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Hyers-Ulam-Rassias Stability of the Apollonius Type Quadratic Mapping in Non–Archimedean Spaces*

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Abstract

In this paper we establish Hyers–Ulam–Rassias stability of the following quadratic mapping of Apollonius type

$$Q(z-x)+Q(z-y)=\frac{1}{2}Q(x-y)+2Q\Big(z-\frac{x+y}{2}\Big) \tag{1}$$

in non-Archimedean spaces. The term Hyers–Ulam–Rassias stability is due to the seminal paper: Th. M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.* **72** (1978) 297–300.

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1. Introduction

The stability problem of functional equations originated from a question of S.M. Ulam [25] concerning the stability of group homomorphisms: Let G_1 be a group and let G_2 be a metric group with the metric $d(.,.)$. Given $\epsilon > 0$, does there exist a $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? In other words, we are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it. If we turn our attention to the case of functional equations, we can ask the question: When the solutions of an equation differing slightly from a given one must be close to the true solution of the given equation. For Banach spaces the Ulam problem was first solved by D.H. Hyers [8] in 1941, which states that if $\delta > 0$ and $f : X \to Y$ is a mapping with X, Y Banach spaces, such that

$$\Big\|f(x+y) - f(x) - f(y)\Big\|_Y \le \delta \tag{2}$$

for all $x, y \in X$, then there exists a unique additive mapping $T : X \to Y$ such that

$$\Big\|f(x) - T(x)\Big\|_Y \le \delta$$

for all $x \in X$. Th.M. Rassias [21] succeeded in extending the result of Hyers by weakening the condition for the Cauchy difference to be unbounded. A number of mathematicians were attracted to this result of Th.M. Rassias and stimulated to investigate the stability problems of functional equations. The stability phenomenon that was introduced and proved by Th.M. Rassias in his 1978 paper is called the *Hyers–Ulam stability.* G.L. Forti [4] and P. Găvruta [7] have generalized the result of Th.M. Rassias, which permitted the Cauchy difference to become arbitrary unbounded. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem. A large list of references can be found, for example, in the papers $[2, 5, 6, 9-19, 21, 22, 23]$. Now, a square norm on an inner product space satisfies the important parallelogram equality

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

for all vectors x, y. The following functional equation, which was motivated by this equation,

$$Q(x+y) + Q(x-y) = 2Q(x) + 2Q(y), \tag{3}$$

is called a *quadratic functional equation,* and every solution of equation (3) is said to be a *quadratic mapping.*

F. Skof [24] proved the Hyers–Ulam stability of the quadratic functional equation (3) for mappings $f : E_1 \to E_2$, where E_1 is a normed space and E_2 is a Banach space. C. Borelli and G.L. Forti [3] generalized the stability result as follows: let G be an abelian group, E a Banach space. Assume that a mapping $f : G \to E$ satisfies the functional inequality

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \le \varphi(x, y)$$

for all $x, y \in G$, and $\varphi : G \times G \to [0, \infty)$ is a function such that

$$\phi(x, y) := \sum_{i=0}^{\infty} \frac{1}{4^{i+1}} \varphi(2^i x, 2^i y) < \infty$$

for all $x, y \in G$. Then there exists a unique quadratic mapping $Q : G \to E$ with the properties

$$\|f(x) - Q(x)\| \le \phi(x, x)$$

for all $x \in G$. Jun and Lee [13] proved the Hyers–Ulam stability of the Pexiderized quadratic equation

$$f(x+y) + g(x-y) = 2h(x) + 2k(y)$$

for mappings f, g, h and k.

In an inner product space, the equality

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{x+y}{2}\right\|^2 \tag{4}$$

holds, and is called the *Apollonius' identity.* The following functional equation, which was motivated by this equation,

$$Q(z-x) + Q(z-y) = \frac{1}{2}Q(x-y) + 2Q\Big(z - \frac{x+y}{2}\Big), \tag{5}$$

is quadratic (see [20]). For this reason, the functional equation (5) is called a *quadratic functional equation of Apollonius type,* and each solution of the functional equation (5) is said to be a *quadratic mapping of Apollonius type.* The quadratic functional equation and several other functional equations are useful to characterize inner product spaces [1].

