Tamsui Oxford Journal of Mathematical Sciences **25(2)** (2009) 167-176
Aletheia University

A New Class Of Meromorphic Multivalent Functions Involving Certain Linear Operator*

S. P. Goyal†

Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India

and

J. K. Prajapat‡

Department of Mathematics, Sobhasaria Engineering College, Gokulpura, NH-11, Sikar-332001, Rajasthan, India

Received August 1, 2007, Accepted September 25, 2007.

Abstract

Making use of certain extended derivative operator of Ruscheweyh type, we introduce a new class $\mathcal{J}_p(\lambda, \mu, \alpha)$ of meromorphic multivalent function in the punctured disk $\mathbb{D} = \{z : \ z \in \mathbb{C}, \ 0 < |z| < 1\}$, and obtain some sufficient conditions for the functions belonging to this class.

Keywords and Phrases: *Meromorphic multivalent functions; Meromorphic starlike function; Meromorphic convex function; Meromorphic close-to-convex function; Hadamard product(or convolutions), Ruscheweyh derivative.*

---

*2000 *Mathematics Subject Classification.* Primary 30C45.

†E-mail: spgoyal@rediffmail.com
‡E-mail: jkp_0007@rediffmail.com

1. Introduction and Definitions

Let $\Sigma(p)$ denote the class of functions of the form

$$f(z) = z^{-p} + \sum_{k=1}^{\infty} a_k z^{k-p} \qquad (p \in \mathbb{N}), \tag{1.1}$$

which are analytic and $p-$valent in the punctured unit disk

$$\mathbb{D} = \{z : \ z \in \mathbb{C}, \ 0 < |z| < 1\}.$$

We denote by $\Sigma^*(p, \alpha)$, $\Sigma_k(p, \alpha)$ and $\Sigma_c(p, \alpha)$, the subclasses of the class $\Sigma(p)$, which are defined (for $0 \leq \alpha < p$, $p \in \mathbb{N}$) as follows:

$$\Sigma^*(p, \alpha) = \left\{ f : \ f \in \Sigma(p), \ \Re\left(-\frac{zf'(z)}{f(z)}\right) > \alpha \right\} \qquad (z \in \mathbb{D}), \tag{1.2}$$

$$\Sigma_k(p, \alpha) = \left\{ f : \ f \in \Sigma(p), \ \Re\left(-1 - \frac{zf''(z)}{f'(z)}\right) > \alpha \right\} \qquad (z \in \mathbb{D}), \tag{1.3}$$

and

$$\Sigma_c(p, \alpha) = \left\{ f : \ f \in \Sigma(p), \ \Re\left(-\frac{f'(z)}{z^{-p-1}}\right) > \alpha \ \right\} \qquad (z \in \mathbb{D}), \tag{1.4}$$

Note thet $\Sigma^*(p, \alpha)$, $\Sigma_k(p, \alpha)$ and $\Sigma_c(p, \alpha)$, are the well known subclasses of $\Sigma(p)$ consisting of meromorphic multivalent functions which are respectively starlike, convex and close-to-convex functions of order $\alpha (0 \leq \alpha < p)$. Furthermore $\Sigma^*(1, \alpha) = \Sigma^*(\alpha)$, $\Sigma_k(1, \alpha) = \Sigma_k(\alpha)$ and $\Sigma_c(1, \alpha) = \Sigma_c(\alpha)$, where $\Sigma^*(\alpha)$, $\Sigma_k(\alpha)$ and $\Sigma_c(\alpha)$ are subclasses of $\Sigma(1)$ consisting meromorphic univalent functions which are respectively starlike, convex and close-to-convex of order $\alpha (0 \leq \alpha < 1)$. We refer to Liu and Srivastava [2], Mogra [4], Raina and Srivastava [5] and Xu and Yang [8] for related work on the subject of meromorphic functions.

