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Stability of Generalized Lie (σ, τ)-Derivations*

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Abstract

Let A be a normed algebra, let σ and τ be two mappings on A and let M be an A-bimodule. A linear mapping $L : A \to M$ is called a generalized Lie (σ, τ)-derivation if

$$L([x, y]) = [L(x), y]_{\sigma,\tau} - [L(y), x]_{\sigma,\tau} + \sigma(x)m\tau(y) - \sigma(y)m\tau(x)$$

for some $m \in M$ and all $x, y \in A$, where $[x, y]_{\sigma,\tau}$ is $x\tau(y) - \sigma(y)x$ and $[x, y]$ is the commutator $xy - yx$ of elements x, y. If $m = 0$, then L is Lie (σ, τ)-derivation. In this paper we investigate the generalized Hyers–Ulam–Rassias stability of generalized Lie (σ, τ)-derivations. We also prove that if the center of A is zero, then every "approximate Lie (I, I)-derivation" is indeed a Lie (I, I)-derivation.

Keywords and Phrases: *Generalized Hyers–Ulam–Rassias stability, Normed algebra, Lie (σ, τ)-derivation, Generalized Lie (σ, τ)-derivation, Superstability.*

1. Introduction

The stability of functional equations was started in 1940 with a problem raised by S. M. Ulam [14]. In 1941 Hyers affirmatively solved the problem of S. M.

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Ulam in the context of Banach spaces. In 1950 T. Aoki [1] extended the Hyers' theorem. In 1978, Th. M. Rassias [11] formulated and proved the following Theorem:

Assume that E_1 and E_2 are real normed spaces with E_2 complete, $f : E_1 \to E_2$ is a mapping such that for each fixed $x \in E_1$ the mapping $t \mapsto f(tx)$ is continuous on $\mathbb{R}$, and let there exist $\varepsilon \geq 0$ and $p \in [0, 1)$ such that $\|f(x+y)-f(x)-f(y)\| \leq \varepsilon(\|x\|^p+\|y\|^p)$ for all $x, y \in E_1$. Then there exists a unique linear mapping $T : E_1 \to E_2$ such that $\|f(x)-T(x)\| \leq \varepsilon\|x\|^p/(1-2^{p-1})$ for all $x \in E_1$.

The inequality $\|f(x+y)-f(x)-f(y)\| \leq \varepsilon(\|x\|^p+\|y\|^p)$ has provided extensive influence in the development of what we now call Hyers–Ulam–Rassias stability of functional equations[3, 5, 8, 12, 13]. In 1994, a generalization of Rassias' theorem was obtained by Găvruta [4], in which he replaced the bound $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function.

Let A be a normed algebra, σ and τ two mappings on A and let M be an A-bimodule. A linear mapping $L : A \to M$ is called a Lie (σ, τ)-derivation if

$$L([x, y]) = [L(x), y]_{\sigma,\tau} - [L(y), x]_{\sigma,\tau}$$

for all $x, y \in A$, where $[x, y]_{\sigma,\tau}$ is $x\tau(y) - \sigma(y)x$ and $[x, y]$ is the commutator $xy - yx$ of elements x, y. Following [2] A linear mapping $L : A \to M$ is called a generalized Lie (σ, τ)-derivation if

$$L([x, y]) = [L(x), y]_{\sigma,\tau} - [L(y), x]_{\sigma,\tau} + \sigma(x)m\tau(y) - \sigma(y)m\tau(x)$$

for some $m \in M$ and all $x, y \in A$.

Also $Z(A) = \{z \in A|\ zx = xz \quad \forall x \in A\}$ denotes the center of algebra A. Throughout in this paper, A denotes a (not necessary unital) normed algebra and M is a Banach A-bimodule. We investigate the generalized Hyers–Ulam–Rassias stability of generalized Lie (σ, τ)-derivations by using some ideas of [6, 7, 9, 10]. We also prove that if the center of A is zero, then every "approximate Lie (I, I)-derivation" is indeed a Lie (I, I)-derivation.

2. Stability of Lie (σ, τ)-Derivations

In this section our aim is to establish the Hyers–Ulam–Rassias stability of Lie (σ, τ)-derivations.

