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On W_2-Curvature Tensor in a Kenmotsu Manifold*

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Abstract

The purpose of this paper is to study some properties of W_2-curvature tensor in Riemannian and Kenmotsu manifolds.

Keywords and phrases: *Riemannian manifold, Kenmotsu manifold, W_2-curvature tensor, Irrotational W_2-curvature tensor, η-Einstein manifold.*

1. Introduction

In 1958, Boothby and Wong [1] studied odd dimensional manifolds with contact and almost contact structures from topological point of view. Sasaki and Hatakeyama [11] re-investigated them using tensor calculus in 1961. In 1972, K. Kenmotsu studied a class of contact Riemannian manifold and call them Kenmotsu manifold [7]. He proved that if Kenmotsu manifold satisfies the condition R(X, Y).R = 0, then the

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manifold is of negative curvature -1, where R is the Riemannian curvature tensor of type (1, 3) and R(X, Y) denotes the derivation of the tensor algebra at each point of the tangent space. The properties of Kenmotsu manifold have been studied by several authors such as De [4], Sinha and Shrivastava [12], Jun, De and Pathak [6], De and Pathak [3], De, Yildiz and Yaliniz [5], Özgur and De [10] and many others. In this paper, we consider Kenmotsu manifold satisfying the conditions $R(\xi, X).W_2 = 0$, $W_2(\xi, X).R = 0$, $P(\xi, X).W_2 = 0$ and $W_2(\xi, X).P = 0$, where W_2 and P denotes the W_2-curvature tensor and projective curvature tensor respectively. Also we have studied the W_2-curvature tensor in a Riemannian manifold and obtained the relation between different curvature tensors. In last section, we have shown that the W_2-curvature tensor in a Kenmotsu manifold M^n is irrotational if and only if $R(X, Y)Z = g(X, Z)Y - g(Y, Z)X + \frac{1}{n-1}\{\eta(X)(\nabla_Z Q)(Y) - \eta(Y)(\nabla_Z Q)(X)\}$.

2. **Preliminaries**

If on an odd dimensional differentiable manifold M^n (n = 2m+1), of differentiability class C^{r+1}, there exists a vector valued real linear function φ, a 1-form η, the associated vector field ξ and the Riemannian metric g satisfying

$$\varphi^2 X = -X + \eta(X)\xi, \tag{1}$$

$$\eta(\varphi X) = 0, \tag{2}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\,\eta(Y), \tag{3}$$

for arbitrary vector fields X and Y, then (M^n, g) is said to be an almost contact metric manifold [2] and the structure (φ, ξ, η, g) is called an almost contact metric structure to M^n.

In view of equations (1), (2) and (3), we have

$$\eta(\xi) = 1,\ g(X, \xi) = \eta(X),\ \varphi(\xi) = 0. \tag{4}$$

An almost contact metric manifold is called Kenmotsu manifold [7] if

$$(\nabla_X \varphi) = -\eta(Y)\varphi X - g(X, \varphi Y)\xi, \tag{5}$$

$$(\nabla_X \xi) = X - \eta(X)\xi, \tag{6}$$

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\,\eta(Y), \tag{7}$$

where ∇ is the Levi-Civita connection of g. Also the following relations hold in Kenmotsu manifold [3], [5], [6]

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{8}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi = - R(X, \xi)Y, \tag{9}$$

$$\eta(R(X, Y)Z) = \eta(Y)g(X, Z) - \eta(X)g(Y, Z), \tag{10}$$

$$S(X, \xi) = -(n-1)\, \eta(X), \tag{11}$$

$$Q\xi = -(n-1)\xi, \tag{12}$$

where Q is the Ricci operator, i.e. g(QX, Y) = S(X, Y) and

$$S(\varphi X, \varphi Y) = S(X, Y) + (n-1)\, \eta(X)\eta(Y), \tag{13}$$

for arbitrary vector fields X, Y, Z on M^n.

A Kenmotsu manifold is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\, \eta(X)\eta(Y) \tag{14}$$

for arbitrary vector fields X and Y, where a and b are smooth functions on M^n.