In [20], C. Park and Th.M. Rassias introduced and investigated a functional equation, which is called the *generalized Apollonius type quadratic functional equation.*

The first author [17], introduced a general version of (5), which is called the *quadratic functional equation of n–Apollonius type.*

We recall some basic facts concerning non-Archimedean spaces and preliminary results.

Definition 1.1. *A field $\mathbb{K}$ equipped with a function(valuation) $|.|$ from $\mathbb{K}$ into $[0,\infty)$ is called a non-Archimedean field if the function $|.| : \mathbb{K} \to [0,\infty)$ satisfies the following conditions:*

1. $|r| = 0$ *if and only if* $r = 0$;
2. $|rs| = |r||s|$;
3. $|r + s| \le \max\{|r|, |s|\}$;

for all $r, s \in \mathbb{K}$.

Clearly $|1| = |-1| = 1$ and $|n| \le 1$ for all $n \in \mathbb{N}$.

Definition 1.2. *Let X be a vector space over scaler field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|.|$. A function $\|.\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:*

$(1)'$ $\|x\| = 0$ *if and only if* $x = 0$;

$(2)'$ $\|rx\| = |r|\|x\|$;

$(3)'$ *the strong triangle inequality; namely,*

$$\|x + y\| \le \max\{\|x\|, \|y\|\}$$

for all $x, y \in X$ and for all $r \in \mathbb{K}$.

The pair $(X, \|.\|)$ is called a *non-Archimedean space* if $\|.\|$ is non–Archimedean norm on X. It follows from $(3)'$ that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j \leq n-1\}$$

for all $x_n, x_m \in X$ and all $m, n \in \mathbb{N}$ with $n > m$. Therefore a sequence $\{x_n\}$ is a Cauchy sequence in non-Archimedean space $(X, \|.\|)$, if and only if, the sequence $\{x_{n+1} - x_n\}$ converges to zero in $(X, \|.\|)$. In a complete non–Archimedean space every Cauchy sequence is convergent.

M.S. Moslehian and Th.M. Rassias in [15] investigated stability of Cauchy functional equation $f(x+y) = f(x) + f(y)$ and quadratic functional equation $f(x+y) + f(x-y) = 2f(x) + 2f(y)$ in non-Archimedean spaces.

In this paper we establish Hyers–Ulam–Rassias stability of the quadratic functional equation of Apollonius type (5) in non-Archimedean spaces.

2. Hyers-Ulam-Rassias Stability

Lemma 2.1. *Let X and Y be real vector spaces. A function $Q : X \to Y$ satisfies (5) if and only if Q is quadratic.*

Proof. (*Necessity*). Letting $x = y = z = 0$ in (5), we get that $Q(0) = 0$. Putting $y = -x$ and $z = x$ in (5), we get $Q(2x) = 4Q(x)$ for all $x \in X$. So by letting $y = -x$ in (5), we have

$$Q(z - x) + Q(z + x) = 2Q(x) + 2Q(z) \tag{6}$$

for all $x, z \in X$. So the mapping $Q : X \to Y$ is quadratic.

(*Sufficiency*). It follows from (6) that Q is even. So (6) implies that

$$Q[(z - x) + (z - y)] + Q(x - y) = 2Q(z - x) + 2Q(z - y) \tag{7}$$

for all $x, y, z \in X$. Since $Q(2x) = 4Q(x)$ for all $x \in X$, hence (7) implies that the function Q satisfies (5). □

Throughout this section, let X be a normed space with norm $\|.\|_X$ and Y be a complete non-Archimedean space with norm $\|.\|_Y$.