Theorem 2.1. *Suppose $f : A \to M$ is a mapping with $f(0) = 0$ for which there exist mappings $g, h : A \to A$ with $g(0) = h(0) = 0$ and a function $\varphi : A \times A \times A \times A \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w) = \frac{1}{2}\sum_{n=0}^{\infty} 2^{-n}\varphi(2^n x, 2^n y, 2^n z, 2^n w) < \infty \tag{2.1}$$

$$\begin{aligned} &\|f(\lambda x + \lambda y + [z, w]) - \lambda f(x) - \lambda f(y) - [f(z), w]_{g,h} + [f(w), z]_{g,h}\| \\ \leq\ &\varphi(x, y, z, w) \end{aligned} \tag{2.2}$$

$$\|g(\lambda x + \lambda y) - \lambda g(x) - \lambda g(y)\| \leq \varphi(x, y, 0, 0) \tag{2.3}$$

$$\|h(\lambda x + \lambda y) - \lambda h(x) - \lambda h(y)\| \leq \varphi(x, y, 0, 0) \tag{2.4}$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and for all $x, y \in A$. Then there exist unique linear mappings σ and τ from A to A satisfying $\|g(x) - \sigma(x)\| \leq \widetilde{\varphi}(x, x, 0, 0)$ and $\|h(x) - \tau(x)\| \leq \widetilde{\varphi}(x, x, 0, 0)$, and there exists a unique Lie (σ, τ)-derivation $L : A \to M$ such that

$$\|f(x) - L(x)\| \leq \widetilde{\varphi}(x, x, 0, 0) \tag{2.5}$$

for all $x \in A$.

Proof. Put $\lambda = 1$ and $z = w = 0$ in (2.2) to obtain

$$\|f(x + y) - f(x) - f(y)\| \leq \varphi(x, y, 0, 0) \qquad (x, y \in A). \tag{2.6}$$

Fix $x \in A$. Replace y by x in (2.6) to get

$$\|f(2x) - 2f(x)\| \leq \varphi(x, x, 0, 0). \tag{2.7}$$

One can use the induction to show that

$$\|\frac{f(2^p x)}{2^p} - \frac{f(2^q x)}{2^q}\| \leq \frac{1}{2}\sum_{k=q}^{p-1} \frac{1}{2^k}\varphi(2^k x, 2^k x, 0, 0) \tag{2.8}$$

for all $x \in A$, and all $p > q \geq 0$. It follows

from the convergence of series (2.1) that the sequence $\{\frac{f(2^n x)}{2^n}\}$ is Cauchy. By the completeness of M, this sequence is convergent. Set

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}. \tag{2.9}$$

Putting $z = w = 0$ and replacing x, y by $2^n x$ and $2^n y$ in (2.2) respectively, and divide the both sides of the inequality by 2^n we get

$$\|2^{-n} f(2^n(\lambda x + \lambda y)) - 2^{-n}\lambda f(2^n x) - 2^{-n}\lambda f(2^n y)\| \leq \frac{1}{2^n}\varphi(2^n x, 2^n y, 0, 0)$$

.

Passing to the limit as $n \to \infty$ we obtain $L(\lambda x + \lambda y) = \lambda L(x) + \lambda L(y)$.

Put $q = 0$ in (2.8) to get

$$\|\frac{f(2^p x)}{2^p} - f(x)\| \leq \frac{1}{2}\sum_{k=0}^{p-1} \frac{1}{2^k}\varphi(2^k x, 2^k x, 0, 0) \tag{2.10}$$

for all $x \in A$.

Taking the limit as $p \to \infty$ we infer that

$$\|f(x) - L(x)\| \leq \tilde{\varphi}(x, x, 0, 0)$$

for all $x \in A$.

Next, let $\gamma \in \mathbb{C}$ ($\gamma \neq 0$) and let N be a positive integer number greater than $|\gamma|$. It is shown that there exist two numbers $\theta_1, \theta_2 \in \mathbb{T}$ such that $2\frac{\gamma}{N} = \theta_1 + \theta_2$. Since L is additive we have $L(\frac{1}{2}x) = \frac{1}{2}L(x)$ for all $x \in A$. Hence

$$\begin{aligned} L(\gamma x) &= L(\frac{N}{2} \cdot 2 \cdot \frac{\gamma}{N} x) = N L(\frac{1}{2} \cdot 2 \cdot \frac{\gamma}{N} x) = \frac{N}{2} L(2 \cdot \frac{\gamma}{N} x) \\ &= \frac{N}{2} L(\theta_1 x + \theta_2 x) = \frac{N}{2}(L(\theta_1 x) + L(\theta_2 x)) \\ &= \frac{N}{2}(\theta_1 + \theta_2) L(x) = (\frac{N}{2} \cdot 2 \cdot \frac{\gamma}{N}) L(x) = \gamma L(x) \end{aligned}$$

for all $x \in A$. Thus L is linear.