Projective curvature tensor P, concircular curvature tensor C and the conformal curvature tensor V are given by [8]

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1}[S(Y, Z)X - S(X, Z)Y]. \tag{15}$$

$$P(\xi, Y)Z = \mathbf{-\{g(Y, Z) + \frac{1}{n-1} S(Y, Z)\}\xi,} \tag{16}$$

$$P(X, Y)\xi = 0. \tag{17}$$

$$C(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}\{ g(Y, Z)X - g(X, Z)Y \}. \tag{18}$$

$$\begin{aligned} V(X, Y)Z \\ = R(X, Y)Z - \frac{1}{n-2}\{S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY\} \\ + \frac{r}{(n-1)(n-2)}\{g(Y, Z)X - g(X, Z)Y\}. \end{aligned} \tag{19}$$

3. W_2 – Curvature Tensor of a Kenmotsu Manifold

Pokhariyal and Mishra [10] have defined a new curvature tensor $'W_2$ as

$$'W_2(X, Y, Z, U) = 'R(X, Y, Z, U) + \frac{1}{n-1}[g(X, Z)S(Y, U) - g(Y, Z)S(X, U)], \tag{20}$$

for arbitrary vector fields X, Y, Z and U, where S is the Ricci tensor of type (0, 2) and

$'W_2(X, Y, Z, U) = g(W_2(X, Y)Z, U)$ and $'R(X, Y, Z, U) = g(R(X, Y)Z, U)$ called W_2-curvature tensor of M^n.

Proposition: *On an n- dimensional Kenmotsu manifold M^n,*

$$\eta(W_2(X, Y)Z) = 0.$$

Proof: From equation (20), we have

$$W_2(X, Y)Z = R(X, Y)Z + \frac{1}{n-1}[g(X, Z)QY - g(Y, Z)QX]. \tag{21}$$

Taking the inner product of above equation with ξ and using equations (10), (11) and (12), we get

$$\eta(W_2(X, Y)Z) = 0. \tag{22}$$

Theorem 1: *On an n- dimensional Kenmotsu manifold M^n,*

$R(\xi, X).W_2 = 0$ if and only if $W_2 = 0$.

Proof: Let on an n- dimensional Kenmotsu manifold $R(\xi, X).W_2 = 0$, then

$$R(\xi, X)W_2(Y, Z)U - W_2(R(\xi, X)Y, Z) U - W_2(Y, R(\xi, X)Z)U - W_2(Y, Z)R(\xi, X)U = 0. \tag{23}$$

From equations (9) and (23), we have

$$\begin{aligned} &\eta(W_2(Y, Z)U)X - 'W_2(Y, Z, U, X)\xi - \eta(Y)W_2(X, Z)U \\ &+ g(X, Y)W_2(\xi, Z)U - \eta(Z)W_2(Y, X)U + g(X, Z)W_2(Y, \xi)U \\ &- \eta(U)W_2(Y, Z)X + \mathbf{g(X, U)W_2(Y, Z)\xi} = 0. \end{aligned} \tag{24}$$

Taking the inner product of above equation with ξ, we get

$$\eta(W_2(Y, Z)U)\,\eta(X) - {}'W_2(Y, Z, U, X) - \eta(Y)\,\eta(W_2(X, Z)U)$$
$$+ g(X, Y)\,\eta(W_2(\xi, Z)U) - \eta(Z)\,\eta(W_2(Y, X)U)$$
$$+ g(X, Z)\,\eta(W_2(Y, \xi)U) - \eta(U)\,\eta(W_2(Y, Z)X) + \mathbf{g(X, U)\,\eta(W_2(Y, Z)\xi)} = 0, \quad (25)$$

which on using equation (22) gives

$$'W_2(Y, Z, U, X) = 0,$$

i.e. $W_2 = 0$.

Conversely, suppose $W_2 = 0$, then from equation (23), we have

$$R(\xi, X).W_2 = 0.$$

This completes the proof.

Theorem 2: *An n-dimensional Kenmotsu manifold M^n satisfying $W_2(\xi, X).R = 0$, is an Einstein manifold.*

Proof: Let $W_2(\xi, X).R = 0$, then we have

$$W_2(\xi, X).R(Y, Z)U - R(W_2(\xi, X)Y, Z)U - R(Y, W_2(\xi, X)Z)U$$
$$- \mathbf{R(Y, Z)\,W_2(\xi, X)U} = 0. \quad (26)$$

Now putting $X = \xi$ in equation (21) and using equations (9) and (12), we obtain

$$W_2(\xi, Y)Z = \eta(Z)[Y + \frac{1}{n-1}QY]. \quad (27)$$

Now from equations (26) and (27), we have

$$\eta(R(Y,Z)U)\{X + \frac{1}{n-1}QX\} - \eta(Y)\{R(X, Z)U + \frac{1}{n-1}R(QX, Z)U\}$$
$$- \eta(Z)\{R(Y, X)U + \frac{1}{n-1}R(Y, QX)U\} - \eta(U)\{R(Y, Z)X + \frac{1}{n-1}R(Y, Z)QX\}$$
$$= 0, \quad (28)$$

which on taking the inner product with ξ and using equations (4), (11) and (12) gives

$$\mathbf{-\eta(U)\{\eta(Z)g(X, Y) - \eta(Y)g(X, Z)\} - \frac{1}{n-1}\{\eta(Z)S(X, Y) - \eta(Y)S(X, Z)\}\eta(U) = 0.} \quad (29)$$

Putting U = Z = ξ in above equation and using equations (4) and (11), we get

$$\mathbf{S(X, Y) = (1-n)\, g(X, Y),} \quad (30)$$

which shows that M^n is an Einstein Manifold.