For a given a mapping $Q : X \to Y$, we define $DQ : X^3 \to Y$ by

$$DQ(x, y, z) := Q(z - x) + Q(z - y) - \frac{1}{2}Q(x - y) - 2Q\Big(z - \frac{x + y}{2}\Big).$$

Theorem 2.2. *Let $\varphi : X^3 \to [0,\infty)$ be a function such that*

$$\lim_{n\to\infty} |4|^n \varphi\Big(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}\Big) = 0 \tag{8}$$

for all $x, y, z \in X$, and let for all $x \in X$ the limit

$$\lim_{n\to\infty} \max\left\{ |4|^j \varphi\Big(\frac{x}{2^j}, -\frac{x}{2^j}, \frac{x}{2^j}\Big) : 0 < j \le n \right\} \tag{9}$$

denoted by $\widetilde{\varphi}(x)$, exists. Suppose that a function $Q : X \to Y$ with $Q(0) = 0$ satisfies the inequality

$$\|DQ(x,y,z)\|_Y \le \varphi(x,y,z) \tag{10}$$

for all $x, y \in X$. Then the limit

$$T(x) := \lim_{n\to\infty} 4^n Q\big(\frac{x}{2^n}\big) \tag{11}$$

exists for all $x \in X$ and $T : X \to Y$ is a quadratic function satisfying

$$\|Q(x) - T(x)\|_Y \le \frac{1}{|2|}\widetilde{\varphi}(x) \tag{12}$$

for all $x \in X$. Moreover, if

$$\lim_{n\to\infty} \lim_{k\to\infty} \max\left\{ |4|^j \varphi\big(\frac{x}{2^j}, -\frac{x}{2^j}, \frac{x}{2^j}\big) : n < j \le n+k \right\} = 0, \tag{13}$$

then T is the unique quadratic mapping satisfying (12).

Proof. Letting $y = -x$ and $z = x$ in (10), we get

$$\|Q(2x) - 4Q(x)\|_Y \le |2|\varphi(x, -x, x) \tag{14}$$

for all $x \in X$. If we replace x in (14) by $\frac{x}{2^n}$ and multiply both sides of (14) to $|4|^{n-1}$, then we have

$$\Big\|4^n Q\big(\frac{x}{2^n}\big) - 4^{n-1} Q\big(\frac{x}{2^{n-1}}\big)\Big\|_Y \le \frac{|4|^n}{|2|}\varphi\big(\frac{x}{2^n}, -\frac{x}{2^n}, \frac{x}{2^n}\big) \tag{15}$$

for all $x \in X$ and all non-negative integers n. It follows from (8) and (15) that the sequence $\{4^n Q(\frac{x}{2^n})\}$ is a Cauchy sequence in Y for all $x \in X$. Since Y is complete, the sequence $\{4^n Q(\frac{x}{2^n})\}$ converges for all $x \in X$.

It follows from (15) that

$$\begin{aligned}\Big\|4^nQ\big(\frac{x}{2^n}\big)-4^mQ\big(\frac{x}{2^m}\big)\Big\|_Y &= \Big\|\sum_{i=m}^{n-1}\Big[4^{i+1}Q\big(\frac{x}{2^{i+1}}\big)-4^iQ\big(\frac{x}{2^i}\big)\Big]\Big\|_Y\\ &\le \max\Big\{\ \big\|4^{i+1}Q\big(\frac{x}{2^{i+1}}\big)-4^iQ\big(\frac{x}{2^i}\big)\big\|_Y : m\le i<n\Big\}\\ &\le \frac{1}{|2|}\max\Big\{|4|^i\varphi\big(\frac{x}{2^i},-\frac{x}{2^i},\frac{x}{2^i}\big) : m<i\le n\Big\}\end{aligned}$$

for all $x\in X$ and non-negative integers m,n with $n>m\ge 0$. Letting $m=0$ and passing the limit $n\to\infty$ in the last inequality and using (9) and (11), we get (12). Now, we show that T is quadratic. It follows from (8), (10) and (11) that

$$\begin{aligned}\|DT(x,y,z)\|_Y &= \lim_{n\to\infty}|4|^n\Big\|DQ\big(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}\big)\Big\|_Y\\ &\le \lim_{n\to\infty}|4|^n\varphi\big(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}\big)=0\end{aligned}$$

for all $x,y,z\in X$. Therefore the mapping $T:X\to Y$ satisfies (5). So by Lemma 2.1 we infer that the mapping $T:X\to Y$ is quadratic.

