Completely 2-primal Ideals in Near-rings*

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Abstract

We introduce the notion of completely 2-primal ideals in near-rings. We have also introduced strongly completely 2-primal near-rings. An ideal I of a near-ring N is said to be completely 2-primal if the prime radical of I, $P(I)$ is completely prime. We have obtained equivalent conditions for an ideal I to be completely 2-primal.

Keywords and Phrases: *2-primal, Completely 2-primal, Strongly completely 2-primal, Prime radical.*

1. Introduction

Throughout this paper N denotes a zero-symmetric near-ring with identity. An ideal P of N is called prime if for any two ideals A and B of N, $AB \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. An ideal P of N is called completely prime if $ab \in P$ implies $a \in P$ or $b \in P$, for any $a,\ b \in N$. An ideal P of N is called completely semiprime if $a^2 \in P$ implies $a \in P$, for any $a \in N$. Given a near-ring N, the intersection of all prime ideals called the prime radical of N is denoted by $P(N)$ and the set of all nilpotent elements is denoted by $N(N)$. $P(I)$ denotes the prime radical of I which is the intersection of all prime ideals containing $I$and $P_c(I)$ denotes the completely prime radical of I which is the intersection

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of all completely prime ideals containing I. An ideal I of a near-ring N is called 2-primal if $P(N/I) = N(N/I)$. N is called 2-primal if the zero ideal of N is 2-primal (Equivalently $P(N) = N(N)$).

An ideal I of N is said to have the insertion of factors property (IFP) if $xy \in I$ implies $xNy \subseteq I$ for $x,\ y \in N$. An ideal I of N has the strict IFP if $xy \in I$ implies $< x > N < y >\subseteq I$ for $x,\ y \in N$. In a ring IFP implies strict IFP but in a near-ring IFP does not imply strict IFP. An ideal I of N is called right (left) symmetric if $xyz \in I$ implies $xzy \in I(yxz \in I)$. An ideal I of N is said to be semi symmetric if for any $x \in N,\ x^n \in I$ for some positive integer n implies $< x >^n\subseteq I$. For an ideal I of N, $\sqrt{I} = \{x \in N | x^n \in I$ for some positive integer $n\}$. If A and B are ideals of N then $(A : B) = \{x \in N | xB \subseteq A\}$ is an ideal of N. A near-ring N is called regular if for every $a \in N$, there exists an $x \in N$ such that $a = axa$. A near-ring N is called a domain if every nonzero element is not a zero divisor.

An ideal I of a near-ring N is called completely 2-primal if $P(I)$ is completely prime. A near-ring N is called completely 2-primal if the zero ideal is completely 2-primal i.e, $P(N)$ is completely prime. It is obvious that domains are completely 2-primal and completely 2-primal near-rings are 2-primal. We use $P_c(N)$ for the intersection of all completely prime ideals of a near-ring N, and define $C(P(I)) = \{n \in N | n + P(I)$ is not a zero divisor $N/P(I)\}$. Birkenmeier-Heatherly-Lee [2] have proved that an ideal I of a near-ring N is 2-primal if and only if $P(I)$ is completely semiprime.

If I is an ideal of a near-ring N and P is a prime ideal of N containing I, then

$N_I(P) = \{a \in N | aN < b >\subseteq P(I)$ for some $b \in N \backslash P\}$;
$N_{IP} = \{a \in N | ab \in P(I)$ for some $b \in N \backslash P\}$;
$\overline{N}_{IP} = \{a \in N | a^m \in N_P$ for some positive integer $m\}$.
If the context is clear we write $N(P)$ (N_P, $\overline{N}_P$) instead of $N_I(P)(N_{IP},\overline{N}_{IP})$

If $I = 0$, we have defined $N(P)$ as $N_I(P) = \{a \in N | aN < b >\subseteq P(N)$ for some $b \in N \backslash P\}$. For rings Kwang-Ho kang, Byung-Ok Kim, Sang-Jig Nam, and Su-Ho Sohn [4] defined $N(P) = \{a \in R | aRb \subseteq P(R)$ for some $b \in R \backslash P\}$. These two definitions coincide in rings. If R is a ring and if $aRb \subseteq P(R)$ then we have $b \in (P(R) : aR)_r = \{x \in R | aRx \subseteq P(R)\}$. Since R is a ring, $(P(R) : aR)_r$ is an ideal. Hence $< b >\subseteq (P(R) : aR)_r$. Thus $aR < b >\subseteq P(R)$.