Theorem 3: *An n-dimensional Kenmotsu manifold M^n satisfying $P(\xi, X).W_2 = 0$, is an Einstein manifold.*

Proof: Let $P(\xi, X).W_2 = 0$, then

$$P(\xi, X)W_2(Y,Z)U - W_2(P(\xi, X)Y, Z)U - W_2(Y, P(\xi, X)Z)U - W_2(Y,Z)P(\xi, X)U = 0. \quad (31)$$

Using equations (11) and (16) in above equation, we have

$$\mathbf{-'W_2(Y, Z, U, X)\xi - \frac{1}{n-1}\,'W_2(Y, Z, U, QX)\xi}$$
$$+ \{g(X, Y) + \frac{1}{n-1}S(X, Y)\}W_2(\xi, Z)U$$
$$+ \{g(X, Z) + \frac{1}{n-1}S(X, Z)\}W_2(Y, \xi)U$$
$$+\{g(X, U) + \frac{1}{n-1}S(X, U)\}W_2(Y, Z)\xi = 0. \quad (32)$$

Now taking the inner product of above equation with ξ and using equation (22), we get

$$'W_2(Y, Z, U, QX) = (1-n)\,'W_2(Y, Z, U, X),$$

which on using equation (21), gives

$$g(QX, R(Y, Z)U) + \frac{1}{n-1}\{g(Y, U)g(QX, QZ) - g(Z, U)g(QX, QY)\}$$
$$= (1-n)[\, g(X, R(Y, Z)U) + \frac{1}{n-1}\{g(Y, U)g(X, QZ) - g(Z, U)g(X, QY)\}].$$

Putting Z = U = ξ in above equation and using equations (4), (8) and (11), we get

$$-S(X, QY) = 2(n-1)S(X, Y) + (n-1)^2 g(X, Y),$$

which on using equation (11), gives

$$S(X, Y) = (1-n)g(X, Y).$$

This completes the proof.

Theorem 4: ***An n-dimensional Kenmotsu manifold M^n satisfying $W_2(\xi, X).P = 0$, is an Einstein manifold.***

Proof: Let $W_2(\xi, X).P = 0$, then we have

$$W_2(\xi, X)P(Y, Z)U - P(W_2(\xi, X)Y, Z)U - P(Y, W_2(\xi, X)Z)U - P(Y, Z)W_2(\xi, X)U = 0, \quad (33)$$

which on using equation (27), gives

$$\eta(P(Y, Z)U)\{X + \frac{1}{n-1}QX\} - \eta(Y)\{P(X, Z)U + \frac{1}{n-1}P(QX, Z)U\}$$

$$- \eta(Z)\{ P(Y, X)U + \frac{1}{n-1} P(Y, QX)U\} - \eta(U)\{P(Y, Z)X + \frac{1}{n-1}P(Y, Z)QX\} = 0. \quad (34)$$

Now taking the inner product of above equation with ξ and using equation (11), we get

$$\eta(Y)\{ \eta(P(X, Z)U) + \frac{1}{n-1} \eta (P(QX, Z)U)\}$$

$$+ \eta(Z)\{ \eta(P(Y, X)U) + \frac{1}{n-1} \eta(P(Y, QX)U)\}$$

$$+ \eta(U)\{ \eta(P(Y, Z)X) + \frac{1}{n-1} \eta(P(Y, Z)QX)\} = 0. \quad (35)$$

Putting $U = Z = \xi$ in above equation and using equations (15), (16) and (17), we get

$$S(QX, Y) = (n-1)^2 g(X, Y),$$

which on using equation (11), gives

$$\mathbf{S(X, Y) = (1-n)g(X, Y).}$$

This completes the proof.