It is known that additive mapping L satisfying (2.5) is unique [4].

Similarly one can use (2.3)and (2.4) to show that there exist unique linear mappings σ and τ defined by $\lim_{n\to\infty} \frac{g(2^n x)}{2^n}$ and $\lim_{n\to\infty} \frac{h(2^n x)}{2^n}$, respectively.

Putting $x = y = 0$ and replacing z, w by $2^n z$ and $2^n w$ in (2.2) respectively, we obtain

$$\|f([2^n z, 2^n w]) - [f(2^n z), 2^n w]_{g,h} + [f(2^n w), 2^n z]_{g,h}\| \leq \varphi(0, 0, 2^n z, 2^n w),$$

then

$$\frac{1}{2^{2n}}\|f([2^n z, 2^n w]) - [f(2^n z), 2^n w]_{g,h} + [f(2^n w), 2^n z]_{g,h}\| \quad \leq \quad \frac{1}{2^n}\varphi(0, 0, 2^n z, 2^n w)$$

for all $z, w \in A$, whence

$$\begin{aligned} & \lim_{n\to\infty} \frac{1}{2^{2n}}\|f([2^n z, 2^n w]) - [f(2^n z), 2^n w]_{g,h} + [f(2^n w), 2^n z]_{g,h}\| \\ \leq & \lim_{n\to\infty} \frac{1}{2^n}\varphi(0, 0, 2^n z, 2^n w) \\ = & \ 0 \end{aligned}$$

Therefore

$$\begin{aligned} L([z, w]) = & = \lim_{n\to\infty} \frac{f(2^{2n}[z, w])}{2^{2n}} \\ & = \lim_{n\to\infty} \frac{f([2^n z, 2^n w])}{2^{2n}} \\ & = \lim_{n\to\infty} (\frac{[f(2^n z), 2^n w]_{g,h} - [f(2^n w), 2^n z]_{g,h}}{2^{2n}} \\ & = \lim_{n\to\infty} (\frac{f(2^n z)h(2^n w) - g(2^n w)f(2^n z) - f(2^n w)h(2^n z) + g(2^n z)f(2^n w)}{2^{2n}}) \\ & = \big(L(z)\tau(w) - \sigma(w)L(z)\big) - \big(L(w)\tau(z) - \sigma(z)L(w)\big) \\ & = [L(z), w]_{\sigma,\tau} - [L(w), z]_{\sigma,\tau} \end{aligned}$$

for each $z, w \in A$. Hence the linear mapping L is a Lie (σ, τ)-derivation. □

Remark 2.2. *In the previous theorem if g and h satisfy the following*

$$\|g(xy) - g(x)g(y)\| \leq \varphi(x, y) \tag{2.11}$$

$$\|h(xy) - h(x)h(y)\| \leq \varphi(x, y) \tag{2.12}$$

then the linear mappings σ and τ on A shall be homomorphisms.

Corollary 2.2. *Suppose $f : A \to M$ is a mapping with $f(0) = 0$ for which there exist mappings $g, h : A \to A$ with $g(0) = h(0) = 0$ and there exits $\beta \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned} & \|f(\lambda x + \lambda y + [z, w]) - \lambda f(x) - \lambda f(y) - [f(z), w]_{g,h} + [f(w), z]_{g,h}\| \\ \leq\ & \beta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p) \end{aligned}$$

$$\|g(\lambda x + \lambda y) - \lambda g(x) - \lambda g(y)\| \leq \beta(\|x\|^p + \|y\|^p)$$

$$\|h(\lambda x + \lambda y) - \lambda h(x) - \lambda h(y)\| \leq \beta(\|x\|^p + \|y\|^p)$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and for all $x, y \in A$. Then there exist unique linear mappings σ and τ from A to A satisfying $\|g(x) - \sigma(x)\| \leq \frac{\beta\|x\|^p}{1-2^{p-1}}$ and $\|h(x) - \tau(x)\| \leq \frac{\beta\|x\|^p}{1-2^{p-1}}$, and there exists a unique Lie (σ, τ)-derivation $L : A \to M$ such that

$$\|f(x) - L(x)\| \leq \frac{\beta\|x\|^p}{1 - 2^{p-1}} \tag{2.13}$$

for all $x \in A$.