For $f(z) \in \Sigma(p)$ given by (1.1) and $g(z) \in \Sigma(p)$ given by

$$g(z) = z^{-p} + \sum_{k=1}^{\infty} b_k z^{k-p} \qquad (p \in \mathbb{N}), \tag{1.5}$$

the Hadamard product (or convolution) of f and g is defined by

$$(f * g)(z) = z^{-p} + \sum_{k=1}^{\infty} a_k \, b_k \, z^{k-p} = (f * g)(z). \tag{1.6}$$

The extended linear derivative operator of Ruscheweyh type for the functions belonging to the class $\Sigma(p)$

$$\mathcal{D}_*^{\lambda,p} \; : \; \Sigma(p) \to \Sigma(p),$$

is defined by the following convolution:

$$\mathcal{D}_*^{\lambda,p} f(z) = \frac{1}{z^p(1-z)^{\lambda+1}} * f(z) \qquad (\lambda > -1; \; f \in \Sigma(p)). \tag{1.7}$$

In terms of binomial coefficients, (1.7) can be written as

$$\mathcal{D}_*^{\lambda,p} f(z) = z^{-p} + \sum_{k=1}^{\infty} \binom{\lambda + k}{k} a_k z^{k-p} \qquad (\lambda > -1; f \in \Sigma(p)). \tag{1.8}$$

In particular when $\lambda = n$ ($n \in \mathbb{N}$), it is easily observed from (1.7) and (1.8) that

$$\mathcal{D}_*^{n,p} f(z) = \frac{z^{-p} \left(z^{n+p} f(z)\right)^{(n)}}{n!} \qquad (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}). \tag{1.9}$$

The definition (1.7) of linear operator $\mathcal{D}_*^{\lambda,p}$ is motivated essentially by familiar Ruscheweyh operator D^{λ}, which has been used widely on the space of analytic and univalent functions (see for details, [7]). A linear operator $D^{\lambda,p}$ analogous to $\mathcal{D}_*^{\lambda,p}$ (defined by (1.7)), was considered recently by Raina and Srivastava [6] on the space of analytic and $p-$valent functions in $\mathbb{U}$ ($\mathbb{U} = \mathbb{D} \cup \{0\}$). We remark in passing that a more general convolution operator then the operator $\mathcal{D}_*^{\lambda,p}$ considered recently by Liu and Srivastava [2].

By using the operator $\mathcal{D}_*^{\lambda,p}$ ($\lambda > -p$; $p \in \mathbb{N}$) given by (1.7), we now introduce a new class of meromorphically p-valent analytic functions defined as follows:

Definition 1. *A function $f(z) \in \Sigma(p)$, is said to be a member of the class $\mathcal{J}_p(\lambda, \mu, \alpha)$ if and only if*

$$\left| \frac{z^{p+1} \left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p \, \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} + p \right| < p - \alpha \tag{1.10}$$

$$(z \in \mathbb{D};\ p \in \mathbb{N};\ \lambda > -1;\ \mu \geq 0;\ 0 \leq \alpha < p).$$

Note that condition (1.10) implies that

$$\Re\left(-\frac{z^{p+1}\ \left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\ \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}}\right) > \alpha. \tag{1.11}$$

It is obvious that

$$\mathcal{J}_p(0,2,\alpha) = \Sigma^*(p,\alpha) \qquad \text{and} \qquad \mathcal{J}_p(0,1,\alpha) = \Sigma_c(p,\alpha).$$

The object in the present paper is to obtain some sufficient conditions of functions belonging to the above defined subclass $\mathcal{J}_p(\lambda,\mu,\alpha)$.

2. Main Results

In our present investigation of the function class $\mathcal{J}_p(\lambda,\mu,\alpha)$ $(\lambda > -1;\ \mu \geq 0;\ 0 \leq \alpha < p)$, we shall require the following Lammas.

Lemma 1(see, [1]). *Let the nonconstant function $w(z)$ be analytic in $\mathbb{U}$, with $w(0) = 0$. If $|w(z)|$ attains its maximum value on the circle $|z| = r < 1$ at a point $z_0 \in \mathbb{U}$, then*

$$z_0 w'(z_0) = k w(z_0),$$

where $k \geq 1$ is a real number.

Lemma 2 (see, [3]). *Let S be a set in the complex plane $\mathbb{C}$ and suppose that $\phi(z)$ is a mapping from $\mathbb{C}^2 \times \mathbb{U}$ to $\mathbb{C}$ which satisfies $\Phi(ix, y; z) \notin S$ for all $z \in \mathbb{U}$, and for all real x, y such that $y \leq -(1+x^2)/2$. If the function $q(z) = 1 + q_1 z + q_2 z^2 + \cdots$ is analytic in $\mathbb{U}$ such that $\phi(q(z),\ zq'(z);\ z) \in S$ for all $z \in \mathbb{U}$, then $\Re\,(q(z)) > 0$.*