Kwang-Ho kang, Byung-Ok Kim, Sang-Jig Nam, and Su-Ho Sohn [4] have obtained equivalent conditions for a ring to be completely 2-primal. We have extended these results to near-rings. We have also obtained more equivalent conditions for a near-ring to be completely 2-primal. For basic notations and terminology we refer to Pilz [6].

Lemma 1. *Let I be an ideal of N. Then the following are equivalent:*
(1) I is 2-primal.
(2) $P(I) = \sqrt{I}$

Proof. (1) $\Rightarrow$ (2) Clearly $P(I) \subseteq \sqrt{I}$. Let $x \in \sqrt{I}$. Then $x^n \in I$ for some positive integer n. Now $x^n + I = I$ implies $x + I \in N(R/I) = P(R/I)$ as I is 2-primal. Let P be an arbitrary prime ideal in N containing I. Then $x + I \in P + I$ as $P + I$ is a prime ideal in R/I. Hence $x \in P$ and thus $x \in P(I)$.
(2) $\Rightarrow$ (1) Clearly $P(N/I) \subseteq N(N/I)$. Let $x + I \in N(N/I)$. Then there exists a positive integer n such that $x^n \in I$. Hence $x \in \sqrt{I}$. Thus $x + I \in P(N/I)$.□

Lemma 2. *If I is an ideal of a near-ring N and P is a prime ideal of N containing I then $N(P) \subseteq P,\ N(N) \subseteq \overline{N}_P$ and $N(P) \subseteq N_P \subseteq \overline{N}_P$.*

Proof. Let $x \in N(P)$. Then $xN < b > \subseteq P(I)$ for some $b \in N \backslash P$. Since N has identity, $x < b > \subseteq P(I) \subseteq P$ for some $b \in N \backslash P$. Hence $x \in (P :< b >)$. Since $(P :< b >)$ is an ideal, $< x > \subseteq (P :< b >)$. Thus $< x >< b > \subseteq P$. Since P is prime, we have $< x > \subseteq P$. Thus $x \in P$.

Let $x \in N(N)$. Then $x^n = 0$ for some positive integer n. Hence $x^n b = 0$ for every $b \in N \backslash P$. Hence $x \in \overline{N}_P$.
Let $x \in N(P)$. Then $xN < b > \subseteq P(I)$ for some $b \in N \backslash P$. Therefore $x < b > \subseteq P(I)$ and hence $xb \in P(I)$. Thus $x \in N_P$. And $x \in N_P$ implies $x \in \overline{N}_P$. □

Lemma 3. *For any ideal I of N, if $ab \in I$ implies $ba \in I$ for any $a,\ b \in N$, then $(I : S)$ is an ideal for any subset S of N.*

Theorem 4. *If I is a 2-primal ideal and P is a prime ideal of N then $N(P) = \cap\{Q$-prime ideal of N containing $I : Q \subseteq P\}$.*