To prove the uniqueness of T, let $U:X\to Y$ be another quadratic mapping satisfying (12). Since

$$\widetilde{\varphi}\big(\frac{x}{2^n}\big)=\frac{1}{|4|^n}\lim_{k\to\infty}\max\Big\{\ |4|^j\varphi\big(\frac{x}{2^j},\frac{-x}{2^j},\frac{x}{2^j}\big) : n<j\le n+k\Big\}$$

for all $x\in X$, then it follows from (11), (12) and (13) that

$$\begin{aligned}\|T(x)-U(x)\|_Y &= \lim_{n\to\infty}|4|^n\Big\|Q\big(\frac{x}{2^n}\big)-U\big(\frac{x}{2^n}\big)\Big\|_Y\\ &\le \frac{1}{|2|}\lim_{n\to\infty}|4|^n\widetilde{\varphi}\Big(\frac{x}{2^n}\Big)\\ &= \frac{1}{|2|}\lim_{n\to\infty}\lim_{k\to\infty}\max\Big\{\ |4|^j\varphi\big(\frac{x}{2^j},\frac{-x}{2^j},\frac{x}{2^j}\big) : n<j\le n+k\Big\}=0\end{aligned}$$

for all $x\in X$. So $T=U$. □

Theorem 2.3. *Let $\psi:X^3\to[0,\infty)$ be a function such that*

$$\lim_{n\to\infty}\frac{1}{|4|^n}\psi(2^nx,2^ny,2^nz)=0 \tag{16}$$

for all $x, y, z \in X$, and let for all $x \in X$ the limit

$$\lim_{n\to\infty} \max\left\{ \frac{1}{|4|^j}\psi(2^j x, -2^j x, 2^j x) : 0 \le j < n \right\} \tag{17}$$

denoted by $\widetilde{\psi}(x)$, exists. Suppose that a function $Q : X \to Y$ with $Q(0) = 0$ satisfies the inequality

$$\|DQ(x, y, z)\|_Y \le \psi(x, y, z) \tag{18}$$

for all $x, y \in X$. Then the limit

$$T(x) := \lim_{n\to\infty} \frac{1}{4^n} Q(2^n x) \tag{19}$$

exists for all $x \in X$ and $T : X \to Y$ is a quadratic function satisfying

$$\|Q(x) - T(x)\|_Y \le \frac{1}{|2|}\widetilde{\psi}(x) \tag{20}$$

for all $x \in X$. Moreover, if

$$\lim_{n\to\infty} \lim_{k\to\infty} \max\left\{ \frac{1}{|4|^j}\psi(2^j x, -2^j x, 2^j x) : n \le j < n + k \right\} = 0, \tag{21}$$

then T is the unique quadratic mapping satisfying (20).

Proof. Letting $y = -x$ and $z = x$ in (18), we get

$$\|Q(2x) - 4Q(x)\|_Y \le |2|\psi(x, -x, x) \tag{22}$$

for all $x \in X$. If we replace x in (22) by $2^n x$ and divide both sides of (22) by $|4|^{n+1}$, then we have

$$\left\| \frac{1}{4^{n+1}} Q(2^{n+1}x) - \frac{1}{4^n} Q(2^n x) \right\|_Y \le \frac{1}{|2|.|4|^n}\psi(2^n x, -2^n x, 2^n x) \tag{23}$$

for all $x \in X$ and all non-negative integers n. It follows from (16) and (23) that the sequence $\{\frac{1}{4^n}Q(2^n x)\}$ is a Cauchy sequence in Y for all $x \in X$. Since Y is complete, the sequence $\{\frac{1}{4^n}Q(2^n x)\}$ converges for all $x \in X$.