Making use of Lemma 1, we first prove

Theorem 1. *Let $p \in \mathbb{N}$, $\gamma \geq 0$, $\lambda > -p$, $\mu \geq 0$ and $0 \leq \alpha < p$. If $f(z) \in \Sigma(p)$ satisfies the following inequality*

$$\left|1 + p + \frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'} - (\mu-1)\left(p + \frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\mathcal{D}_*^{\lambda,p} f(z)}\right) - \gamma\left(\frac{z^{p+1}\ \left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\ \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} + p\right)\right|$$

$$< \frac{(p-\alpha)(\gamma(2p-\alpha)+1)}{2p-\alpha}, \tag{2.1}$$

then $f(z) \in \mathcal{J}_p(\lambda, \mu, \alpha)$.

Proof. Define the function $w(z)$ by

$$\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} = -p + (\alpha - p) w(z), \tag{2.2}$$

then $w(z)$ is analytic in $\mathbb{U}$ and $w(0) = 0$. Differentiating logarithmically both sides of (2.2) with respect to z, we get

$$p+1+\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'} - (\mu-1)\left(p+\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\mathcal{D}_*^{\lambda,p} f(z)}\right) = \frac{(p-\alpha) z w'(z)}{p+(p-\alpha)w(z)}. \tag{2.3}$$

Now using (2.2) in (2.3), we find that

$$p+1+\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'} - (\mu-1)\left(p+\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\mathcal{D}_*^{\lambda,p} f(z)}\right) - \gamma\left(\frac{z^{p+1}\ \left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\ \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} + p\right)$$

$$= \gamma(p-\alpha)w(z) + \frac{(p-\alpha) z w'(z)}{p+(p-\alpha)w(z)}. \tag{2.4}$$

Let us suppose that there exist $z_0 \in \mathbb{U}$ such that

$$max_{|z|<|z_0|}\ |w(z)| = |w(z_0)| = 1,$$

and apply Lemma 1, we find that

$$z_0 w'(z_0) = k w(z_0) \qquad (k \geq 1) \tag{2.5}$$

writting $w(z) = e^{i\theta}$ $(0 \leq \theta < 2\pi)$ and setting $z = z_0$ in (2.4), we get

$$\left| p+1+\frac{z_0\left(\mathcal{D}_*^{\lambda,p} f(z_0)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z_0)\right)'} - (\mu-1)\left(p+\frac{z_0\left(\mathcal{D}_*^{\lambda,p} f(z_0)\right)'}{\mathcal{D}_*^{\lambda,p} f(z_0)}\right) - \gamma\left(\frac{z_0^{p+1}\ \left(\mathcal{D}_*^{\lambda,p} f(z_0)\right)'}{\left(z_0^p\ \mathcal{D}_*^{\lambda,p} f(z_0)\right)^{\mu-1}} + p\right) \right|$$

$$
\begin{aligned}
&= \left|\gamma(p-\alpha)e^{i\theta} + \frac{(p-\alpha)ke^{i\theta}}{p+(p-\alpha)e^{i\theta}}\right| \\
&\geq \Re\left(\gamma(p-\alpha) + \frac{(p-\alpha)k}{p+(p-\alpha)e^{i\theta}}\right) \\
&> \gamma(p-\alpha) + \frac{(p-\alpha)}{2p-\alpha} \\
&= \frac{(p-\alpha)(\gamma(2p-\alpha)+1)}{2p-\alpha},
\end{aligned}
$$

which contradicts our assumption (2.1). Therefore, we have $|w(z)| < 1$ in $\mathbb{U}$. Finally, we have

$$
\begin{aligned}
\left|\frac{z^{p+1}(\mathcal{D}_*^{\lambda,p} f(z))'}{(z^p \mathcal{D}_*^{\lambda,p} f(z))^{\mu-1}} + p\right| &= |(p-\alpha)w(z)| = (p-\alpha)|w(z)| \\
&< p-\alpha \qquad (z \in \mathbb{U}), \qquad (2.6)
\end{aligned}
$$

that is $f(z) \in \mathcal{J}_p(\lambda, \mu, \alpha)$. This proves the Theorem 1.

Theorem 2. *Let $p \in \mathbb{N}$, $\lambda > -p$, $\mu \geq 0$ and $0 \leq \delta < p$. If $f(z) \in \Sigma(p)$ satisfies the following inequality*

$$
\Re\left[\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\, \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}}\left(\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\, \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} + (\mu-1)\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\mathcal{D}_*^{\lambda,p} f(z)} - \frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'} - 1\right)\right]
$$

$$
> \delta\left(\delta + \frac{1}{2}\right) + \left(\delta(\mu-2) - \frac{1}{2}\right), \qquad (2.7)
$$

then $f(z) \in \mathcal{J}_p(\lambda,\ \mu,\ \delta)$.