Proof. Put $\varphi(x, y, z, w) = \beta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.1. □

In the next result we shall show that under certain assumptions an "approximate Lie (σ, τ)-derivations" is indeed a Lie (σ, τ)-derivations. This phenomenon is called the superstability of Lie (σ, τ)-derivations.

Corollary 2.3. *Suppose $f : A \to M$ is a mapping with $f(0) = 0$ and there exist $\beta \geq 0$ and $p \in [0, 1)$ such that*

$$\|f(\lambda x+\lambda y+[z, w])-\lambda f(x)-\lambda f(y)-[f(z), w]_{I,I}+[f(w), z]_{I,I}\| \leq \beta(\|x\|^p+\|y\|^p+\|z\|^p+\|w\|^p)$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and for all $x, y, z, w \in A$. If $Z(A) = \{0\}$, then f a Lie (I, I)-derivation.

Proof. Put $g = h = I$ in Corollary 2.2, then there is a unique Lie (I, I)-derivation $L : A \to M$ such that

$$\|L(x) - f(x)\| \leq \frac{\beta\|x\|^p}{1 - 2^{p-1}} \tag{2.14}$$

for all $x \in A$.

We will show that $L(x) = f(x)$ for all $x \in A$.

Let m, n be two nonnegative integers and let $x, y \in A$. Then

$$\begin{aligned}
& \|2^n\big([x, 2^m f(y)] - [x, f(2^m y)]\big)\| \\
= \ & \|[2^n x, 2^m f(y)] - [2^n x, f(2^m y)]\| \\
\leq \ & \|f[2^n x, 2^m y] - [f(2^n x), 2^m y] - [2^n x, f(2^m y)]\| \\
& + \|f[2^n x, 2^m y] - [f(2^n x), 2^m y] - [2^n x, 2^m f(y)]\| \\
\leq \ & \beta\big(\|2^n x\|^p + \|2^m y\|^p\big) + \|L[2^n x, 2^m y] - f[2^n x, 2^m y]\| \quad \text{(by Corollary 2.2)} \\
& + \|L[2^n x, 2^m y] - [f(2^n x), 2^m y] - [2^n x, 2^m f(y)]\| \\
\leq \ & \beta\big(\|2^n x\|^p + \|2^m y\|^p\big) + \beta\frac{\|[2^n x, 2^m y]\|^p}{1 - 2^{p-1}} \quad \text{(by Corollary 2.2)} \\
& + 2^m\|L[2^n x, y] - [f(2^n x), y] - [2^n x, f(y)]\| \quad \text{(by linearity } L) \\
\leq \ & \beta\big(\|2^n x\|^p + \|2^m y\|^p\big) + \beta\frac{\|[2^n x, 2^m y]\|^p}{1 - 2^{p-1}} + 2^m\big(\|L(2^n x, y] - f[2^n x, y]\| \\
& + \|f[2^n x, y] - [f(2^n x), y] - [2^n x, f(y)]\|\big) \\
\leq \ & \beta\big(\|2^n x\|^p + \|2^m y\|^p\big) + \beta\frac{\|[2^n x, 2^m y]\|^p}{1 - 2^{p-1}} \\
& + 2^m\big(\frac{\beta(\|[2^n x, y]\|^p)}{1 - 2^{p-1}} + \beta(\|2^n x\|^p + \|y\|^p\big)\,.
\end{aligned}$$

Fix m, and divide the both sides of the last inequality by 2^n and let n tends to infinity to obtain $\|[x, 2^m f(y)] - [x, f(2^m y)]\| \leq 0$ for all m and all $x, y \in A$. Hence by continuty norm and commutator we have $\|[x, \frac{f(2^m y)}{2^m} - f(y)]\| = 0$ for all m and for all $x, y \in A$. Letting m tends to infinity we get

$$[x, L(y) - f(y)] = 0 \quad (x, y \in A).$$

Hence $L(y) - f(y) \in Z(A) = \{0\}$ and so $L(y) = f(y)$ for all $y \in A$. Thus f is a Lie (I, I)-derivation. □

3. Generalized Lie (σ,τ)-Derivation

In this section we investigate the Hyers–Ulam–Rassias stability of generalized Lie (σ,τ)-derivation.