It follows from (23) that

$$\begin{aligned}\Big\|\frac{1}{4^n}Q(2^nx)-\frac{1}{4^m}Q(2^mx)\Big\|_Y &= \Big\|\sum_{i=m}^{n-1}\Big[\frac{1}{4^{i+1}}Q(2^{i+1}x)-\frac{1}{4^i}Q(2^ix)\Big]\Big\|_Y\\ &\le \max\Big\{\Big\|\frac{1}{4^{i+1}}Q(2^{i+1}x)-\frac{1}{4^i}Q(2^ix)\Big\| : m\le i<n\Big\}\\ &\le \frac{1}{|2|}\max\Big\{\frac{1}{|4|^i}\psi(2^ix,-2^ix,2^ix) : m\le i<n\Big\}\end{aligned}$$

for all $x\in X$ and all non-negative integers m,n with $n>m\ge 0$. Letting $m=0$ and passing the limit $n\to\infty$ in the last inequality and using (17) and (19), we obtain (20).

The rest of the proof is similar to the proof of Theorem 2.2. □

Corollary 2.4. *Let $\omega:[0,\infty)\to[0,\infty)$ be a function satisfying*

$$\omega(|2|^{-1}t)\le\omega(|2|^{-1})\omega(t)\quad (t\ge 0),\qquad \omega(|2|^{-1})<|2|^{-r}$$

where r is a fixed real number in $(-\infty,2]$. Let $\theta>0$ and let $Q:X\to Y$ be a function with $Q(0)=0$ and satisfying the inequality

$$\|DQ(x,y,z)\|_Y\le\theta\big[\omega(\|x\|)+\omega(\|y\|)+\omega(\|z\|)\big] \tag{24}$$

for all $x,y,z\in X$. Then there exists a unique quadratic mapping $T:X\to Y$ such that

$$\|Q(x)-T(x)\|_Y\le 3\,\theta\,|2|^{1-r}\omega(\|x\|)$$

for all $x\in X$.

Proof. Let $\varphi:X^3\to[0,\infty)$ defined by

$$\varphi(x,y,z)=\theta\big[\omega(\|x\|)+\omega(\|y\|)+\omega(\|z\|)\big].$$

Since $|4|\omega(|2|^{-1})<|2|^{2-r}\le 1$, we have

$$\lim_{n\to\infty}|4|^n\varphi\big(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}\big)\le\lim_{n\to\infty}|4|^n\omega(|2|^{-1})^n\varphi(x,y,z)=0,$$

for all $x,y,z\in X$. Also

$$\widetilde{\varphi}(x)=\lim_{n\to\infty}\max\Big\{|4|^j\varphi\big(\frac{x}{2^j},\frac{-x}{2^j},\frac{x}{2^j}\big) : 0<j\le n\Big\}=|4|\varphi\big(\frac{x}{2},\frac{-x}{2},\frac{x}{2}\big)$$

and

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\Big\{\,|4|^j\varphi\big(\frac{x}{2^j},\frac{-x}{2^j},\frac{x}{2^j}\big):n<j\le n+k\,\Big\}$$
$$=\lim_{n\to\infty}|4|^{n+1}\varphi\big(\frac{x}{2^{n+1}},\frac{-x}{2^{n+1}},\frac{x}{2^{n+1}}\big)=0$$

for all $x\in X$. Hence the result follows by Theorem 2.2. □

Corollary 2.5. *Let $\omega:[0,\infty)\to[0,\infty)$ be a function satisfying*

$$\omega(|2|t)\le\omega(|2|)\omega(t)\quad(t\ge 0),\qquad \omega(|2|)<|2|^r$$

where r is a fixed real number in $[2,\infty)$. Let $\theta>0$ and let $Q:X\to Y$ be a function with $Q(0)=0$ and satisfying the inequality (24) for all $x,y,z\in X$. Then there exists a unique quadratic mapping $T:X\to Y$ such that

$$\|Q(x)-T(x)\|_Y\le\frac{3\,\theta}{|2|}\omega(\|x\|)$$

for all $x\in X$.