Theorem 5: *The W_2-curvature tensor and projective curvature tensor of the Riemannian manifold M^n are linearly dependent if and only if M^n is an Einstein Manifold.*

Proof: Let

$$W_2(X, Y)Z = \alpha P(X, Y)Z,$$

where α being any non-zero constant. In view of equations (15) and (21), above equation assumes the form

$$(1-\alpha) R(X, Y)Z + \frac{1}{n-1}\{g(X, Z)QY - g(Y, Z)QX\} + \frac{\alpha}{n-1}\{S(Y, Z)X - S(X, Z)Y\} = 0,$$

which can be written as

$$(1-\alpha)\, 'R(X, Y, Z, U) + \frac{1}{n-1}\{g(X, Z)S(Y, U) - g(Y, Z)S(X, U)\}$$
$$+ \frac{\alpha}{n-1}\{S(Y, Z)g(X, U) - S(X, Z)g(Y, U)\} = 0.$$

Now putting $X = U = e_i$ in above equation and taking summation over i, $1 \le i \le n$, we get

$$(1-\alpha) S(Y, Z) + \frac{1}{n-1}\{S(Y, Z) - r\, g(Y, Z)\} + \frac{\alpha}{n-1}\{n\, S(Y, Z) - S(Y, Z)\} = 0,$$

i.e. $$S(Y, Z) = \frac{r}{n} g(Y, Z) \Rightarrow QY = \frac{r}{n} Y,$$

which shows that M^n is an Einstein manifold.

Conversely, let M^n be an Einstein manifold, i.e. $S(Y, Z) = \frac{r}{n} g(Y, Z)$ and $QY = \frac{r}{n} Y$, then from equations (15) and (21), we have

$$W_2(X, Y)Z = \alpha P(X, Y)Z.$$

Theorem 6: *A necessary and sufficient condition for a Riemannian manifold M^n to be an Einstein manifold is that the W_2-curvature tensor and concircular curvature tensor C are linearly dependent.*

Proof: Let $W_2(X, Y)Z = \alpha C(X, Y)Z$,

where α being any non-zero constant. In consequence of equations (18) and (21), we have

$$(1-\alpha) R(X, Y)Z + \frac{1}{n-1}\{ g(X, Z)QY - g(Y, Z)QX \}$$

$$+ \frac{\alpha r}{n(n-1)} \{ g(Y, Z)X - g(X, Z)Y \} = 0,$$

from which, we get

$$(1-\alpha)\, 'R(X, Y, Z, U) + \frac{1}{n-1} \{ g(X, Z)S(Y, U) - g(Y, Z)S(X, U) \}$$

$$+ \frac{\alpha r}{n(n-1)} \{ g(Y, Z)g(X, U) - g(X, Z)g(Y, U) \} = 0.$$

Now putting $X = U = e_i$ in above equation and taking the summation over i, $1 \le i \le n$, we get

$$\frac{(n-1)(1-\alpha)+1}{(n-1)} S(Y, Z) + \frac{\{(n-1)\alpha - n\} r}{n(n-1)} g(Y, Z) = 0,$$

which can be written as

$$S(Y, Z) = \frac{k}{n} g(Y, Z),$$

where $k = \frac{\{(n-1)\alpha - n\} r}{\{(n-1)(\alpha-1)+1\}}$. This shows that Riemannian manifold is an Einstein manifold. Converse part is obvious from the equations (18) and (21).

Theorem 7: *A Riemannian manifold M^n becomes an Einstein manifold if and only if conformal curvature tensor and W_2-curvature tensor of the manifold are linearly dependent.*

Proof: Let $W_2(X, Y)Z = \alpha\, V(X, Y)Z$.

The above equation on straight forward calculations, gives

$$(1-\alpha)\, 'R(X, Y, Z, U) + \frac{1}{n-1} \{g(X, Z)\, S(Y, U) - g(Y, Z)\, S(X, U)\}$$

$$+ \frac{\alpha}{(n-2)} \{ S(Y, Z)\, g(X, U) - S(X, Z)\, g(Y, U) + g(Y, Z)S(X, U)$$

$$- g(X, Z)S(Y, U)\} - \frac{\alpha r}{(n-1)(n-2)} \{g(Y, Z)g(X, U) - g(X, Z)g(Y, U)\} = 0.$$

Now putting $X = U = e_i$ in above equation and taking summation over i, $1 \le i \le n$, we get

$$(1-\alpha)\, S(Y, Z) + \frac{1}{n-1}\{S(Y, Z) - r\, g(Y, Z)\}$$
$$+ \frac{\alpha}{(n-2)}\{n\, S(Y, Z) - S(Y, Z) + r\, g(Y, Z) - S(Y, Z)\}$$
$$- \frac{\alpha r}{(n-1)(n-2)}\{n\, g(Y, Z) - g(Y, Z)\} = 0,$$

which reduces to

$$S(Y, Z) = \frac{r}{n}\, g(Y, Z) \Rightarrow QY = \frac{r}{n} Y.$$

This shows that Riemannian manifold is an Einstein manifold. Converse part is obvious from equations (19) and (21).