Theorem 3.1. *Suppose $f : A \to M$ is a mapping with $f(0) = 0$ for which there exist mappings $g, h : A \to A$ with $g(0) = h(0) = 0$ and a function $\varphi : A \times A \times A \times A \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w) = \frac{1}{2}\sum_{n=0}^{\infty} 2^n \varphi(2^{-n}x, 2^{-n}y, 2^{-n}z, 2^{-n}w) < \infty \tag{3.1}$$

$$\lim_{n\to\infty} 2^{2n}\varphi(2^{-n}x, 2^{-n}y, 2^{-n}z, 2^{-n}w) = 0$$

$$\|f(\lambda x + \lambda y + [z, w]) - \lambda f(x) - \lambda f(y) - [f(z), w]_{g,h} + [f(w), z]_{g,h} + g(z)mh(w) - g(w)mh(z)\| \leq \varphi(x, y, z, w) \tag{3.2}$$

$$\|g(\lambda x + \lambda y) - \lambda g(x) - \lambda g(y)\| \leq \varphi(x, y, 0, 0) \tag{3.3}$$

$$\|h(\lambda x + \lambda y) - \lambda h(x) - \lambda h(y)\| \leq \varphi(x, y, 0, 0) \tag{3.4}$$

for some $m \in M$ and for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $x, y, z, w \in A$. Then there exist unique linear mappings σ and τ from A to A satisfying $\|g(x)-\sigma(x)\| \leq \widetilde{\varphi}(x, x, 0, 0)$ and $\|h(x)-\tau(x)\| \leq \widetilde{\varphi}(x, x, 0, 0)$, and there exists a unique generalized Lie (σ, τ)-derivation $L : A \to M$ such that

$$\|f(x) - L(x)\| \leq \widetilde{\varphi}(x, x, 0, 0) \tag{3.5}$$

for all $x \in A$.

Proof. Put $\lambda = 1$ and $z = w = 0$ in (3.2).Fix $x \in A$. Replace both y and x by $x/2$ in (3.2) to get

$$\|f(x) - 2f(\frac{x}{2})\| \leq \varphi(\frac{x}{2}, \frac{x}{2}, 0, 0). \tag{3.6}$$

Using the same method as in the proof of Theorem 2.1 one can show that there is a unique linear mapping

$$L(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n}) \tag{3.7}$$

and there exist unique linear mappings σ and τ defined by $\lim_{n\to\infty} 2^n g(\frac{x}{2^n})$ and $\lim_{n\to\infty} 2^n h(\frac{x}{2^n})$, respectively, for all $x \in A$.

Putting $x = y = 0$, and replacing z, w by $2^{-n}z$ and $2^{-n}w$ in (3.2), respectively, we have

$$\begin{aligned} & \|f([2^{-n}z, 2^{-n}w]) - [f(2^{-n}z), 2^{-n}w]_{g,h} + [f(2^{-n}w), 2^{-n}z]_{g,h} \\ & +g(2^{-n}z)mh(2^{-n}w) - g(2^{-n}w)mh(2^{-n}z)\| \\ \leq \; & \varphi(0, 0, 2^{-n}z, 2^{-n}w)\|, \end{aligned}$$

whence

$$\begin{aligned} & 2^{2n}\|f([2^{-n}z, 2^{-n}w]) - [f(2^{-n}z), 2^{-n}w]_{g,h} + [f(2^{-n}w), 2^{-n}z]_{g,h} \\ & +g(2^{-n}z)mh(2^{-n}w) - g(2^{-n}w)mh(2^{-n}z)\| \\ \leq \; & 2^{2n}\varphi(0, 0, 2^{-n}z, 2^{-n}w) \end{aligned}$$