Proof. The proof is similar to the proof of Corollary 2.4 and the result follows from Theorem 2.3. □

Corollary 2.6. *Let $\omega:[0,\infty)\to[0,\infty)$ be a function satisfying*

$$\omega(|2|^{-1}t)\le\omega(|2|^{-1})\omega(t)\quad(t\ge 0),\qquad \omega(|2|^{-1})<|2|^{-r}$$

where r is a fixed real number in $(-\infty,\frac{2}{3}]$. Let $\theta>0$ and let $Q:X\to Y$ be a function with $Q(0)=0$ and satisfying the inequality

$$\|DQ(x,y,z)\|_Y\le\theta\,\omega(\|x\|)\omega(\|y\|)\omega(\|z\|)\tag{25}$$

for all $x,y,z\in X$. Then there exists a unique quadratic mapping $T:X\to Y$ such that

$$\|Q(x)-T(x)\|_Y\le\theta\,|2|^{1-3r}\omega(\|x\|)^3$$

for all $x\in X$.

Proof. Define $\varphi:X^3\to[0,\infty)$ by $\varphi(x,y,z)=\theta\,\omega(\|x\|)\omega(\|y\|)\omega(\|z\|)$. Since $|4|\omega(|2|^{-1})^3<|2|^{2-3r}\le 1$, we have

$$\lim_{n\to\infty}|4|^n\varphi\big(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}\big)\le\lim_{n\to\infty}|4|^n\omega(|2|^{-1})^{3n}\varphi(x,y,z)=0,$$

for all $x, y, z \in X$. Also

$$\widetilde{\varphi}(x) = \lim_{n\to\infty} \max\Big\{ |4|^j \varphi\big(\frac{x}{2^j}, \frac{-x}{2^j}, \frac{x}{2^j}\big) : 0 < j \leq n \Big\} = |4|\varphi\big(\frac{x}{2}, \frac{-x}{2}, \frac{x}{2}\big)$$

and

$$\lim_{n\to\infty} \lim_{k\to\infty} \max\Big\{ |4|^j \varphi\big(\frac{x}{2^j}, \frac{-x}{2^j}, \frac{x}{2^j}\big) : n < j \leq n+k \Big\}$$
$$= \lim_{n\to\infty} |4|^{n+1}\varphi\big(\frac{x}{2^{n+1}}, \frac{-x}{2^{n+1}}, \frac{x}{2^{n+1}}\big) = 0$$

for all $x \in X$. Hence the result follows by Theorem 2.2. □

Corollary 2.7. *Let $\omega : [0, \infty) \to [0, \infty)$ be a function satisfying*

$$\omega(|2|t) \leq \omega(|2|)\omega(t) \quad (t \geq 0), \qquad \omega(|2|) < |2|^r$$

where r is a fixed real number in $[\frac{2}{3}, \infty)$. Let $\theta > 0$ and let $Q : X \to Y$ be a function with $Q(0) = 0$ and satisfying the inequality (25) for all $x, y, z \in X$. Then there exists a unique quadratic mapping $T : X \to Y$ such that

$$\|Q(x) - T(x)\|_Y \leq \frac{\theta}{|2|}\omega(\|x\|)^3$$

for all $x \in X$.

Proof. The proof is similar to the proof of Corollary 2.6 and the result follows from Theorem 2.3. □

Remark 2.8. *If we let $\omega(0) = 0$ in Corollaries 2.6 and 2.7, then (by letting $z = 0$ and replacing x and y by $-x$ and $-y$ in (25), respectively) we have*

$$Q(x) + Q(y) = \frac{1}{2}Q(y - x) + 2Q\big(\frac{x + y}{2}\big) \tag{26}$$

for all $x, y \in X$. It is clear that (26) implies that $Q(0) = 0$. Letting $x = 0$ in (26), we get that $4Q(y/2) = Q(y)$ for all $y \in X$. So (26) means the following equality

$$Q(y - x) + Q(y + x) = 2Q(x) + 2Q(y)$$

for all $x, y \in X$. Therefore Q is a quadratic function.

Remark 2.9. *The classical example of the function ω in Corollaries 2.4 and 2.6 (respectively, Corollaries 2.5 and 2.7) is the mapping $\omega(t) = t^p$ $(t \geq 0)$, where $p < r$ (respectively, $p > r$) with the further assumption that $|2| < 1$.*

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