
0000001010000110	110111100110
0000001011010000	000001011010
0000001010101000	000001010101
0000001010010100	100101110110
0000001010001010	010011000011
0000001010000101	101100111101
0000000100000000	000000100000
0000000100100000	000000100100
0000000100010000	000000100010
0000000100001000	000000100001
0000000100000100	100100000100
0000000100110000	000000100110
0000000100011000	000000100011
0000000100001100	100100000101
0000000100000110	110110010110
0000000100101000	000000100101
0000000100010100	100100000110
0000000100001010	010010110011
0000000100000101	101101001101
0000000110000000	000000110000
0000000110100000	000000110100
0000000110010000	000000110010
0000000110001000	000000110001
0000000110000100	100100010100
0000000110110000	000000110110
0000000110011000	000000110011
0000000110001100	100100010101
0000000110000110	110110000110
0000000110101000	000000110101

Error vectors	Syndromes
0000000110010100	100100010110
0000000110001010	010010100011
0000000110000101	101101011101
0000000101000000	000000101000
0000000101100000	000000101100
0000000101010000	000000101010
0000000101001000	000000101001
0000000101000100	100100001100
0000000101110000	000000101110
0000000101011000	000000101011
0000000101001100	100100001101
0000000101000110	110110011110
0000000101101000	000000101101
0000000101010100	100100001110
0000000101001010	010010111011
0000000101000101	101101000101
0000000010000000	000000010000
0000000010010000	000000010010
0000000010001000	000000010001
0000000010000100	100100110100
0000000010011000	000000010011
0000000010001100	100100110101
0000000010000110	110110100110
0000000010010100	100100110110
0000000010001010	010010000011
0000000010000101	101101111101
0000000011000000	000000011000
0000000011010000	000000011010
0000000011001000	000000011001
0000000011000100	100100111100
0000000011011000	000000011011
0000000011001100	100100111101
0000000011000110	110110101110
0000000011010100	100100111110
0000000011001010	010010001011
0000000011000101	101101110101
0000000010100000	000000010100

(Contd.)

Error vectors	Syndromes
0000000010110000	000000010110
0000000010101000	000000010101
0000000010100100	100100110000
0000000010111000	000000010111
0000000010101100	100100110001
0000000010100110	110110100010
0000000010110100	100100110010
0000000010101010	010010000111
0000000010100101	101101111001
0000000001000000	000000001000
0000000001001000	000000001001
0000000001000100	100100101100
0000000001001100	100100101101
0000000001000110	110110111110
0000000001001010	010010011011
0000000001000101	101101100101
0000000001100000	000000001100
0000000001101000	000000001101
0000000001100100	100100101000
0000000001101100	100100101001
0000000001100110	110110111010
0000000001101010	010010011111
0000000001100101	101101100001
0000000001010000	000000001010
0000000001011000	000000001011
0000000001010100	100100101110
0000000001011100	100100101111
0000000001010110	110110111100
0000000001011010	010010011001
0000000001010101	101101100111
0000000000100000	000000000100
0000000000100100	100100100000
0000000000100110	110110110010
0000000000100101	101101101001
0000000000110000	000000000110
0000000000110100	100100100010
0000000000110110	110110110000

Error vectors	Syndromes
0000000000110101	101101101011
0000000000101000	000000000101
0000000000101100	100100100001
0000000000101110	110110110011
0000000000101101	101101101000
0000000000010000	000000000010
0000000000011000	000000000011
0000000000010100	100100100110
0000000000001000	000000000001
0000000000001100	100100100101
0000000000001010	010010010011
0000000000000100	100100100100
0000000000000110	110110110110
0000000000000101	101101101101

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