Proof. Let Q be a prime ideal of N containing I such that $Q \subseteq P$. Then $N(P) \subseteq N(Q) \subseteq Q$. So we have $N(P) \subseteq \cap\{Q$-prime ideal of N containing $I : Q \subseteq P\}$. Suppose $a \notin N(P)$. We shall find a prime ideal Q of N containing I such that $a \notin Q$ and $Q \subseteq P$. If $S = \{a,\ a^2,\ a^3, \ldots\}$ then S is a multiplicative system that does not contain 0. Clearly $L = N \backslash P$ is an m-system. Let T be the set of all non-zero elements of N of the form $a^{t_0} x_1 a^{t_1} x_2 a^{t_2} \ldots x_n a^{t_n}$, where $x_i \in L$ and $t_i \in \{0\} \cup Z^+$, where Z^+ is the set of all positive integers. Let $M = S \cup T$. Note that $L \subseteq T$. We claim that M is an m-system. If $x,\ y \in S$ then $xay \in S$. Let $x \in S,\ y \in T$ with $x = a^s,\ y = a^{t_0}x_1a^{t_1}x_2a^{t_2}\ldots x_na^{t_n}$. If $xay \neq 0$ then $xay \in T$. Suppose $xay = 0$. Since $x_1,\ x_2 \in L$ then there exists $x_1' \in <x_1>$ and there exists $x_2' \in <x_2>$ such that $x_1'x_2' \in L$. Since $x_1'x_2', x_3 \in L$ there exists $x_{12}' \in < x_1'x_2' > \subseteq << x_1 >< x_2 >>$ and there exists $x_3' \in < x_3 >$ such that $x_{12}'x_3' \in L$. Continuing this process we arrive at $x_{123\ldots n-2}'x_{n-1}',\ x_n \in L$. Now there exists $x_{123\ldots n-1}' \in < x_{123\ldots n-2}'x_{n-1}' > \subseteq < \ldots <<< x_1 >< x_2 >< x_3 >> \ldots < x_{n-1} >>$ and there exists $x_n' \in < x_n >$ such that $w = x_{123\ldots n-1}'x_n' \in L$. Since $xay = 0,\ xay \in P(I)$. This implies $a^saa^{t_0}x_1a^{t_1}x_2a^{t_2}\ldots x_na^{t_n} \in P(I)$. Since I is 2-primal, $P(I)$ is completely semiprime and hence $x_1x_2\ldots x_na^{1+s+t_0+\ldots+t_n} \in P(I)$. Choose $m = 1+s+t_0+\ldots+t_n$. Then $x_1x_2\ldots x_na^m \in P(I)$. By Lemma 3, $< x_1 >< x_2 > \ldots < x_n >< a^m > \subseteq P(I)$. Again $<< x_1 >< x_2 >>< x_3 > \ldots < x_n >< a^m > \subseteq P(I)$. Again applying Lemma 3, $<<< x_1 >< x_2 >>< x_3 >>< x_4 > \ldots < x_n >< a^m > \subseteq P(I)$. Continuing this process we arrive at $< \ldots <<< x_1 >< x_2 >>< x_3 >> \ldots < x_{n_1} >>< x_n >< a^m > \subseteq P(I)$ and so $x_{123\ldots n-1}x_n'a^m \in P(I)$. Hence $wa^m \in P(I)$. Since $P(I)$ is completely semiprime, we have $(aw)^m \in P(I)$ and hence $aw \in P(I)$. Therefore $a \in N_P = N(P)$, a contradiction. Similarly if $x,\ y \in T$ then $xay \neq 0$ and $xay \in T$. This shows that M is an m-system that is disjoint from (0). Hence there is a prime ideal Q that is disjoint from M. Then $a \notin Q$ and $Q \subseteq P$, completing the proof. □

Theorem 5. *I is 2-primal if and only if every minimal prime ideal of N containing I is completely prime.*

Proof. Assume that I is 2-primal. Let P be a minimal prime ideal of N containing I and let M be the multiplicative subsemigroup of N generated by $N\backslash P$. We claim that $P(I) \cap M = \phi$. If not choose an element $y \in P(I) \cap M$. Since $y \in M$, there exists $x_1, x_2, \ldots, x_n \in N\backslash P$ such that $y = x_1x_2\ldots x_n \in P(I)$. Since I is 2-primal, $< x_1 >< x_2 > \ldots < x_n > \subseteq P(I) \subseteq P$. Thus $< x_i > \subseteq P$ for some i.