for all $z, w \in A$. Hence

$$\begin{aligned} L([z, w]) &= \lim_{n\to\infty} 2^{2n} f(\frac{[z, w]}{2^{2n}}) \\ &= \lim_{n\to\infty} 2^{2n} f([2^{-n}z, 2^{-n}w]) \\ &= \lim_{n\to\infty} 2^{2n}[f(2^{-n}z), 2^{-n}w]_{g,h} - [f(2^{-n}w), 2^{-n}z]_{g,h} \\ &+ (g(2^{-n}z)mh(2^{-n}w) - g(2^{-n}w)mh(2^{-n}z)) \\ &= [L(z), w]_{\sigma,\tau} - [L(w), z]_{\sigma,\tau} + \sigma(z)m\tau(w) - \sigma(w)m\tau(z) \end{aligned}$$

for all $z, w \in A$ and $m \in M$. □

Corollary 3.2. *Suppose $f : A \to M$ is a mapping with $f(0) = 0$ for which there exist mappings $g, h : A \to A$ with $g(0) = h(0) = 0$ and there exist $\beta \geq 0$ and $p > 1$ such that*

$$\begin{aligned} & \|f(\lambda x + \lambda y + [z, w]) - \lambda f(x) - \lambda f(y) - [f(z), w]_{g,h} + [f(w), z]_{g,h} \\ & +g(z)mh(w) - g(w)mh(z)\| \leq (\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p) \end{aligned} \tag{3.8}$$

$$\|g(\lambda x + \lambda y) - \lambda g(x) - \lambda g(y)\| \leq \beta(\|x\|^p + \|y\|^p) \tag{3.9}$$

$$\|h(\lambda x + \lambda y) - \lambda h(x) - \lambda h(y)\| \leq \beta(\|x\|^p + \|y\|^p) \tag{3.10}$$

for some $m \in M$ and for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $x, y, z, w \in A$. Then there exist unique linear mappings σ and τ from A to A satisfying

$\|g(x)-\sigma(x)\| \le \frac{\beta\|x\|^p}{1-2^{1-p}}$ and $\|h(x)-\tau(x)\| \le \frac{\beta\|x\|^p}{1-2^{1-p}}$, and there exists a unique generalized Lie (σ,τ)-derivation $L : A \to M$ such that

$$\|f(x)-L(x)\| \le \frac{\beta\|x\|^p}{1-2^{1-p}} \tag{3.11}$$

for all $x \in A$.

Proof. Put $\varphi(x,y,z,w) = \beta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 3.1. □

References

[1] T. Aoki, On the stability of the linear transformation in Banach spaces, *J. Math. Soc. Japan,* **2**(1950) 64–66.

[2] N. Argac and E. Albas, On generalized of (σ,τ)-derivations, *Siberian Math. Journal*, **43** (2002), 977–984.

[3] S. Czerwik (ed.), *Stability of Functional Equations of Ulam–Hyers–Rassias Type*, Hadronic Press, 2003.

[4] P. Găvruta, A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings, *J. Math. Anal. Appl.*, **184** (1994), 431–436.

[5] S.-M. Jung, *Hyers–Ulam–Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic press, Palm Harbor, Florida, 2001.

[6] M. S. Moslehian, Hyers–Ulam–Rassias stability of generalized derivations, *Internat J. Math. Math. Sci.*, **2006** (2006), 93942, 1-8.

[7] M. S. Moslehian, Almost derivations on C^*-ternary rings, *Bull. Belg. Math. Soc.*, **14** (2007), 135–142.

[8] M. S. Moslehian, Ternary derivations stability and physical aspects, *Acta Applicandae Math.*, **100** (2008), 187–199.

[9] C. Park, Lie *-homomorphisms between Lie C^*-algebras and Lie $*$-derivations on Lie C^*-algebras, *J. Math. Anal. Appl.*, **293** (2004), 419–434.

[10] C. Park, An. Jong Su and Cui. Jianlian, Isomorphisms and derivations in Lie C^*-algebras, *Abstr. Appl. Anal.*, **85737** (2007), pp.14.

[11] Th. M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.*,**72** (1978), 297–300.

[12] Th. M. Rassias, On the stability of functional equations and a problem of Ulam, *Acta Appl. Math.*, **62** (2000), 23–130.

[13] Th. M. Rassias (ed.), *Functional Equations and Inequalities*, Kluwer Academic Publishers, Dordrecht, Boston, London, 2000.

[14] S. M. Ulam, *Problems in modern mathematics*, Chapter VI,Science Editions, Wiley, New York, 1964.