Proof. Define the functions $q(z)$ by

$$
\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\left(z^p\, \mathcal{D}_*^{\lambda,p} f(z)\right)^{\mu-1}} = -\delta + (\delta - p)q(z), \qquad (2.8)
$$

then we see that $q(z) = 1 + q_1 z + q_2 z^2 + ...$ is analytic in $\mathbb{U}$. Now differentiating both sides of (2.8) with respect to z logarithmically, we get

$$
\delta + (p-\delta)q(z)\left(1 + \frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'} + (1-\mu)\frac{z\left(\mathcal{D}_*^{\lambda,p} f(z)\right)'}{\mathcal{D}_*^{\lambda,p} f(z)}\right)
$$

$$= (p-\delta)zq'(z) + p(\mu-2)[\delta + (p-\delta)q(z)]. \tag{2.9}$$

¿From (2.8) and (2.9) we have

$$-\frac{z^{p+1}\left(D_*^{\lambda,p}f(z)\right)'}{\left(z^p\ D_*^{\lambda,p}f(z)\right)^{\mu-1}}\left(1+\frac{z\left(\mathcal{D}_*^{\lambda,p}f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}+(1-\mu)\frac{z\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}{\mathcal{D}_*^{\lambda,p}f(z)}\right)$$
$$= (p-\delta)zq'(z) + p(\mu-2)[\delta + (p-\delta)q(z)]. \tag{2.10}$$

Again using (2.8) in (2.10), we find that

$$\left(\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}{\left(z^p\ \mathcal{D}_*^{\lambda,p}f(z)\right)^{\mu-1}}\right)^2-\frac{z^{p+1}\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}{\left(z^p\mathcal{D}_*^{\lambda,p}f(z)\right)^{\mu-1}}\left(1+\frac{z\left(\mathcal{D}_*^{\lambda,p}f(z)\right)''}{\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}+(1-\mu)\frac{z\left(\mathcal{D}_*^{\lambda,p}f(z)\right)'}{\mathcal{D}_*^{\lambda,p}f(z)}\right)$$

$$\begin{aligned} &= (p-\delta)zq'(z) + (p-\delta)^2q^2(z) + (p-\delta)[2\delta + p(\mu-2)]q(z) + p\delta(\mu-2) + \delta^2 \\ &= \phi(q(z), zq'(z); z), \end{aligned}$$

where

$$\phi(r,s,z) = (p-\delta)s + (p-\delta)^2r^2 + (p-\delta)[2\delta + p(\mu-2)]r + p\delta(\mu-2) + \delta^2. \tag{2.11}$$

For all real x, y satisfying $y \le -(1+x^2)/2$, we have

$$\begin{aligned} \Re\ (\phi(ix,y,z)) &= (p-\delta)y + (p-\delta)^2x^2 + p\ \delta(\mu-2) + \delta^2 \\ &\le -\frac{1}{2}(p-\delta)(1+x^2) - (p-\delta)^2x^2 + \delta p(\mu-2) + \delta^2 \\ &= -\frac{1}{2}(p-\delta) - (p-\delta)\left(\frac{1}{2}+p-\delta\right)x^2 + \delta p(\mu-2) + \delta^2 \\ &\le \delta p(\mu-2) + \delta^2 - \frac{1}{2}(p-\delta) \\ &= \delta\left(\delta+\frac{1}{2}\right) + p\left(\delta(\mu-2) - \frac{1}{2}\right). \end{aligned}$$

Let

$$S = \left\{w:\ \ \Re(w) > \delta\left(\delta+\frac{1}{2}\right) + p\left(\delta(\mu-2)-\frac{1}{2}\right)\right\},$$

then $\phi(q(z), zq'(z); z) \in S$ and $\phi(ix, y; z) \notin S$ for all real x and $y < -(1+x^2)/2$, $z \in \mathbb{U}$. By using Lemma 2, we have $\Re\ (q(z)) > 0$, that is $f(z) \in \mathcal{J}_p(\lambda,\ \mu,\ \delta)$. This proves the Theorem 2.