Hence $x_i \in P$ for some i which contradicts our assumption. Let $K = \{J | J$ is an ideal of N containing I and $J \cap M = \phi\}$. K is not empty as $P(I) \in K$. By Zorn's lemma K contains a maximal element say Q. Hence $Q \subseteq N\backslash M$. Now we show that Q is prime. Otherwise there exists ideals A and B such that $AB \subseteq Q,\ A \nsubseteq Q,\ B \nsubseteq Q$. Choose $y \in A\backslash Q$ and $z \in B\backslash Q$. By the maximality of Q, we have $(Q+ < y >) \cap M \neq \phi$ and $(Q+ < z >) \cap M \neq \phi$. Let $a \in (Q+ < y >) \cap M$ and $b \in (Q+ < z >) \cap M$. Then $ab \in M$. Let

$a = p + r$ and $b = q + s$ for some $p,\ q \in Q$ and $r \in < y >$ and $s \in < z >$. Then $ab = (p + r)(q + s) = p(q + s) + r(q + s) - rs + rs \in Q$, for $rs \in < y >< z > \subseteq AB \subseteq Q$. Thus $ab \in Q \cap M$, a contradiction. Hence Q is a prime ideal. Now $P(I) \subseteq Q \subseteq N \backslash M \subseteq P$. By the minimality of P we have $Q = N \backslash M = P$. Since M is a multiplicative semigroup, P is a completely prime ideal.

Conversely assume that every minimal prime ideal of N containing I is completely prime. Clearly $P(N/I) \subseteq N(N/I)$. Let $x + I \in N(N/I)$. Then $x^n \in I$ for some positive integer n. Let $P + I$ be a minimal prime ideal in N/I. Then P is a minimal prime ideal of N containing I and hence $x \in P$. Thus $x + I \in P_m(N/I)$ which is the intersection of all minimal prime ideals in N/I. Since the intersection of all minimal prime ideals in N/I coincides with the intersection of all prime ideals in $N/I,\ x + I \in P(N/I)$. Hence I is 2-primal. □

Theorem 6. *Given an ideal I of a near-ring N the following conditions are equivalent:*

1. *I is completely 2-primal;*
2. *I is 2-primal and $P_c(I)$ is completely prime;*
3. *Every minimal prime ideal of N containing I and $P_c(I)$ are both completely prime;*
4. *$C(P(I)) = N \backslash P(I)$;*
5. *$N(P) = N_P = \overline{N}_P = P(I)$ for any minimal prime ideal P of N containing I;*
6. *$N(P) = N_P = \overline{N}_P = P(I)$ for any minimal completely prime ideal P of N containing I.*

Proof. (1) ⇒ (2) Clearly, I is 2-primal. Therefore $P(I) = P_c(I)$. Since I is completely 2-primal, $P(I)$ is completely prime and hence $P_c(I)$ is completely prime.

(2) ⇒ (1) Since I is 2-primal, $P(I)$ is completely prime. Therefore I is completely 2-primal.

(2) ⇒ (3) Since I is 2-primal by Theorem 5, every minimal prime ideal of N containing I is completely prime.

(3) ⇒ (2) Again using Theorem 5, we have I is 2-primal.

(1) $\Leftrightarrow$ (4) is a restatement of the definition.

(1) $\Rightarrow$ (5) Let P be a minimal prime ideal of N containing I. Clearly $N(P) \subseteq N_P \subseteq \overline{N}_P$. Let $a \in \overline{N}_P$. Then $a^n b \in P(I)$ for some $b \in N \backslash P$ and for a positive integer n. Since $P(I)$ is completely prime $a \in P(I)$. Thus $a \in N(P)$. Hence $N(P) = N_P = \overline{N}_P$. By Theorem 4, $N(P) = P$. Since $P(I)$ is completely prime, $P(I)$ is the unique minimal prime ideal of N containing I and thus $N(P) = N_P = \overline{N}_P = P(I)$.

(5) $\Rightarrow$ (6) Let $a + I \in N(N/I)$. Then $a^n \in I$, for some positive integer n. Thus $a \in \overline{N}_P = N_P = N(P) = P(I)$ for any minimal prime ideal P of N containing I. Hence I is 2-primal. So every minimal prime ideal of N containing I is completely prime by Theorem 5. Thus every minimal completely prime ideal of N containing I is a minimal prime ideal of N containing I.