Corollary: *In an n-dimensional Riemannian manifold M^n, the following statements are equivalent-*

(i) M^n is an Einstein manifold,

(ii) W_2- curvature tensor and projective curvatures are linearly dependent,

(iii) W_2- curvature tensor and concircular curvature tensors are linearly dependent,

(iv) W_2- curvature tensor and conformal curvature tensors are linearly dependent.

4. The Irrotational W_2-Curvature Tensor

Definition: Let ∇ be a Riemannian connection. The rotation (Curl) of W_2-curvature tensor on Riemannian manifold M^n is defined as

$$\text{Rot}W_2 = (\nabla_U W_2)(X, Y)Z + (\nabla_X W_2)(U, Y)Z$$
$$+ (\nabla_Y W_2)(X, U)Z - (\nabla_Z W_2)(X, Y)U. \tag{36}$$

In consequence of Bianchi's second identity for Riemannian connection ∇, equation (36) becomes

$$\text{Rot}W_2 = - (\nabla_Z W_2)(X, Y)U. \tag{37}$$

If the W_2-curvature tensor is irrotational, then curl $W_2 = 0$ and hence

$$(\nabla_Z W_2)(X, Y)U = 0,$$

which gives

$$\nabla_Z (W_2(X, Y)U) = W_2(\nabla_Z X, Y)U + W_2(X, \nabla_Z Y)U + W_2(X, Y)\nabla_Z U. \quad (38)$$

Theorem 8: *The W_2-curvature tensor in a Kenmotsu manifold M^n is irrotational if and only if*

$$R(X, Y)Z = g(X, Z)Y - g(Y, Z)X + \frac{1}{n-1}\{ \eta(X)(\nabla_Z Q)(Y) - \eta(Y)(\nabla_Z Q)(X) \}.$$

In particular, if $\eta(X)(\nabla_Z Q)(Y) = \eta(Y)(\nabla_Z Q)(X)$, then the manifold is locally isometric to the hyperbolic space $H^n(-1)$.

Proof: Let W_2-curvature tensor in M^n be irrotational then putting $U = \xi$ in equation (38), we get

$$\nabla_Z (W_2(X, Y)\xi) = W_2(\nabla_Z X, Y)\xi + W_2(X, \nabla_Z Y)\xi + W_2(X, Y)\nabla_Z \xi. \quad (39)$$

Putting $Z = \xi$ in equation (21) and using equations (4) and (8), we get

$$W_2(X, Y)\xi = \{ \eta(X)Y - \eta(Y)X\} + \frac{1}{n-1}\{ \eta(X)QY - \eta(Y)QX \}. \quad (40)$$

Using above equation in equation (39), we obtain

$$(\nabla_Z \eta)(X)Y - (\nabla_Z \eta)(Y)X + \frac{1}{n-1}\{(\nabla_Z \eta)(X)QY - (\nabla_Z \eta)(Y)QX$$

$$+ \eta(X)(\nabla_Z Q)(Y) - \eta(Y)(\nabla_Z Q)(X) \}$$

$$= W_2(X, Y)Z - \eta(Z)[\{ \eta(X)Y - \eta(Y)X\} + \frac{1}{n-1}\{ \eta(X)QY - \eta(Y)QX \}], \quad (41)$$

which on using equation (7) gives

$$g(X, Z)Y - g(Y, Z)X + \frac{1}{n-1}\{g(X, Z)QY - g(Y, Z)QX + \eta(X)(\nabla_Z Q)(Y)$$

$$- \eta(Y)(\nabla_Z Q)(X) \} = W_2(X, Y)Z. \quad (42)$$

Using equation (21) in above equation, we have

$$R(X, Y)Z = g(X, Z)Y - g(Y, Z)X + \frac{1}{n-1}\{ \eta(X)(\nabla_Z Q)(Y) - \eta(Y)(\nabla_Z Q)(X) \}. \quad (43)$$

Conversely, retreating the steps, we can show that W_2-curvature tensor is an irrotational.

Now if $\eta(X)(\nabla_Z Q)(Y) = \eta(Y)(\nabla_Z Q)(X)$, then equation (43) reduces to

$$R(X, Y)Z = -(g(Y, Z)X - g(X, Z)Y),$$

which shows that Kenmotsu manifold is locally isometric to hyperbolic space $H^n(-1)$.

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