3. Some Consequences Of Main Results

Among various intersting and importent consequences of our Theorems 1 and 2 we mentioned here some of the corollaries relating subclasses $\Sigma^*(\alpha), \Sigma_c(\alpha), \Sigma^*$ and Σ_c which are easily deduciable from the main results.

Firstly, if we take $\lambda = 0$ and $\mu = \gamma = p = 1$, then Theorem 1 gives the following result

Corollary 1. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} - z^2 f'(z) + 1 \right| < \frac{(1-\alpha)(3-\alpha)}{2-\alpha} \qquad (0 \leq \alpha < 1), \tag{3.1}$$

then $f(z) \in \Sigma_c(\alpha)$.

Further setting $\alpha = 0$ in Corollary 1, we get

Corolary 2. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} - z^2 f'(z) + 1 \right| < \frac{3}{2}, \tag{3.2}$$

then $f(z) \in \Sigma_c$ $(\Sigma_c = \Sigma_c(0))$.

Again if we set $\lambda = \gamma = \alpha = 0$ and $\mu = p = 1$ in theorem 1, we get

Corolary 3. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} + 2 \right| < \frac{1}{2}, \tag{3.3}$$

then $f(z) \in \Sigma_c$.

Also for $\lambda = \delta = 0$ and $\mu = p = 1$, Theorem 2 gives

Corolary 4. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\Re \left[z^2 \{ f'(z)(z^2 f'(z) - 1) - zf''(z) \} \right] > -\frac{1}{2}, \tag{3.4}$$

then $f(z) \in \Sigma_c$.

Again on setting $\mu = 2,\ \gamma = p = 1$ and $\lambda = 0$ in Theorem 1, we get

Corollary 5. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} - \frac{2zf'(z)}{f(z)} \right| < \frac{(1-\alpha)(3-\alpha)}{2-\alpha} \qquad (0 \le \alpha < 1), \tag{3.5}$$

then $f(z) \in \Sigma^*(\alpha)$.

On further setting $\alpha = 0$ in Corollary 5, we get

Corollary 6. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} - \frac{2zf'(z)}{f(z)} \right| < \frac{3}{2}, \tag{3.6}$$

then $f(z) \in \Sigma^*$ $(\Sigma^* = \Sigma^*(0))$.

Also let $\mu = 2,\ p = 1$ and $\gamma = \alpha = \lambda = 0$ in Theorem 1, we have

Corollary 7. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\left| \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)} + 1 \right| < \frac{1}{2}, \tag{3.7}$$

then $f(z) \in \Sigma^*$.

For $\mu = 2,\ p = 1$ and $\delta = \lambda = 0$, Theorem 2 gives

Corollary 8. *If $f(z) \in \Sigma$ satisfies the following inequality*

$$\Re \left[\frac{zf'(z)}{f(z)} \left\{ \frac{2zf'(z)}{f(z)} - \frac{zf''(z)}{f'(z)} - 1 \right\} \right] > -\frac{1}{2}, \tag{3.8}$$

then $f(z) \in \Sigma^*$.

References

[1] Jack, I. S., Functions starlike and convex of order α, *J. Londan Math. Soc.* **3(2)**(1971), 469-474.

[2] Liu, J. L. and Srivastava, H. M., A linear operator and associated families of meromorphically multivalent functions, *J. Math. anal. appl.* **259**(2001), 566-581.

[3] Miller, S. S. and Mocanu, P. T., Differential subordinations and inequalities in the complex plane, *J. Differ. Equations.* **67**(1987), 199-211.

[4] Mogra, M. L., Hadamard product of certain meromorphic univalent functions, *J. Math. Anal. Appl.* **157**(1991), 10-16.

[5] Raina, R. K. and Srivastava, H. M., A new class of meromorphically multivalent functions with applications of generalized hypergeometric functions, *Math. Comput. Modelling* **43**(2006), 350-356.

[6] Raina, R. K. and Srivastava, H. M., Inclusion and neighborhoods properties of some analytic and multivalent functions, *J. Inequal. Pure and Appl. Math.* **7(1)**(2006), Article 5, 1-6 (electronic).

[7] Rusheweyh, S., New criteria for univalent functions, *Proc. Amer. Math. Soc.* **49**(1975), 109-115.

[8] Xu, N. and Yang, D., On starlikeness and close to convexity of certain meromorphic function, *J. Korean Soc. Math. Edus. Ser B : Pure appl. Math.* **10**(2003), 566-581.