(6) $\Rightarrow$ (1) Let P be a minimal completely prime ideal of N containing I. Hence I is 2-primal and so any minimal completely prime ideal containing I is a minimal prime ideal containing I by [Theorem, 5]. So P is a minimal prime ideal of N containing I and $N(P) = P$. Hence we have $P = N(P) = N_P = \overline{N}_P = P(I)$ proving that I is completely 2-primal. □

Corollary 7 (4, Proposition 1)**.** *Given a ring R the following conditions are equivalent:*

1. *R is completely 2-primal;*
2. *R is 2-primal and $P_c(R)$ is completely prime;*
3. *Every minimal prime ideal of R and $P_c(R)$ are both completely prime;*
4. *$C(P(R)) = R \backslash P(R)$;*
5. *$N(P) = N_P = \overline{N}_P = P(R)$ for any minimal prime ideal P of R;*
6. *$N(P) = N_P = \overline{N}_P = P(R)$ for any minimal completely prime ideal P of R.* □

Theorem 8. *Let I be an ideal of a near-ring N. Then the following are equivalent:*

1. *I is completely 2-primal.*
2. *$P(I)$ is a left and right symmetric ideal of N and $P(I)$ is prime.*

3. *$xy \in P(I)$ implies $yNx \subseteq P(I)$ for every $x, y \in N$ and $P(I)$ is prime.*

4. *$xy \in P(I)$ implies $yx \in P(I)$ for every $x,\ y \in N$ and $P(I)$ is prime.*

5. *$P(I)$ has the strong IFP and $P(I)$ is prime.*

6. *$x_1x_2x_3 \ldots x_n \in P(I)$ implies $< x_1 >< x_2 >< x_3 > \ldots < x_n >\subseteq P(I)$ for all $x_1, x_2, x_3, \ldots, x_n \in N$ and $P(I)$ is prime.*

7. *$P(I)$ is a semi-symmetric ideal of N and $P(I)$ is prime.*

Proof. (1) $\Rightarrow$ (2) Clearly, $P(I)$ is prime. If $xyz \in P(I)$, then $(zxy)^2 = zxyzxy \in P(I)$ and so $zxy \in P(I)$. This implies that $(xyxz)^2 \in P(I)$. Thus $xyxz \in P(I)$ and $(yxzyx)^2 \in P(I)$ and hence $yxzyx \in P(I)$. Therefore $(xzy)^3 \in P(I)$ and so $xzy \in P(I)$. Moreover $xzy \in P(I)$ implies $(yxz)^2 \in P(I)$ and so $yxz \in P(I)$. Therefore $P(I)$ is left and right symmetric.

(2) $\Rightarrow$ (3) Let $xy \in P(I)$. Since N has an identity, we have $yx \in P(I)$. Let $n \in N$. Now $nyx \in P(I)$. Since $P(I)$ is left symmetric $ynx \in P(I)$. Therefore $yNx \subseteq P(I)$.

(3) $\Rightarrow$ (4) is obvious.

(4) $\Rightarrow$ (5) If $xy \in P(I)$ then by Lemma 3, $< x > y \subseteq P(I)$. Hence $y < x >\subseteq P(I)$. Again by Lemma 3, $< y >< x >\subseteq P(I)$. Hence $< x >< y >\subseteq P(I)$ and so $< x > N < y >\subseteq< x >< y >\subseteq P(I)$.

(5) $\Rightarrow$ (6) Let $a^2 \in P(I)$. Since $P(I)$ has the strong IFP and N has identity, we have $< a >^2\subseteq P(I)$. Since $P(I)$ is prime $< a >\subseteq P(I)$ and so $a \in P(I)$. If $xy \in P(I)$ then $(yx)^2 = yxyx \in P(I)$ and so $yx \in P(I)$. Assume $x_1x_2 \ldots x_n \in P(I)$. Then $x_1 \in (P(I) : x_2x_3 \ldots x_n)$. Hence by Lemma 3, $< x_1 >\subseteq (P(I) : x_2x_3 \ldots x_n)$ and $< x_1 > x_2x_3 \ldots x_n \subseteq P(I)$. Hence $x_2x_3 \ldots x_n < x_1 >\subseteq P(I)$ and $x_2 \in (P(I) : x_3 \ldots x_n < x_1 >)$. So $< x_2 >\in (P(I) : x_3 \ldots x_n < x_1 >)$ and $< x_2 > x_3 \ldots x_n < x_1 >\subseteq P(I)$. Hence $x_3 \ldots x_n < x_1 >< x_2 >\subseteq P(I)$. Proceeding in this manner, we get that $< x_1 >< x_2 > \ldots < x_n >\subseteq P(I)$.

(6) $\Rightarrow$ (7)is obvious.

(7) $\Rightarrow$ (6) Let $ab \in P(I)$. Then $(ba)^2 = baba \in P(I)$. Hence $< ba >^2\subseteq P(I)$. Since $P(I)$ is prime $< ba >\subseteq P(I)$ and hence $ba \in P(I)$. Thus $x_1x_2x_3 \ldots x_n \in P(I)$ implies $< x_1 >< x_2 >< x_3 > \ldots < x_n >\subseteq P(I)$.

(6) $\Rightarrow$ (1) Let $ab \in P(I)$. Then $< a >< b >\subseteq P(I)$. Since $P(I)$ is prime, $< a >\subseteq P(I)$ or $< b >\subseteq P(I)$. Hence $a \in P(I)$ or $b \in P(I)$, showing that I is completely 2-primal. □

Definition 9. *A near-ring N is said to be strongly completely 2-primal if N is completely 2-primal and it has only one non zero idempotent.*

Note: The notion of strongly completely 2-primal and completely 2-primal coincide in rings.

Completely 2-primal near-rings need not be strongly completely 2-primal as the following example shows.

Example 10. Let $N = \{0,\ a,\ b,\ c\}$ be the Klein's four group. Define multiplication in N as follows.

.	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	a	b	b
c	0	a	b	c

Then $(N,\ +,\ .)$ is a near-ring (see Pilz [6, p. 408] scheme 8). N is completely 2-primal and has two nonzero idempotents, b and c.

Theorem 11. *N is strongly completely 2-primal and regular if and only if N is a near-field.*

Proof. Assume that N is strongly completely 2-primal and regular. Let $a \in N$. Then there exists $x \in N$ such that $a = axa$. Clearly xa and ax are nonzero idempotents and therefore $xa = ax = 1$, showing that N is a near-field. The converse is obvious. □

Corollary 12. *Let R be a ring. Then R is completely 2-primal and regular if and only if R is a division ring.*

Note: The above corollary will fail for near-rings as the following example shows.

Example 13. Let $N = \{0,\ a,\ b,\ c\}$ be the Klein's four group. Define multiplication in N as follows.

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Then $(N, +, .)$ is a near-ring (see Pilz [6, p. 408] scheme 1). N is completely 2-primal and regular. But N is not a near-field.

Theorem 14. *N is completely 2-primal if and only if $P(N)$ is both prime and 2-primal*

Proof. Assume that N is completely 2-primal. Then $P(N)$ is completely prime and hence $P(N)$ is prime. Now let us show that $P(N)$ is 2-primal. Now $P(P(N)) = P(N)$ is completely semiprime. Hence $P(N)$ is 2-primal, by [2, Lemma 2.2(v)].

Conversely assume that $P(N)$ is both prime and 2-primal. Let $x^2 \in P(N)$. Then $x \in P(N)$, by [2, Lemma 2.2(ii)]. Hence $P(N)$ is completely semiprime. Let $xy \in P(N)$. Then $x \in (P(N) : y)$. Since $P(N)$ is completely semiprime, $(P(N) : y)$ is an ideal and hence $< x > \subseteq (P(N) : y)$. Then $< x > y \subseteq P(N)$. Now $y < x > \subseteq P(N)$ and hence $y \in (P(N) :< x >)$. Thus $< y >< x > \subseteq P(N)$. Since $P(N)$ is prime $x \in P(N)$ or $y \in P(N)$, showing that N is completely 2-primal. □

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